

0/05/2020

Lecture - no 18

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Q9  $y = (x-1)(x+2)$  Sketch the curve.  
 $= x^2 + x - 2$

Step (1) put  $x=0 \Rightarrow y=-2$

$\Rightarrow$  Curve intersect at  $y=-2$

put  $y=0 \Rightarrow x=1, -2 \Rightarrow$  Curve intersect  $x$ -axis at  $x=1, -2$

Step (2) (3) find  $f'(x)$  &  $f''(x)$  & critical pt

$$f'(x) = 2x+1 \quad f''(x) = 2 > 0$$

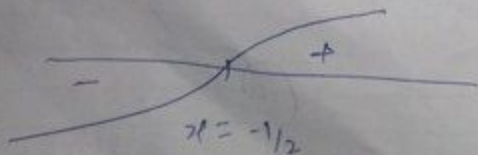
$$f'(x) = 0 \Rightarrow x = -1/2 \text{ critical pt}$$

Step (4) find the interval in which  $f(x)$  is increasing and ~~decreasing~~ decreasing

$$f'(x) = 2x+1 = g(x)$$

$$g(x) = 2x+1 = 0 \Rightarrow x = -1/2$$

check the behavior of  $g(x)$



$$\Rightarrow g(x) > 0 \quad \forall x \in (-1/2, \infty)$$

$\Rightarrow f(x)$  is increasing on  $x \in (-\frac{1}{2}, \infty)$

$\therefore g(x) = f'(x) > 0 \quad \forall x \in (-\frac{1}{2}, \infty)$

~~$\Rightarrow$~~   $g(x) < 0 \quad \forall x \in (-\frac{1}{2}, \infty)$

$\therefore f'(x) = g(x) < 0 \quad \forall x \in (-\frac{1}{2}, \infty) \Rightarrow f(x)$   
is decreasing  $\forall x \in (-\frac{1}{2}, \infty)$

Step (5) Identify any point of inflection

$\therefore f''(x) = 2 \neq 0 > 0 \Rightarrow$  no point of inflection

Step (6) find the interval on which the  $f^n$  is concave up or concave down

$\therefore f''(x) = 2 > 0 \Rightarrow f^n$  is concave up  $\forall x$

Step (7) find the maximum and minimum value of the  $f^n$  if exist

$\therefore f'(x) = 2x - 1 = 0 \quad x = -\frac{1}{2}$

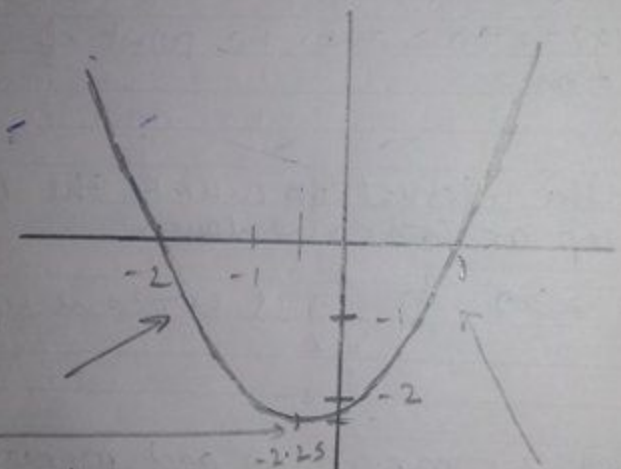
$f''(x) = 2 > 0 \quad f''(-\frac{1}{2}) = 2 > 0$

$\Rightarrow f^n$  has minimum value at  $x = -\frac{1}{2}$

but fn has no maximum value  
 $f(-\frac{1}{2}) = (-\frac{1}{2} - 1)(-\frac{1}{2} + 2) = (-\frac{3}{2})(\frac{3}{2}) = -\frac{9}{4} = -2.25$

step 8 fn has no asymptote  
(find yourself)

step 9 sketch the curve



fn is dec  $\forall$   
 $x \in (-\infty, -\frac{1}{2})$

fn is inc  $\forall$   
 $x \in (-\frac{1}{2}, \infty)$

fn is concave up  $\forall$   $x$

Qn 1  $y = (x-1)(x+3)$  draw the graph

[Do your self]

Qn 2  $y = \cancel{x^2+1}$   $y = (x+2)(x-4)$  draw the graph [Do your self]

Qn 3  $y = (x^2-1)(x+2)$  draw the graph [Do your self]

Qn 4  $y = (x+2)(x+3)(x-1)$  (Sketch graph) [do your self]

Qn 5  $y = (x-1)^2(x-2)$  draw the graph [Do your self]

