Qn Sketch graph for $f(x) = \dfrac{x+1}{x^2+3} = y$

fn is define $\forall\, x \in \mathbb{R}$

Step 1 put $x = 0 \Rightarrow y = \frac{1}{3} \Rightarrow$ curve intersect y-axis at $y = \frac{1}{3}$

put $y = 0 \Rightarrow x = -1 \Rightarrow$ curve intersect ~~at~~ x-axis at $x = -1$

Step(2) (3) $$f'(x) = -\frac{(x+3)(x-1)}{(x^2+3)^2}$$

And $$f''(x) = \frac{2(x^3+3x^2-9x-3)}{(x^2+3)^3}$$

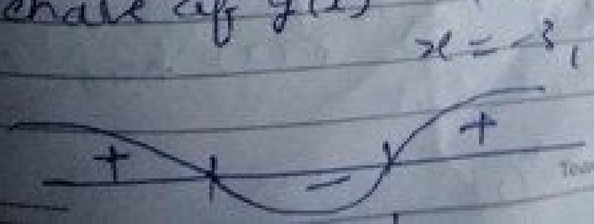
Critical points $f'(x) = 0 \Rightarrow (x+3)(x-1) = 0$
$\Rightarrow x = -3, 1$

Step(4) find the interval at which fn is increasing or decreasing

$f'(x) = -\dfrac{(x+3)(x-1)}{(x^2+3)^2}$ let $g(x) = \dfrac{(x+3)(x-1)}{(x^2+3)^2}$

$\Rightarrow f'(x) = -g(x)$

find behave of $g(x)$ Now $g(x) = 0$
$x = -3, +1$



$\Rightarrow g(x) > 0 \; \forall x \in (-\infty, -3) \cup (1, \infty)$

$\Rightarrow -g(x) < 0 \; \forall x \in (-\infty, -3) \cup (1, \infty)$

$\Rightarrow f'(x) < 0 \; \forall x \in (-\infty, -3) \cup (1, \infty)$

$\Rightarrow f(x)$ decreasing on $(-\infty, -3) \cup (1, \infty)$

$\because g(x) < 0 \; \forall x \in (-3, 1)$

$\Rightarrow -g(x) > 0 \; \forall x \in (-3, 1) \Rightarrow f'(x) > 0 \; \forall x \in (-3, 1)$

$\Rightarrow f(x)$ is increasing on ~~$(-\infty, -3) \cup (1, \infty)$~~
$\forall x \in (-3, 1)$

Step (5) find the point of inflection

$$f''(x) = \frac{2(x^3+3x^2-9x-3)}{(x^2+3)^3} = h(x)$$

$$h(x) = \frac{2(x^3+3x^2-9x-3)}{(x^2+3)^3}$$

$h(x)$ has atleast one real root
[$\because$ odd degree poly has atleast one real root]

take only $r(x) = x^3 + 3x^2 - 9x - 3$

$r(-1) = +8 \qquad r(0) = -3$

$\Rightarrow r(-1) \cdot r(0) < 0 \Rightarrow r(x)$ has atleast one root b/w -1 & 0

Now $\frac{-1+0}{2} = -\frac{1}{2}$

$r(-1/2) = \frac{23}{8}$ & $r(0) = -8$ $\Rightarrow$ $r(-1/2)\cdot r(0) < 0$

$r(x)$ has at least one real root b/w $(-1/2, 0)$

now $\frac{-\frac{1}{2}+0}{2} = -\frac{1}{4}$ $\quad r\left(-\frac{1}{4}\right) = -\frac{37}{64} < 0$

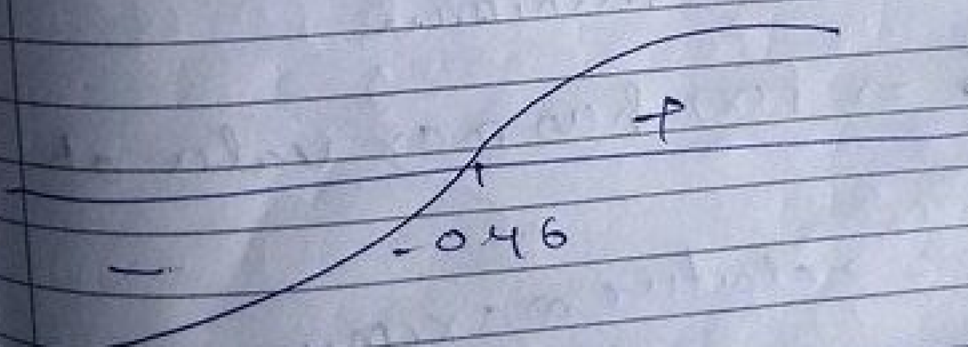
& $r(-1/2) = \frac{23}{8} > 0$

$r\left(-\frac{1}{4}\right)\cdot r(-1/2) < 0$ $\Rightarrow$ root lie b/w $\left(-\frac{1}{4}, -1/2\right)$

now $\frac{-\frac{1}{4}-\frac{1}{2}}{2} = -\frac{3}{8}$ now $r\left(-\frac{3}{8}\right) = \frac{381}{512} > 0$

