→ Defn: Limit as x - approaches to ∞

Let f be a fn defined on an interval (a, ∞), we say that $f(x)$ has the limit L as x - approaches ∞ and write

$$\lim_{x \to \infty} f(x) = L$$

If for each $\epsilon > 0$, there exist a corresponding number $M > 0$ such that

$|f(x) - L| < \epsilon$ whenever $x > M$

in figure 1 ⇒ If $x > M$ then

~~when x~~. $|f(x) - L| < \epsilon$

Q. Prove that $\lim_{x \to \infty} \frac{1}{x} = 0$

Soln Let for each $\epsilon > 0$ be given we find a number $M > 0$ such that $\forall x$

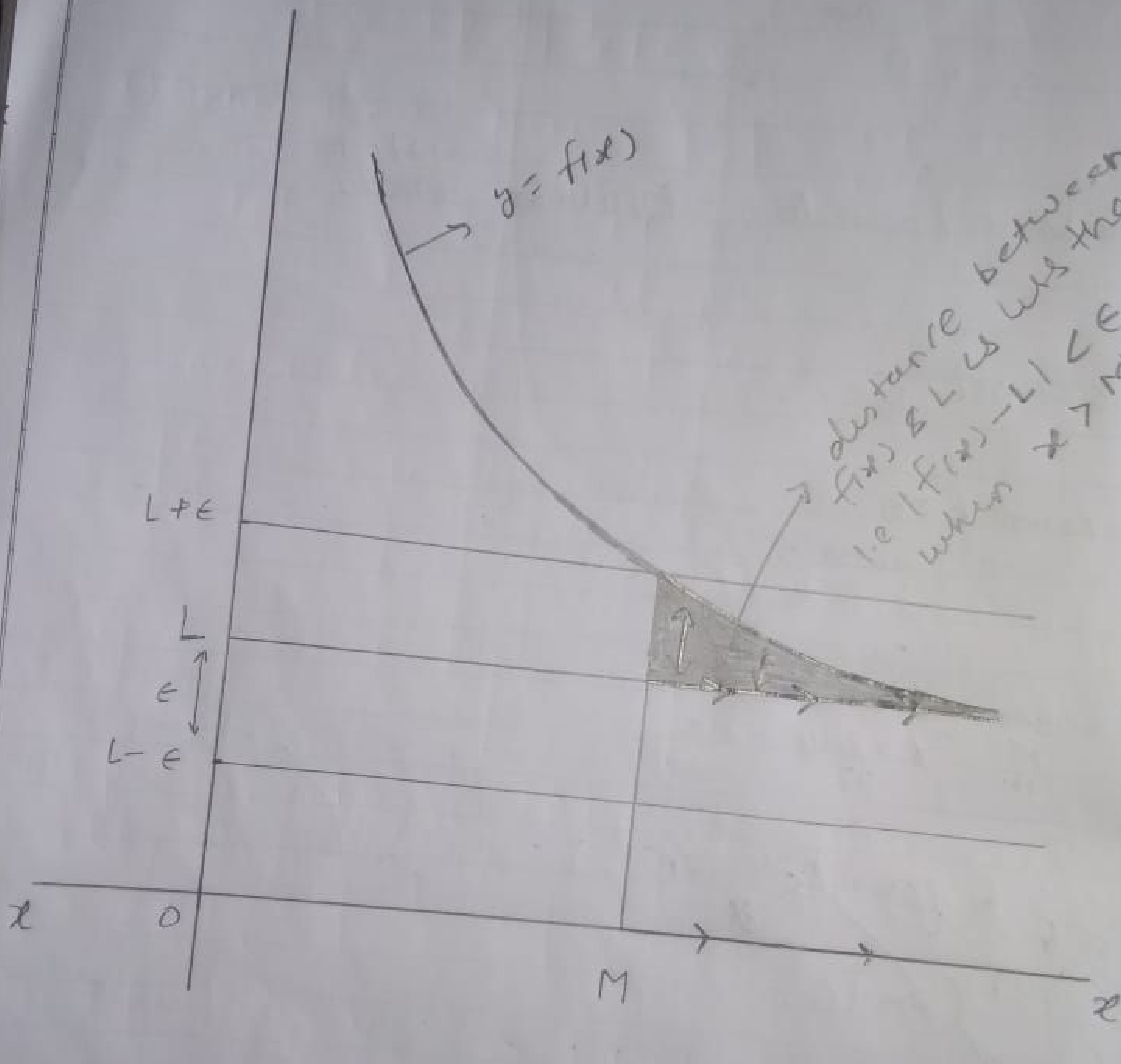

y = f(x)
distance between f(x) & L is less than ε
i.e |f(x) − L| < ε
when x > M
L + ε
L
ε
L − ε
x
O
M
x

figure-1

If $x > M \Rightarrow |f(x) - 2| < \epsilon$

i.e If $|\frac{1}{x} - 0| < \epsilon$

i.e If $|\frac{1}{x}| < \epsilon$

assume $x > 0$ then $|\frac{1}{x}| = +\frac{1}{x}$