Qn 6  $y = (x-1)(x-1)(x-3)$   
draw the graph [Do your self]

$$y = -(x+1)(x+3) = -(x^2+4x+3)$$

Step (1) put  $x=0 \Rightarrow y=-3$   
 $\Rightarrow$  curve intersect at y-axis at  $y=-3$

$$\text{put } y=0 \Rightarrow x=-1, -3$$

$\Rightarrow$  curve intersect at x-axis at  $x=-1, -3$

Step (2) (3)  $f(x) = -(2x+4) = 0$   
 $x = -2$  is critical point

$$f''(x) = -2 < 0$$

Step (4) find the interval at which  $f$  is increasing or decreasing

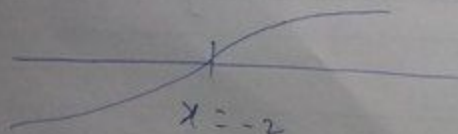
$$f'(x) = -(2x+4) = -2x-4 = 0$$

$$\text{let } g(x) = f(x) = -2x-4 = -(2x+4)$$

$$\text{let } g(x) = 2x+4$$

check the behavior of  $g(x)$  when  $g(x)$  is +ve or -ve

$$g(x) = 0 \Rightarrow x = -2$$





$\Rightarrow f(x) > 0$  on  $x \in (-2, 0)$   
 $\Rightarrow -g(x) < 0$  on  $x \in (-2, \infty)$

$\Rightarrow -f'(x) = -g(x) < 0 \quad \forall x \in (-2, \infty)$

$\Rightarrow f_n$  is increasing on  $(-2, \infty)$

$\therefore g(x) < 0 \quad \forall x \in (-\infty, -2)$

$\Rightarrow -f'(x) = g(x) > 0 \quad \forall x \in (-\infty, -2)$

$\Rightarrow f(x)$  is ~~decreasing~~ <sup>increasing</sup> on  $x \in (-\infty, -2)$

Step (5) Identify the inflection point

$\because f''(x) = -2 \neq 0 \Rightarrow f_n$  has no inflection point

Step (6) Find out the intervals on which  $f_n$  is ~~not~~ concave up or concave down

$\because f''(x) = -2 < 0 \quad \forall x$

$\Rightarrow f_n$  is ~~not concave~~ concave down  $\forall x$

Step (7) Find the maximum and minimum value of  $f_n$

$\because f(x) = -2x - 4 = 0 \Rightarrow x = -2$

$f''(x) = -2 < 0$

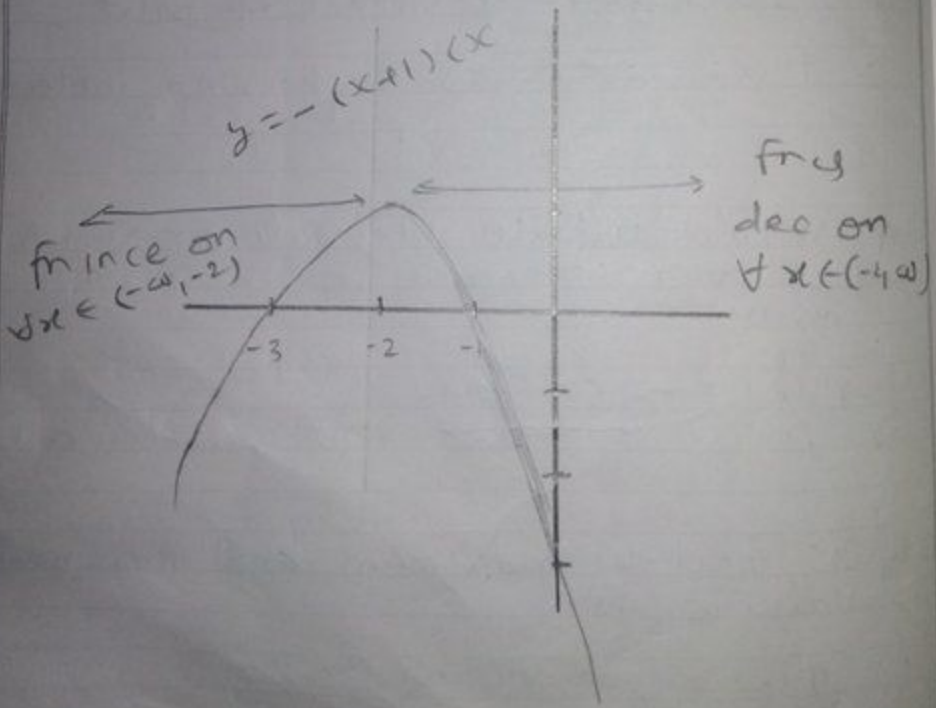
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max

$$f(x) = -(x+1)(x+3)$$

$\Rightarrow f(x)$  has ~~max~~ values  $\forall x$   
 $f(x)$  has no ~~maximum~~ minimum value  
 maximum value  $f(-2) = -(2+1)(-2+3) = 1$

Step (8) for  $f(x)$  has no asymptote



$$y = (x-1)(x-2)(x-3) = x^3 - 6x^2 + 11x - 6$$

put  $y=0 \Rightarrow x=1, 2, 3 \Rightarrow$  Curve intersect at  $x=1, 2, 3$  at  $x$ -axis

put  $x=0 \Rightarrow y=-6 \Rightarrow$  Curve intersect at  $y$ -axis at  $x=-6$

