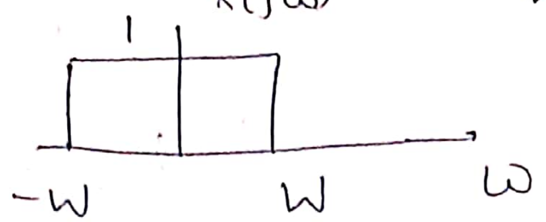
Ex 4.5 If $X(j\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases}$

find Inverse FT.

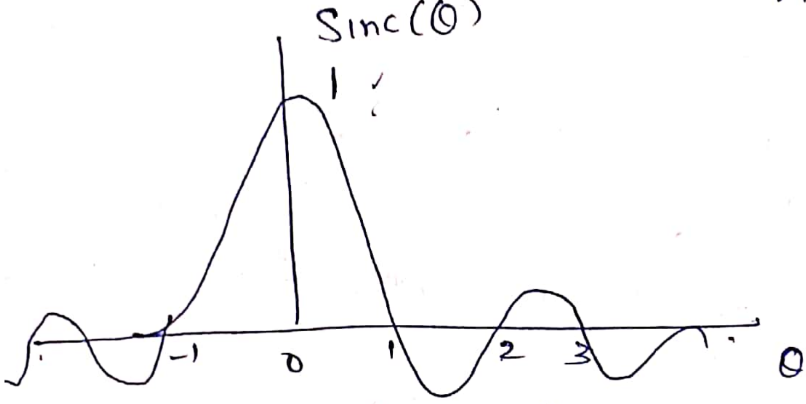


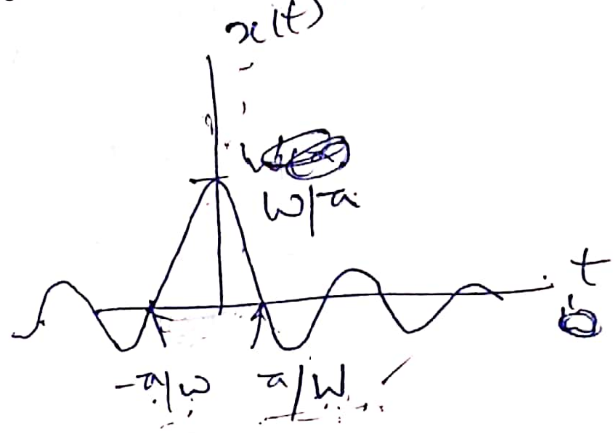

then $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$ → synthesis eq.

$= \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega t}\, d\omega = \frac{\sin Wt}{\pi t}$

Since sinc function is defined as $\mathrm{sinc}(\theta) = \frac{\sin \pi\theta}{\pi\theta}$

∴ $x(t) = \frac{\sin Wt}{\pi t} = \frac{W}{\pi}\,\mathrm{sinc}\left(\frac{Wt}{\pi}\right)$



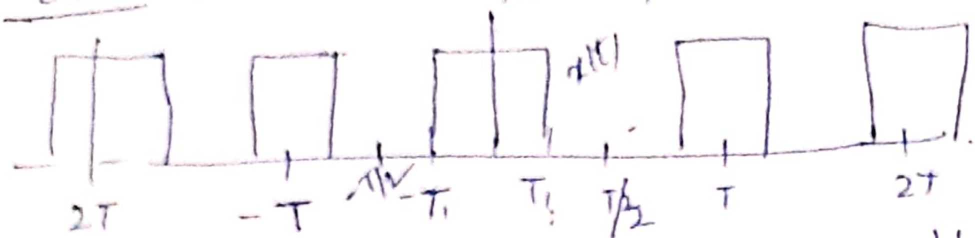




Clear from above figures as W increases X(jω) becomes broader, while the main peak of x(t) at t = 0 becomes higher and the first lobe of this signal becomes narrower. In fact, in the limit as $W \to \infty$, $X(j\omega) = 1$ for all ω and x(t) will converge to an impulse as $W \to \infty$.

related frequencies and for which area of impulse at $k$th harmonic freq $= \cancel{k\omega_0 \times 2\pi a_k}$

$k$th $= k\omega_0 \times 2\pi a_k$.

Ex 4.6 Find $X(j\omega)$ fourier transform of $x(t)$




Sol. We can write $x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T/2 \end{cases}$

$$a_k = \frac{1}{T} \int_{-T_1}^{T_1} x(t) e^{-jk\omega_0 t}\, dt = \frac{1}{j\omega_0 T k} e^{-jk\omega_0 t} \Big|_{-T_1}^{T_1}$$

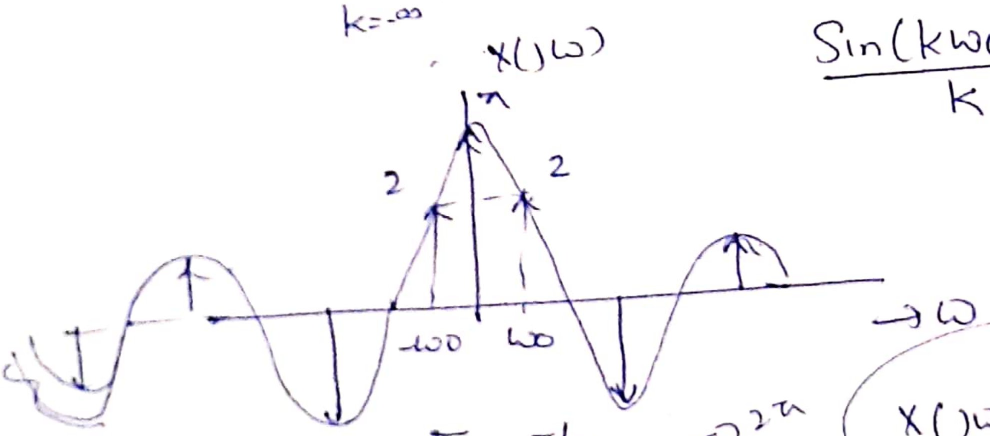
$$a_k = \frac{2}{k\omega_0 T}\left[\frac{e^{j\omega_0 k T_1} - e^{-jk\omega_0 T_1}}{2j}\right] = \frac{2}{k\omega_0 T} \sin(k\omega_0 T_1)$$

