Find Fourier series coefficients?

$$x[N] = 1 + \sin\left(\frac{4\pi}{N}n\right) + 2\cos\left(\frac{4\pi}{N}\right) + \cos\left(\frac{8\pi}{N}n + \pi/2\right)$$

$$= \frac{1}{2j}\left[e^{j\left(\frac{4\pi}{N}\right)n} - e^{-j\left(\frac{4\pi}{N}\right)n}\right]$$

$$+ \left[e^{j\left(\frac{4\pi}{N}\right)n} + e^{-j\left(\frac{4\pi}{N}\right)n}\right]$$

$$+ \frac{1}{2}\left[e^{j\pi/2} \cdot e^{j4\left(\frac{2\pi}{N}\right)n} + e^{-j\frac{\pi}{2}} \cdot e^{-j4\left(\frac{2\pi}{N}\right)n}\right]$$

So fourier series coefficients are

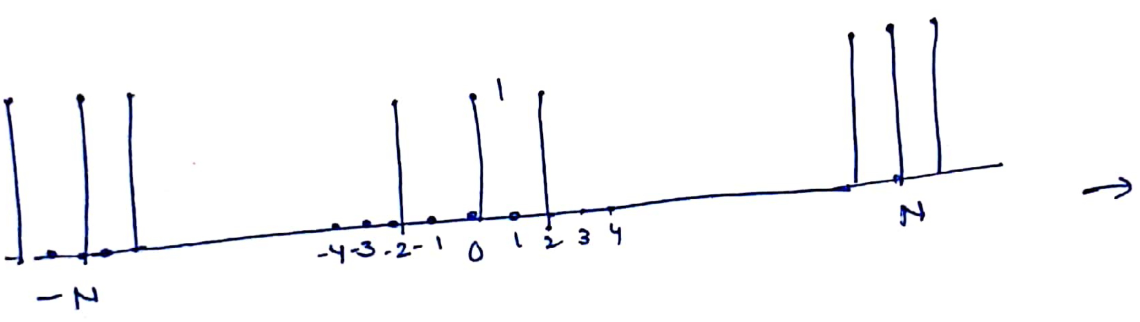
$a_0 = 1$, $a_2 = 1 + \frac{1}{2j}$, $a_{-2} = 1 + \frac{1}{2}j$

$a_4 = \frac{1}{2}e^{j\pi/2} = \frac{1}{2}\left(\cos\pi/2 + j\sin\pi/2\right)$

$= \frac{1}{2}j$

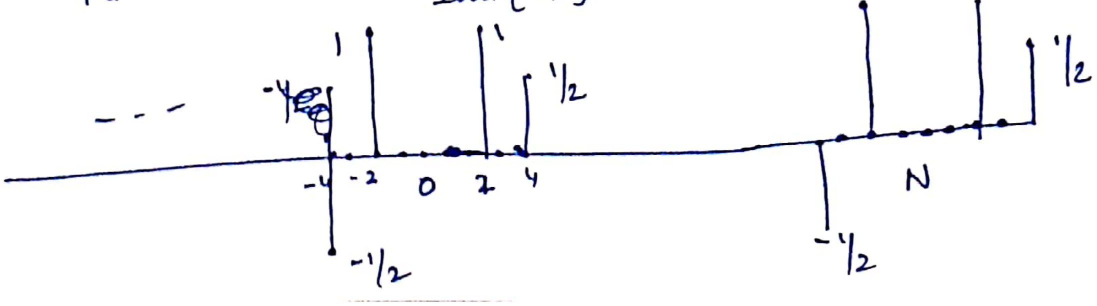
$a_{-4} = -\frac{1}{2}j$

$Re\{a_k\}$



These fourier coefficients repeat after period N.

$Im[a_k]$



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Fourier Transform of a function $x(t)$ is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \quad (1)$$

This is also called Fourier Integral

Inverse Fourier Transform is given by

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega. \quad (2)$$

Eq (1) & (2) are called Fourier Transform pair.

$$a_k = \frac{1}{T} X(j\omega)$$

~~Convergen~~.