Step (2), (3)  $f'(x) = 3x^2 - 12x + 11$

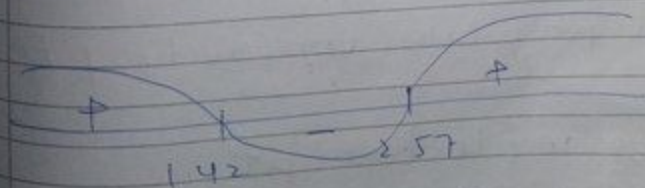
$$f''(x) = 6x - 12$$

Critical point  $f'(x) = 0 \Rightarrow 3x^2 - 12x + 11 = 0$   
 $x = \frac{6 \pm \sqrt{3}}{3} = \frac{6 + \sqrt{3}}{3}, \frac{6 - \sqrt{3}}{3}$

$$x = 2.57, 1.422$$

Step (4) find the interval at which  $f$  is increasing or decreasing

put  $f'(x) = 0 \Rightarrow x = 2.57, 1.422$



$$\Rightarrow f'(x) > 0 \quad \forall x \in (-\infty, 1.42) \cup (2.57, \infty)$$

$$\Rightarrow f \text{ is increasing on } \forall x \in (-\infty, 1.42) \cup (2.57, \infty)$$



$$f(x) < 0 \quad \forall x \in (1.42, 2.57)$$

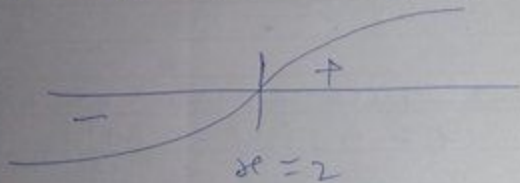
$\Rightarrow$  f is dec on  $x \in (1.42, 2.57)$

Step (3) Identify the pt of inflection

$$f''(x) = 6x - 12 = 0 \quad x = 2$$

Find the behavior of  $f''(x)$

$$\text{put } f''(x) = 0 \Rightarrow x = 2$$



$f''(x) > 0 \quad \forall x \in (2, \infty) \Rightarrow$  f is concave up  $\forall x \in (2, \infty)$

$f''(x) < 0 \quad \forall x \in (-\infty, 2) \Rightarrow$  f is concave down  $\forall x \in (-\infty, 2)$

$\Rightarrow x = 2$  is pt where concavity of f changed

$\Rightarrow x = 2$  is inflection pt

Now pt of inflection on graph

$$f(2) = 0 \quad (2, 0) \text{ is the pt of}$$

# inflection on graph

step 7 maximizes min value of  $f_n$

$$f(x) = 0 \Rightarrow x = 1.42, 2.57$$

$$f''(x) = 6x - 12$$

$$f''(1.42) = 0.048 > 0$$

~~$f_n$  has minimum~~

$$f''(2.57)$$

$$f''\left(\frac{6-\sqrt{3}}{3}\right) = -2\sqrt{3} < 0$$

$$\Rightarrow \text{maximum at } x = \frac{6-\sqrt{3}}{3}$$

$$f''\left(\frac{6+\sqrt{3}}{2}\right) = 2\sqrt{3} > 0$$

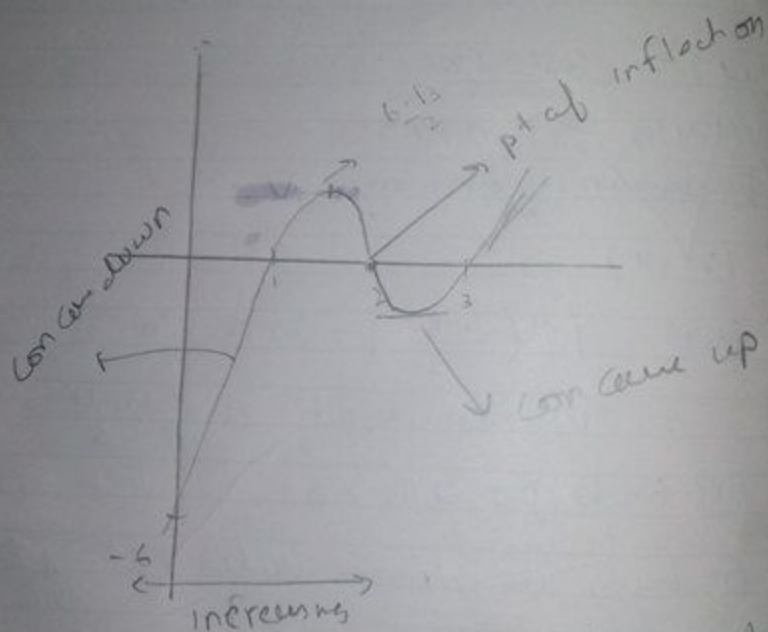
$\Rightarrow f_n$  has minimum value at  $x = \frac{6+\sqrt{3}}{3}$

maxim value

$$f\left(\frac{6-\sqrt{3}}{3}\right) = 0.397$$

$$\text{min value } f\left(\frac{6+\sqrt{3}}{3}\right) = -0.3848$$

step 8)  $f(x)$  has no asymptote



on  $x \in (-\infty, 1.42) \cup (2.57, \infty)$

$f(x)$  is dec on  $\forall x \in (1.42, 2.57)$