GE-02 Linear Algebra

PRACTICE SET

Q.1 (a) Cauchy - Schwarz Inequality
Let x and y be vectors in $\mathbb{R}^n$. Then
$$|x \cdot y| \leq \|x\| \, \|y\|$$

(b) (i) Define the projection vector of a vector b onto a vector a (where a is Non-zero vector).

(ii) For vector $a = [2, -3, 4]$ and $b = [-6, 2, 7]$ find $\text{Proj}_a b$ and verify that $b - \text{Proj}_a b$ is orthogonal to a.

Q.2. Solve the following system of equations, using Gauss - Jordan Reduction method.
$$x_1 + 3x_2 + x_3 = 6$$
$$3x_1 - 2x_2 - 8x_3 = 7$$
$$4x_1 + 5x_2 - 3x_3 = 17.$$

Q.3: Determine whether the vector $x = [5, 2, 2]$ is in row space of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$
If so, then express $x = [5, 2, 2]$ as a linear combination of the rows of A.

Q.4: Find the characteristic polynomial of the matrix $A = \begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$. Also find all the eigen values and Eigen spaces for A.

Q.5: Use the diagonalization method to determine whether each of the following matrices is diagonalizable. If so, specify the matrices D and P and verify that $P^{-1}AP = D$.

(a) $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 4 & 0 & -2 \\ 6 & 2 & -6 \\ 4 & 0 & -2 \end{bmatrix}$

Q.6 Consider the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 5 & -4 & 3 \end{bmatrix}$

Using Rank of A, determine whether the homogeneous system $Ax = 0$ has a non-trivial sol or Not

Q.7: Determine which of the following functions are linear transformations

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T([x, y]) = [2x - 3y, 3x + 4y]$

(b) $T : \mathbb{R}^4 \to \mathbb{R}$ given by $T([x_1, x_2, x_3, x_4]) = |x_1|$

Q.8: Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator given by $L([x, y]) = [2x - y, x - 3y]$. Find the matrix for L with respect to the basis $\{[4, -1], [-7, 2]\}$ using the method of similarity.

Q.9: Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator given by $T([x, y, z]) = [-6x + 4y - z, -2x + 3y - 5z, 3x - y + 7z]$

Find the matrix for T with the standard basis for $\mathbb{R}^3$.

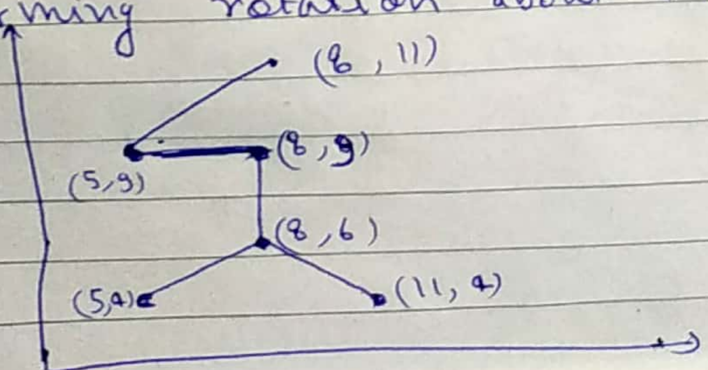
Q.10. State the Dimension Theorem. Let $T : P_2 \to \mathbb{R}^2$ be the linear transformation given by $T(P) = [P(1), P'(1)]$. Find a basis for $\ker(T)$ and a basis for range(T). Also, verify the Dimension Thm.

Q.11: Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator given by

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3 & 1 & -3 \\ 2 & 1 & -1 \\ 2 & 3 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find a basis for $\ker(L)$ and a basis for range(L). Also, verify the Dimension Theorem

Q.12: For the Graphic in fig *, use homogeneous coordinates to find new vertices after performing rotation about $(8, 9)$ through $\theta = 120^\circ$



Q.12. For each of the following subspaces W of $\mathbb{R}^n$ and for a given vector $v \in \mathbb{R}^n$ Find $\mathrm{proj}_W v$ and decompose v into $w_1 + w_2$ where $w_1 \in W$ and $w_2 \in W^\perp$

(a) in $\mathbb{R}^3$, $W = \mathrm{span}(\{[2, 1, -1]\})$, $v = [1, 4, -2]$

(b) in $\mathbb{R}^3$, $W =$ The plane $3x - y + 4z = 0$

$v = [2, 2, -3]$

Q.13: Find the minimum distance from the point $P(1, 4, -2)$ to the subspace

$W = \mathrm{span}\{[x, y, z] : -2x + 5y - z = 0\}$ in $\mathbb{R}^3$.

Q.14: Find a least square sol^n to the inconsistent system $Ax = b$, where

$$A = \begin{bmatrix} 2 & 1 \\ -2 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} -5 \\ 8 \\ 1 \end{bmatrix}$$

Also verify that the least-squares sol^n v satisfies $\|Av - b\| \leq \|Az - b\|$ for the vector $z = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$.

Q.15: For the subspaces $W = \{[x, y, z] \in \mathbb{R}^3 : 2x - 5y + z = 0\}$ of $\mathbb{R}^3$, find a basis for the orthogonal complement $W^\perp$ & verify that

$\dim(W) + \dim(W^\perp) = \dim(\mathbb{R}^3)$.