$\because$ $|x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$

$\Rightarrow$ similarly

$|\frac{1}{x}| = \left\{ \begin{array}{ll} \frac{1}{x} & \text{if } x > 0 \\ -\frac{1}{x} & \text{if } x < 0 \end{array} \right\}$

$\Rightarrow$ $|\frac{1}{x}| = \frac{1}{x} < \epsilon$ iff $x > 1/\epsilon$

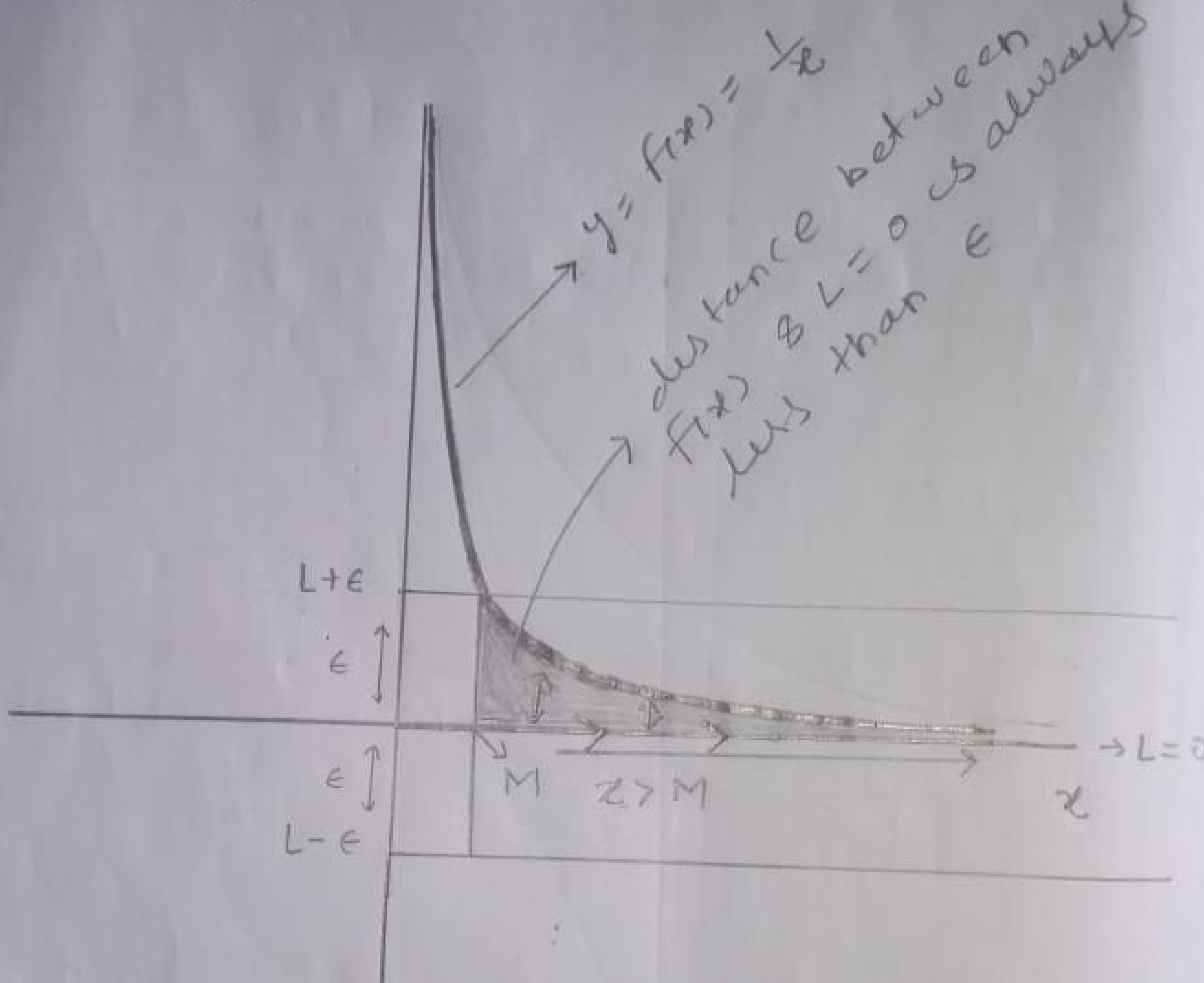
$\Rightarrow$ If we choose $M = 1/\epsilon$ then for all x

