

## Assignment - 03 GE-02

## Linear Algebra

Q.1. Suppose  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  is a linear transformation with  $L([1, -1, 0]) = [2, 1]$ ,  $L([0, 1, -1]) = [-1, 3]$  and  $L([1, 0, 0]) = [0, 1]$ . Find  $L([-1, 1, 2])$ . Also give a formula for  $L([x, y, z])$ , for any  $[x, y, z] \in \mathbb{R}^3$ .

Q.2. Let  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear operator that performs a counterclockwise rotation through an angle  $\theta = 30^\circ$ . Find the matrix  $A$  for  $L$  with respect to the standard ordered basis for  $\mathbb{R}^2$ . Hence or otherwise find the matrix for  $L$  with respect to the basis  $\{[4, -3], [3, -2]\}$  similar to  $A$ .

Q.3. Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be the linear transformation given by  $T([x, y]) = [13x - 9y, -x - 2y, -11x + 6y]$ . Find the matrix for  $T$  with respect to the ordered bases  $B = \{[2, 3], [-3, -4]\}$  for  $\mathbb{R}^2$  and  $C = \{[-1, 2, 2], [-4, 1, 3], [1, -1, -1]\}$  for  $\mathbb{R}^3$ .

Q.4. Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear operator given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 5 & 1 & -1 \\ -3 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find a basis for  $\ker(T)$  and a basis for  $\text{Range}(T)$ . Also verify dimension Theorem.

Q.5. Find a least-squares sol<sup>n</sup> to the inconsistent system  $Ax = b$ , where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \\ 4 & 3 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 12 \\ 15 \\ 4 \end{bmatrix}$$