Dirichlet Conditions :- Conditions for Convergence of Fourier Transforms

They ~~requi~~ are.

(1) $x(t)$ should be absolutely integrable.

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

(2) It should have a finite no. of maxima and minima in a finite inverval.

(3) $x(t)$ should have a finite no. of discontinuites in a finite inverval

Each of these ~~dos~~ discontinuities must be finite

① Find Fourier transform of.

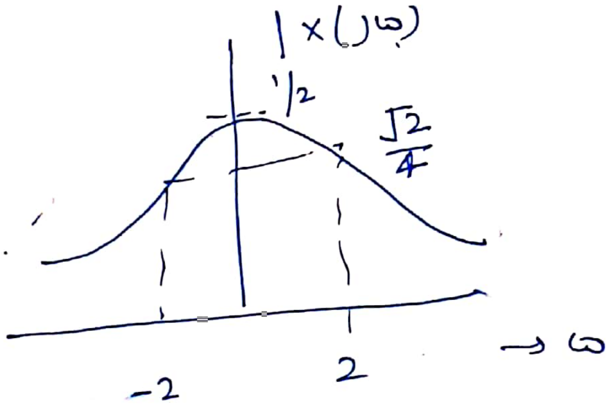
a) $x(t) = e^{-2t}u(t)$

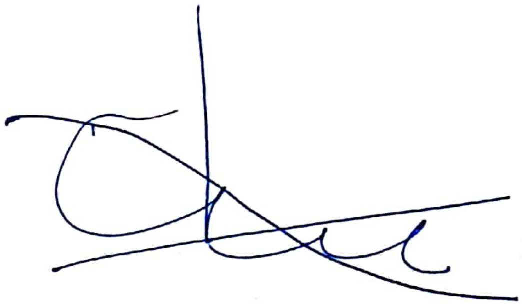
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-2t} \cdot e^{-j\omega t}\, dt$$

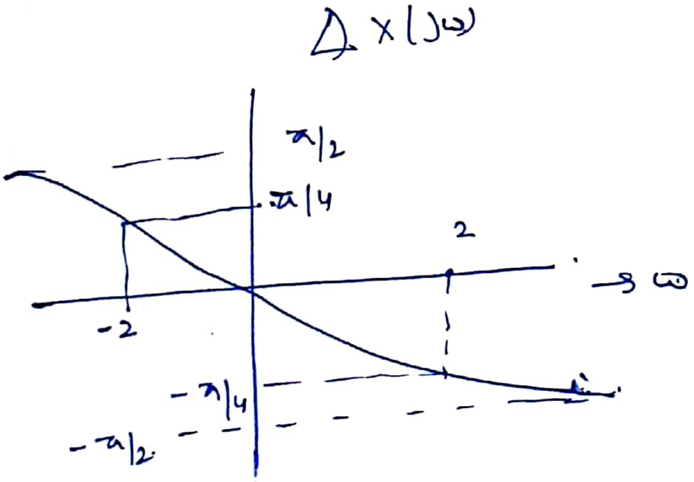
$$= \frac{-1}{2+j\omega}\, e^{-(2+j\omega)t}\Big|_{0}^{\infty} = \frac{1}{2+j\omega}$$

