

# > Deduction of Wien's displacement law from Planck's law; (3)

Planck's radiation law is

$$E_{\lambda} d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{(e^{hc/\lambda KT} - 1)}$$

When this equation is differentiated with respect to  $\lambda$  and equated to zero, the Wien's displacement law ( $\lambda T = \text{constant}$ ) is obtained for wavelength ~~at~~ which corresponds to maximum energy emission for a given value of  $T$ . that is

$$\frac{dE_{\lambda}}{d\lambda} = \frac{1}{(e^{hc/\lambda KT} - 1)} \times \left[ \frac{-5(8\pi hc)}{\lambda^6} + \frac{8\pi hc}{\lambda^5} \times \frac{hc}{\lambda^2 KT} e^{hc/\lambda KT} \right]$$

$$= \frac{1}{(e^{hc/\lambda KT} - 1)} \times \frac{-40\pi hc}{\lambda^6} + \frac{8\pi hc}{\lambda^5} \times \frac{hc}{\lambda^2 KT} \times \frac{e^{hc/\lambda KT}}{(e^{hc/\lambda KT} - 1)^2}$$

For  $E_{\lambda}$  maximum the value of its first derivative should be zero

$$\text{So } \frac{8\pi hc}{(e^{hc/\lambda KT} - 1)} \lambda^6 \left[ -5 + \frac{hc}{\lambda KT} \times \frac{e^{hc/\lambda KT}}{(e^{hc/\lambda KT} - 1)} \right] = 0$$

$$= \left[ -5 + \frac{hc}{\lambda KT} \frac{e^{hc/\lambda KT}}{(e^{hc/\lambda KT} - 1)} \right] = 0$$

$$\text{Let } \frac{hc}{\lambda KT} = y$$

Then  $-5 + \frac{hc}{\lambda KT} \frac{e^{hc/\lambda KT}}{(e^{hc/\lambda KT} - 1)}$  becomes

(4)

$$\left[ -5 + y \frac{e^y}{(e^y - 1)} \right] = 0$$

solving this for  $y$  then we get

$$y = 4.965 = \frac{hc}{\lambda KT}$$

$$\frac{hc}{\lambda KT} = 4.965$$

$$\lambda_m T = \left( \frac{hc}{4.965 k} \right) \Rightarrow \text{constant}$$

$$\boxed{\lambda_m T = b} \quad \text{where } b = \frac{hc}{(4.965 k)}$$

This is Wien's displacement law.