$r\left(-\frac{1}{4}\right) < 0$ & $r\left(-\frac{3}{8}\right) > 0$ $\Rightarrow$ root lie b/w $\left(-\frac{1}{2}, -\frac{3}{8}\right)$

let $x = -0.46$ is root of $r(x)$ assum



~~xxx~~ behave of $r(x)$ & $h(x)$ is same

$r(x) > 0 \ \forall x \in (-0.46, \infty) \Rightarrow h(x) > 0 \ \forall x \in (-0.46, \infty)$

$\Rightarrow f''(x) > 0 \ \forall x \in (-0.46, \infty) \Rightarrow f$ is concave up on $x \in (-0.46, \infty)$

$r(x) < 0 \ \forall x \in (-\infty, -0.46) \Rightarrow h(x) < 0 \ \forall x \in (-\infty, -0.46) \Rightarrow f''(x) < 0 \ \forall x \in (-\infty, -0.4$

$\Rightarrow f$ is concave down on $x \in (-\infty, -0.46)$

$\Rightarrow x \approx -0.46$ is point of inflection

point of inflection on graph is

$f(-0.46) = 0.1681$

ep6 find maximum & minimum value of fn

$$f''(x) = \frac{2(x^3 + 3x^2 - 9x - 3)}{(x^2+3)^2}$$

$f''(-3) = \frac{1}{36} > 0 \Rightarrow$ fn has minimum at $x = -3$

$f(\cancel{-1/3}) \cancel{=}$

$f(-3) = -1/6$ = relative minimum

$f''(1) = -\frac{1}{4} < 0 \Rightarrow f(x)$ has max value at $x = 1$

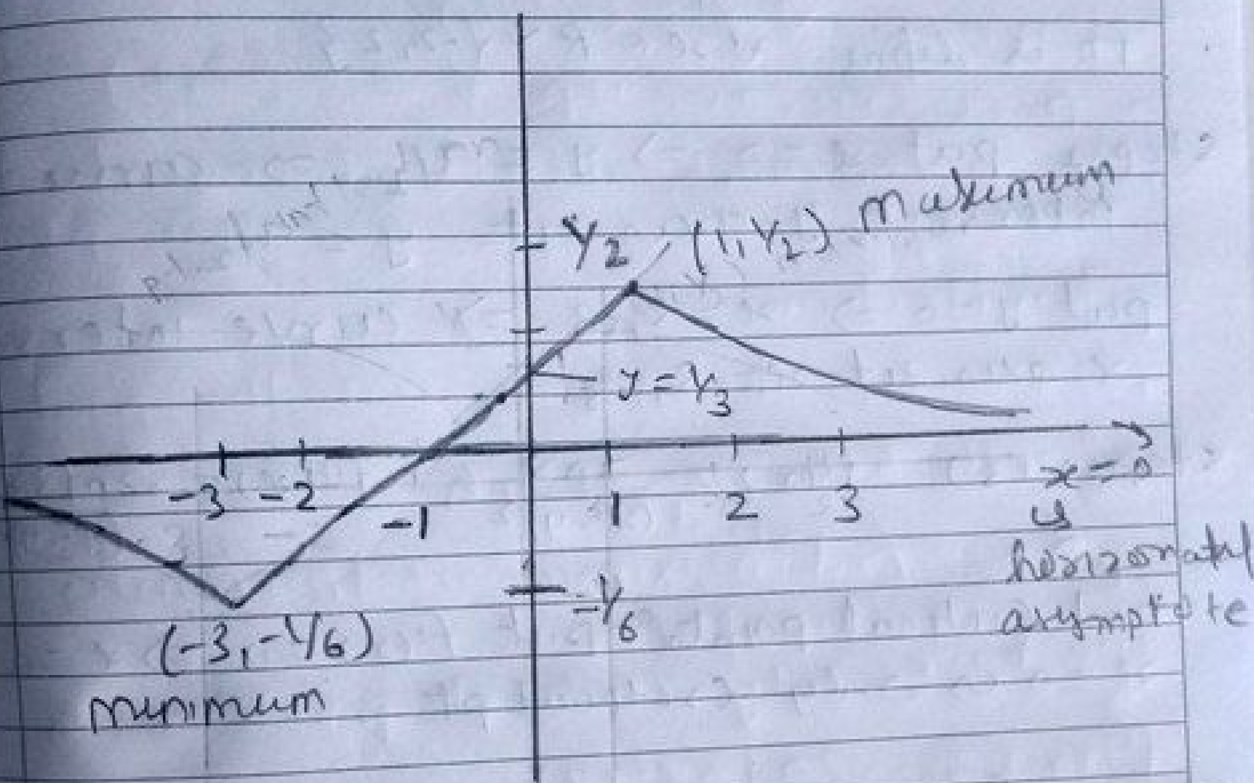
$f(1) = \frac{2}{4} = \frac{1}{2}$ = relative maximum

ep 7 fn has no vertical asymptote By d^n

Horizontal asymptote

$$\lim_{x \to \infty} \frac{x+1}{x^2+3} \cdot \left[\frac{\infty}{\infty}\right] = \lim_{x \to \infty} \frac{1}{2x+0} \to 0$$

$y = 0$ is horizontal asymptote $\Rightarrow$
$= 0$ represent x-axis is Horizontal asymptote



fn is increasing $\forall x \in (-3, 1)$

fn is dec on $\forall x \in (-\infty, -3] \cup (1, \infty)$

fn is concave up $\forall x \in (-\infty, -0.46)$

fn is concave down $\forall x \in (-0.46, \infty)$

Qn Draw the curve $y = 3x^2 - 2x^3$. [Do yourself]