$x > M \Rightarrow |\frac{1}{x} - 0| = \frac{1}{x} < \epsilon$

i.e If $x > 1/\epsilon \Rightarrow |f(x) - 2| < \epsilon$

$\Rightarrow \lim_{x \to \infty} \frac{1}{x} = 0$

By figure



By figure if $x > M \Rightarrow |f(x) - L| < \epsilon$

when $M = 1/\epsilon$

$$\Rightarrow \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{1}{x} = 0$$

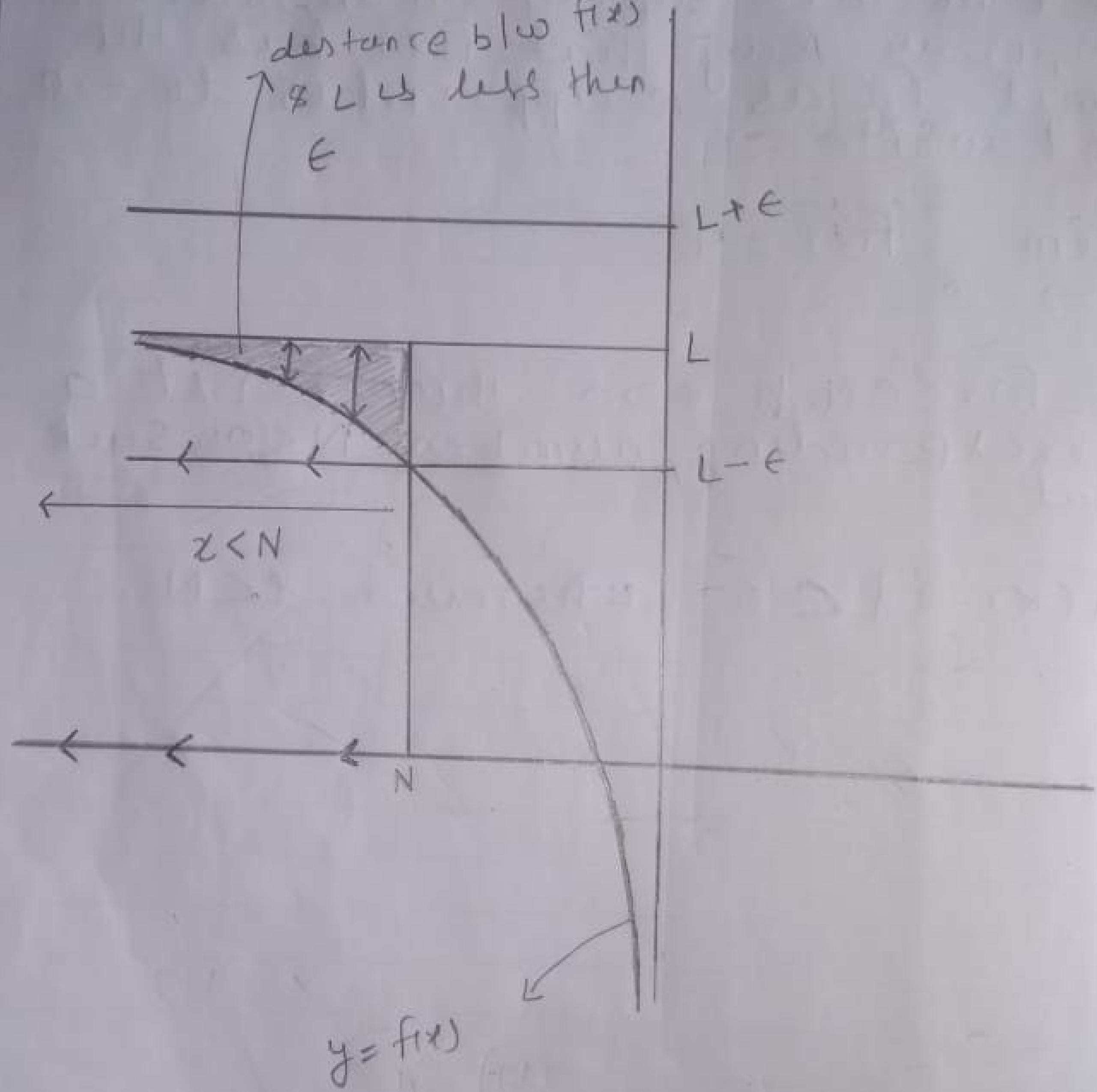
Def^n limit as x - approaches to $-\infty$

let f be a fn defined on an interval $(-\infty, a)$, we say that $f(x)$ has the limit L as x approaches to $-\infty$ and write

$$\lim_{x \to -\infty} f(x) = L$$

If for each $\epsilon > 0$, there exist a corresponding number $N < 0$ such that

$|f(x) - L| < \epsilon$ whenever $x < N$



ie if $x < N \Rightarrow |f(x) - L| < \epsilon$

Q) Prove that $\lim_{x \to -\infty} \frac{1}{x} = 0$

Soln Let $\epsilon > 0$ be given, we find a number $N < 0$ such that $\forall x$

If $x < N \Rightarrow |f(x) - L| < \epsilon \quad \forall x$ By defn

$\Rightarrow |f(x) - 0| < \epsilon \quad \forall x$

$\Rightarrow |\frac{1}{x}| < \epsilon \quad \forall x$

$$\left|\frac{1}{x}\right| = \begin{cases} \frac{1}{x}, & x > 0 \\ -\frac{1}{x} & x < 0 \end{cases}$$