$$|X(j\omega)| = \frac{1}{\omega^2+4}$$

$$\measuredangle X(j\omega) = -\tan^{-1}(\omega/2)$$







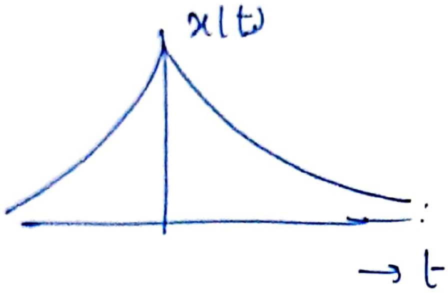
(b) $x(t) = e^{-a|t|}$ $\quad a > 0$

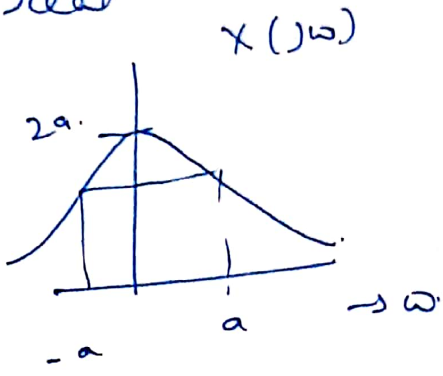
$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{at} \cdot e^{-j\omega t} dt + \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} dt$$

$$= \frac{1}{a - j\omega} + \frac{1}{a + j\omega} = \frac{2a}{a^2 + \omega^2}$$

In this $X(j\omega)$ is real





(c) $x(t) = \delta(t)$

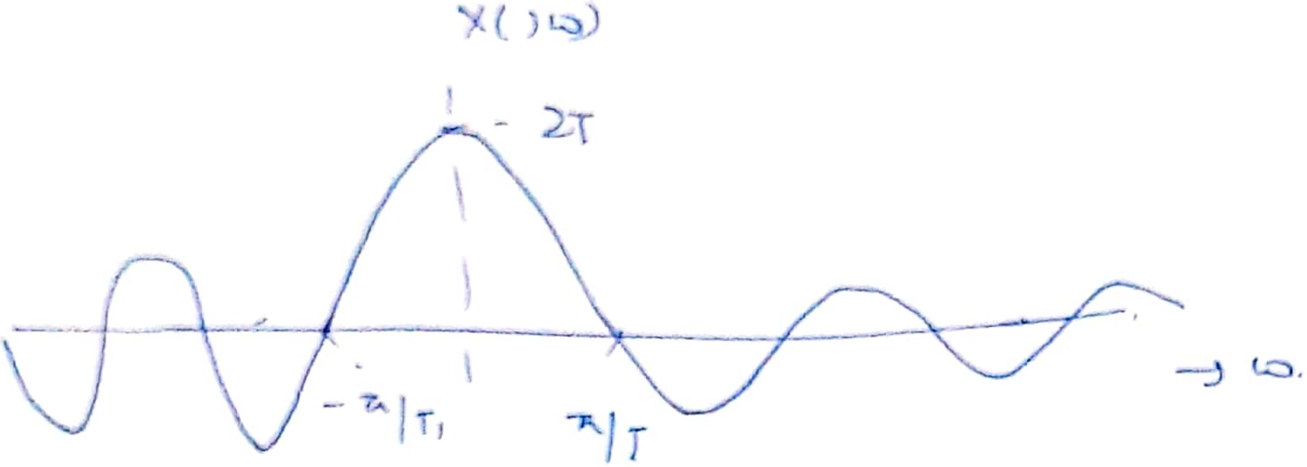
$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} = 1.$$

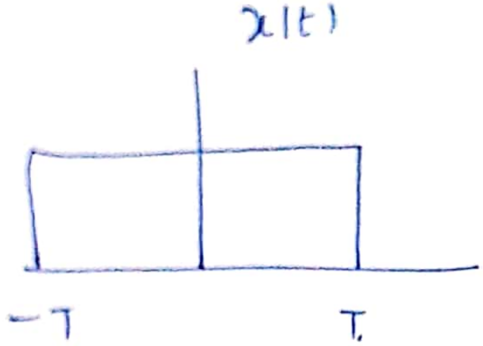
(d) $x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T. \end{cases}$

$$X(j\omega) = \int_{-T}^{T} e^{-j\omega t} dt$$

$$= \frac{-1}{j\omega} \left[e^{-j\omega T} - e^{j\omega T} \right] \times \frac{2}{2}$$

$$= \frac{2 \sin \omega T}{\omega}$$


X(jω)
2T
-π/T
π/T
→ ω


x(t)
-T
T