

ANSWER ANY FIVE QUESTIONS :-

1. Show that the set of vectors of the form $[a, b, \frac{1}{2}a - 2b]$ in \mathbb{R}^3 is a subspace of \mathbb{R}^3 under the usual operations.

2. Check whether following is a linearly independent set or not :-

$$\left\{ \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 6 & -1 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} -11 & 3 \\ -2 & 2 \end{bmatrix} \right\}$$

3. Examine if $B = \{[-1, 2, -3], [3, 1, 4], [2, 1, 6]\}$ is a basis of \mathbb{R}^3 .

4. Let $S = \{[1, 0], [1, -3]\}$ & $T = \{[3, 0], [4, -1]\}$ be two bases for \mathbb{R}^2

(i) Find coordinate vector $[v]_T$ of v w.r.t. T -basis.

(ii) Compute transition matrix $P_{S \leftarrow T}$ from the T -basis to the S -basis.

(iii) Find coordinate vector $[v]_S$ of v w.r.t. S -basis using $P_{S \leftarrow T}$.

5. Let $S = \{[-1, 2, -3], [3, 1, 4], [2, -1, 6]\}$ and $T = \{[1, 1, 0], [2, 0, 1], [0, 1, 2]\}$ be two bases of \mathbb{R}^3 . Show $Q_{T \leftarrow S} = (P_{S \leftarrow T})^{-1}$ for the above two bases.

Sign.

6. Prove that \mathbb{R}^2 is a vector space using the operations \oplus & \odot given by

$$[x, y] \oplus [w, z] = [x+w+1, y+z-2]$$

$$\& \quad a \odot [x, y] = [ax+a-1, ay-2a+2]$$

Find the zero vector $\mathbf{0}$ in V & the additive inverse $-v$ for any vector v in V .