$\Rightarrow$ assume $x < 0$ then

$\left|\frac{1}{x}\right| = -\frac{1}{x} < \epsilon \quad \because x < 0$

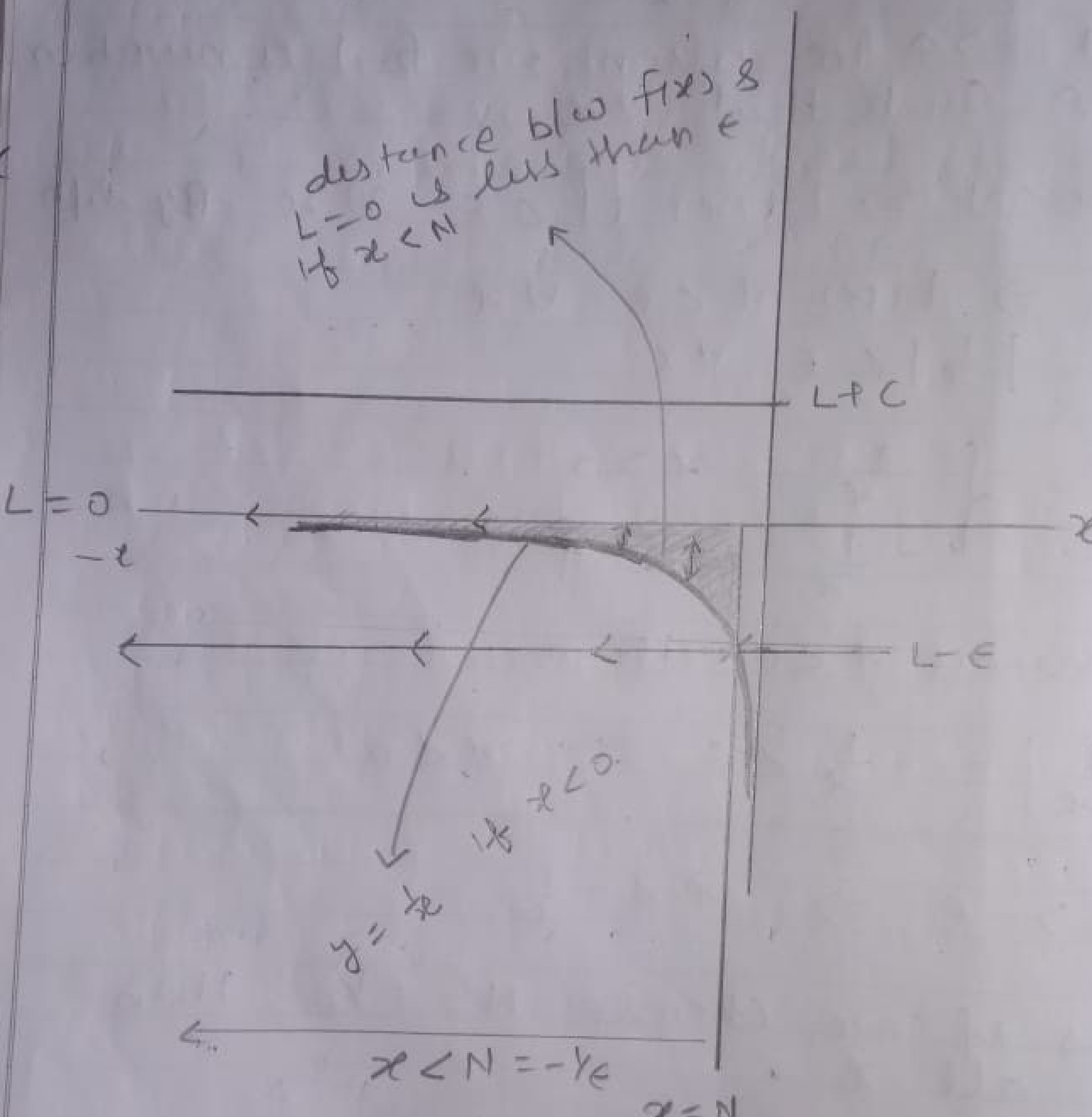
$\Rightarrow \frac{1}{x} > -\epsilon \Rightarrow x < -\frac{1}{\epsilon}$

Thus if we choose $N = -1/\epsilon$ then for all x.

$x < N \Rightarrow \left|\frac{1}{x} - 0\right| = \left|\frac{1}{x}\right| = -\frac{1}{x} < \epsilon$

If $x < -\frac{1}{\epsilon} \Rightarrow |f(x) - L| < \epsilon$

$\Rightarrow \lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{1}{x} = 0$



If $x < N = -1/\epsilon$

$\Rightarrow |f(x) - L| < \epsilon$