Since $\omega_0 = \frac{2\pi}{T} \Rightarrow \omega_0 T = 2\pi$

$$\therefore a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}$$

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

$$= \sum_{k=-\infty}^{\infty} \frac{2\pi}{k\pi} \sin(k\omega_0 T_1)\, \delta(\omega - k\omega_0)$$ exists at $\omega = k\omega_0$




$$\frac{\sin(k\omega_0 T_1)}{k} = \cancel{\text{Sinc}}\, 1 = \frac{\sin \pi \left(\frac{k\omega_0 T_1}{\pi}\right)}{\pi \left(\frac{k\omega_0 T_1}{\pi k\omega_0}\right)}$$

Since $T_1 = T/4$

$2\omega_0 T_1 = 2\omega_0 T/4 = \pi$

$$X(j\omega) = \cancel{2 \sin \pi} = 2\omega_0 T_1 \text{Sinc}\left(\frac{k\omega_0 T_1}{\pi}\right)$$

1 (a) $x(t) = \sin w_0 t = \frac{1}{2j}\left(e^{jw_0 t} - e^{-jw_0 t}\right)$. Find F.T.

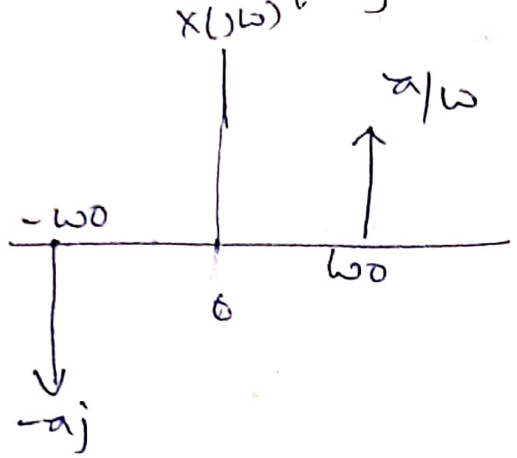
From eq. 3-3 oppenheim

$a_1 = \frac{1}{2j}$, $a_{-1} = \frac{-1}{2j}$, $a_k = 0$ $k \neq \pm 1$

Since $X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(w - kw_0)$

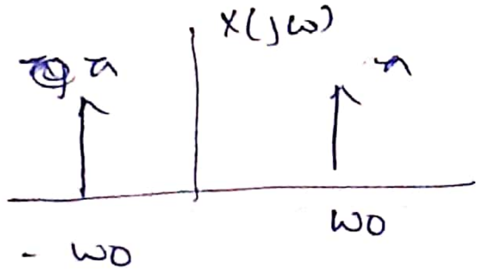
$= 2\pi\left[a_1 \delta(w - w_0) + a_{-1}\delta(w + w_0)\right]$

$= 2\pi\left[\frac{1}{2j}\delta(w - w_0) - \frac{1}{2j}\delta(w + w_0)\right]$




$x(t) = \cos w_0 t = \frac{1}{2}\left(e^{jw_0 t} + e^{-jw_0 t}\right)$

Here $a_1 = a_{-1} = 1/2$, $a_k = 0$ for $k \neq \pm 1$



## Properties of Continuous Time Fourier Transform

Since $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$ synthesis eq.

$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ Analysis eq.

and $x(t) \xleftrightarrow{F} X(j\omega)$

Since $\frac{1}{a+j\omega} = F\{e^{-at}u(t)\}$

$e^{-at}u(t) = F^{-1}\{\frac{1}{a+j\omega}\}$

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and $e^{-at}u(t) \xleftrightarrow{F} \frac{1}{a+j\omega}$.

<u>Linearity</u>

$$x(t) \xleftrightarrow{F} X(j\omega)$$

and $y(t) \xleftrightarrow{F} Y(j\omega)$

then $ax(t)+by(t) \xleftrightarrow{F} aX(j\omega)+bY(j\omega)$

<u>Time shifting</u> $x(t) \xleftrightarrow{F} X(j\omega)$

then $x(t-t_0) \xleftrightarrow{F} e^{-j\omega t_0} X(j\omega)$

<u>Proof</u>

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

Replacing $t$ by $t-t_0$

$$x(t-t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega(t-t_0)}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(X(j\omega)e^{-j\omega t_0}\right)e^{j\omega t}\,d\omega$$

Clearly $F\{x(t-t_0)\} = e^{-j\omega t_0}X(j\omega)$

* If we express $X(j\omega)$ in polar form as

$F\{x(t)\} = X(j\omega) = |X(j\omega)|e^{j\measuredangle X(j\omega)}$

~~then~~ $\cancel{F\{x(t-t_0)\} = e^{-j\omega t_0}X(j\omega)}$

then $F\{x(t-t_0)\} = e^{-j\omega t_0}X(j\omega) = |X(j\omega)|e^{j\{\measuredangle X(j\omega)-\omega t_0\}}$

So the effect of time shift on a signal is to introduce into its transform a phase shift

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$-\omega t_0$. It does not alter the magnitude of Fourier transform.

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