Q. sketch the graph of the function

$$f(x) = \frac{2(x^2-9)}{x^2-4}$$

· fn is define $\forall x \in R \sim \{-2, 2\}$

step 1 put $x = 0 \Rightarrow y = 9/2 \Rightarrow$ curve intersect y-axis at $y = 9/2$

put $y = 0 \Rightarrow x = 3, -3 \Rightarrow$ curve intersect x-axis at $x = 3, -3$

step (2) (3) $f'(x) = \frac{20x}{(x^2-4)^2}$, $f''(x) = \frac{-20(3x^2+4)}{(x^2-4)^3}$

find critical points put $f'(x) = 0 \Rightarrow x = 0$

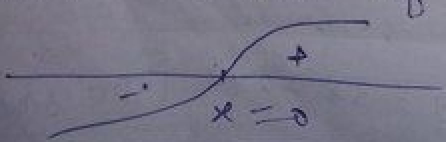
$x = 0$ is only critical pt

Step 4 find interval at which fn is increasing or decreasing

let $f'(x) = \frac{20x}{(x^2-4)^2} = 0$

$\Rightarrow 20x = 0 \quad x = 0$

find the behave of f(x) ~~when f(x)~~



$\Rightarrow f'(x) > 0 \;\forall\; x \in (0, \infty) \Rightarrow$ fn is increasing on $\forall\; x \in (0, \infty)$

$f'(x) < 0 \;\forall\; x \in (-\infty, 0) \Rightarrow$ fn is decreasing on $\forall\; x \in (-\infty, 0)$

Step 5 Find the point of inflection and concavity

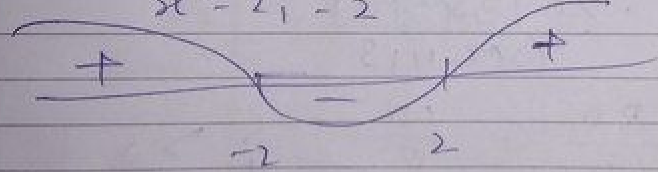
$$f''(x) = -\frac{20(3x^2+4)}{(x^2-4)^3}$$

let $$g(x) = \frac{20(3x^2+4)}{(x+2)^3(x-2)^3}$$

$\Rightarrow f''(x) = -g(x)$

for behave of ~~g(x)~~ $g(x)$ ~~f(x)~~

$x = 2, -2$



$\Rightarrow g(x) > 0 \;\forall\; x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow -g(x) < 0 \;\forall\; x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow f''(x) = -g(x) < 0 \;\forall\; x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow f''(x) < 0 \;\forall\; x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow$ fn is concave down on $\forall\; x \in (-\infty, -2) \cup (2, \infty)$

$\because$ $g(x)<0$ $\forall x \in (-2,2)$

$\Rightarrow -g(x)>0$ $\forall x \in (-2,2)$

$\Rightarrow f''(x) = -g(x) > 0$ $\forall x \in (-2,2)$

$\Rightarrow f''(x) > 0$ $\forall x \in (-2,2)$

$\Rightarrow$ fn is concave up $\forall x \in (-2,2)$

~~so~~ $\because$ concavity of fn is changed at $x = 2, -2$ $\Rightarrow -2, 2$ are the pt of inflection

$f(2) = \infty$, $f(-2) = \infty$ are the point of inflection on graph

Step 6 find local maximum and minimum value of fn

we have only critical point $x=0$

$$f''(x) = \frac{-20(3x^2+4)}{(x^2-4)^3}$$

$$f''(0) = \frac{-80}{-64} = \frac{10}{8} = \frac{5}{4} > 0$$

$\Rightarrow$ fn has local minimum value at $x=0$

$$f(0) = \frac{2(0-9)}{0-4} = 9/2 = 4.5$$

$f(0) = \frac{9}{2}$ ~~is minimum value of fn~~

$f(0) = \frac{9}{2}$ is local minimum

Horizontal asymptotes

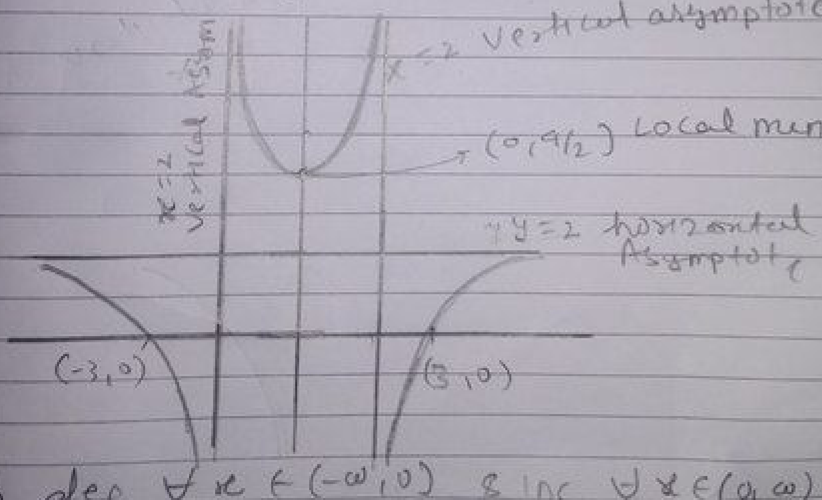
$\lim_{x \to \infty} \frac{2(x^2-9)}{x^2-4} = 2$, $\lim_{x \to -\infty} \frac{2(x^2-9)}{x^2-4} = 2$

$\Rightarrow y=2$ is horizontal asymptote

vertical asymptote

$\lim_{x \to 2^{-,+}} \frac{2(x^2+4)}{x^2-4} = -\infty$, $\lim_{x \to 2^{-+}} \frac{2(x^2-9)}{x^2-4} = \infty$

$\Rightarrow$ line $x=2$ and $x=-2$ are the vertical Asymptote of the Fn



Fn is dec $\forall x \in (-\infty, 0)$ & inc $\forall x \in (0, \infty)$
Fn is concave down $\forall x \in (-\infty, -2) \cup (2, \infty)$
Fn is concave up $\forall x \in (-2, 2)$

Date ______ Teacher's Signature : ______