ARSD College, University of Delhi

Model Course Handout/Lesson Plan

Course Name : B.Sc. (Hons) Mathematics						
Semester	**Course Code**	**Course Title**	**Lecture (L)**	**Tutorial (T)**	**Practical (P)**	**Credit (C)**
VI	32351602	Ring Theory and Linear Algebra - II	5	1	0	6
Teacher/Instructor(s)		Ashutosh Meena				
Session		2021-22				

Course Objective: This course introduces the basic concepts of ring of polynomials and irreducibility tests for polynomials over ring of integers, used in finite fields with applications in cryptography. This course emphasizes the application of techniques using the adjoint of a linear operator and their properties to least squares approximation and minimal solutions to systems of linear equations.

Course Learning Outcomes: On completion of this course, the course will enable the students to:

1. Appreciate the significance of unique factorization in rings and integral domains.
2. Compute the characteristic polynomial, eigenvalues, eigenvectors, and eigen spaces, as well as the geometric and the algebraic multiplicities of an eigenvalue and apply the basic diagonalization result.
3. Compute inner products and determine orthogonality on vector spaces, including Gram−Schmidt orthogonalization to obtain orthonormal basis.
4. Find the adjoint, normal, unitary and orthogonal operators.

Lesson Plan:

Unit No.	Learning Objective	Lecture No.	Topics to be covered
1.	**Unit 1: Polynomial Rings and Unique Factorization Domain (UFD)**	1-2	Polynomial rings over commutative rings,
		3-4	Division algorithm and consequences,
		5	Principal ideal domains,
		6-7	Factorization of polynomials,
		8-9	Reducibility tests,
		10-11	Irreducibility tests,
		12-13	Eisenstein's criterion,
		14-15	Unique factorization in [x],

		16-17	Divisibility in integral domains,
		18-19	Irreducibles,
		`120-21	Primes,
		22-23	Unique factorization domains,
		24-25	Euclidean domains.
2.	**Unit 2: Dual Spaces and Diagonalizable Operators**	26-27	Dual spaces, Double dual,
		28-29	Dual basis, Transpose of a linear transformation and its matrix in dual basis,
		30	Annihilators,
		31-32	Eigenvalues, Eigenvectors,
		33-34	Eigenspaces and characteristic polynomial of a linear operator,
		35-36	Diagonalizability,
		37-38	Invariant subspaces and Cayley−Hamilton theorem,
		39-40	Minimal polynomial for a linear operator.
3.	**Unit 3: Inner Product Spaces**	41-45	Inner product spaces and norms,
		46-47	Orthonormal basis,
		48-50	Gram−Schmidt orthogonalization process,
		51-52	Orthogonal complements,
		53-55	Bessel's inequality.
4.	**Unit 4: Adjoint Operators and Their Properties**	56-57	Adjoint of a linear operator and its properties,
		58	Least squares approximation,
		59-60	Minimal solutions to systems of linear equations,
		61-62	Normal operators and their properties,
		63-64	Self-adjoint operators and their properties,
		65-67	Unitary operators and their properties,
		68-70	Orthogonal operators and their properties.

Evaluation Scheme:

No.	Component	Duration	Marks
1.	Internal Assessment		25
	• Quiz		
	• Class Test		
	• Attendance		
	• Assignment		
2.	End Semester Examination	3 hr	75

Details of the Course

Unit	Contents	Contact Hours
1	Polynomial rings over commutative rings, Division algorithm and consequences, Principal ideal domains, Factorization of polynomials, Reducibility tests, Irreducibility tests, Eisenstein's criterion, Unique factorization in [x]; Divisibility in integral domains, Irreducibles, Primes, Unique factorization domains, Euclidean domains.	25
2	Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators; Eigenvalues, Eigenvectors, Eigenspaces and characteristic polynomial of a linear operator; Diagonalizability, Invariant subspaces and Cayley−Hamilton theorem; Minimal polynomial for a linear operator.	15
3	Inner product spaces and norms, Orthonormal basis, Gram−Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality.	15
4	Adjoint of a linear operator, Least squares approximation, Minimal solutions to systems of linear equations, Normal, self-adjoint, unitary and orthogonal operators and their properties.	15
	Total	**70**

Suggested Books:

Sl. No.	Name of Authors/Books/Publishers	Year of Publication/ Reprint
1.	Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2003). Linear Algebra (4th ed.). Prentice-Hall of India Pvt. Ltd. New Delhi.	2003
2.	Gallian, Joseph. A. (2013). Contemporary Abstract Algebra (8th ed.). Cengage Learning India Private Limited. Delhi. Fourth impression, 2015.	2015
3.	Herstein, I. N. (2006). Topics in Algebra (2nd ed.). Wiley Student Edition. India.	2006
4.	Hoffman, Kenneth, & Kunze, Ray Alden (1978). Linear Algebra (2nd ed.). PrenticeHall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.	2015
5.	Lang, Serge (1987). Linear Algebra (3rd ed.). Springer.	1987

Mode of Evaluation:	Internal Assessment / End Semester Exam