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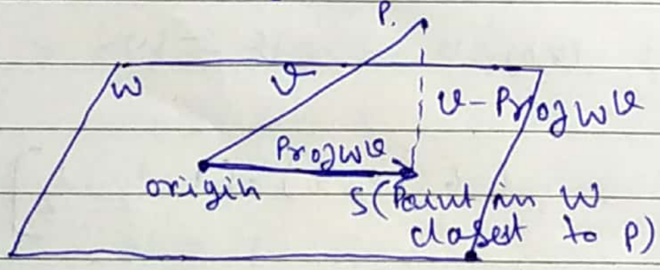
Distance from a Point to a subspace : An Application

Def. : Let W be a subspace of $\mathbb{R}^n$, and assume all vectors in W have initial point at the origin. Let P be any point in n-dimensional space. Then the minimum distance from P to W is the shortest distance between P and the terminal point of any vector in W.

Th^m: Let W be a subspace of $\mathbb{R}^n$, and let P be any Point in n-dimensional space. If v is the vector from origin to P, then the minimum distance from P to W is $\| v - \text{proj}_W v \|$

Note: (1) Notice that if S is the terminal point of $\text{proj}_W v$, then $\| v - \text{proj}_W v \|$ represents the distance from P to S



(2) We Note that the minimum distance can also be obtained using $\| \text{proj}_{W^\perp} v \|$

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Q.1. Let W be a subspace of $\mathbb{R}^3$ whose vectors lie in the plane $2x+y+z=0$. Find the minimum distance from the point $P(-6,10,5)$ to W

Solⁿ: Let v be the vector from origin to the point $P(-6,10,5)$.

Then the minimum distance from $P(-6,10,5)$ to W is $\| v - proj_W v \|$

In Q. 2 (Lecture Note : 03) page / 23, we calculated that

$$v - proj_W v = \left[1, \frac{1}{2}, \frac{1}{2}\right]$$

$\therefore$ minimum distance from $P(-6,10,5)$ to W is

$$\| v - proj_W v \| = \sqrt{1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{2}}$$

OR : $W = \{ [x,y,z] : [x,y,z] \cdot [2,1,1] = 0 \}$

$\Rightarrow$ W is the set of all vectors orthogonal to $[2,1,1]$

$\Rightarrow$ orthogonal to the subspace $Y = span\{[2,1,1]\}$

$\therefore$ $W = Y^{\perp}$ (by def of orthogonal compliment)

$\Rightarrow$ $W^{\perp} = (Y^{\perp})^{\perp} = Y = span\{[2,1,1]\}$

$\Rightarrow$ $W^{\perp} = span\left(\underbrace{\frac{1}{\sqrt{6}}[2,1,1]}_{u_1 \text{ (say)}}\right)$

Note : here we have normalized $[2,1,1]$ to get an orthonormal basis for Y.

$\therefore$ $proj_{W^{\perp}} v = proj_Y v = (v \cdot u_1)\, u_1$

$= \left([-6,10,5] \cdot \frac{1}{\sqrt{6}}[2,1,1]\right)\left(\frac{1}{\sqrt{6}}[2,1,1]\right)$

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$\Rightarrow \text{Proj}_{W^\perp} v = \frac{1}{6}(3)[2,1,1] = \left[1, \frac{1}{2}, \frac{1}{2}\right]$

$\therefore$ the minimum distance from $P(-6,10,5)$ is

$\|\text{Proj}_{W^\perp} v\| = \sqrt{1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{2}}$

Q.2. Find the minimum distance from the point $P(1,4,-2)$ to the subspace $W = \text{span}\{[x,y,z] : -2x+5y-z=0\}$ in $\mathbb{R}^3$.

H.W.

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