Important

LECTURE — II

> Derivation of Plank's law :→

The distribution of energy in a Black-body radiation, plank's proposed the following :- (i) A black-body is made up of a number of oscillating particles called harmonic oscillators, known as plank's oscillators.

(ii) An oscillator emits radiation energy of freq. ν, when it drops from one energy state to the other state. Each discrete bundle has energy $h\nu$ or multiples of $h\nu$. it is given by

$$E = nh\nu \quad , \text{where } n = 0,1,2,3 \cdots \text{———} (1)$$

$$h = 6.6 \times 10^{-34} \text{ J-S (plank's constant)}$$

Let us Assume that the number of vibrating particles (plank's oscillators) in a body as $N_0, N_1, N_2 \cdots N_n$.

So Energy of the each oscillator can be written as

$$0, h\nu, 2h\nu \cdots Nh\nu \quad \text{as eqn from (1)}$$

OR

$$0, \varepsilon, 2\varepsilon, 3\varepsilon \cdots n\varepsilon \text{ ----- } (2)$$

Total energy ⇒ $E_T = 0 + \varepsilon + 2\varepsilon + 3\varepsilon \text{ ---- } + n\varepsilon \text{ ---- } (3)$

Therefore, average energy of a particle is given by

$$\bar{\varepsilon} = \frac{E}{N} = E = N\bar{\varepsilon} \text{ ---- } (4)$$

According to maxwell's distribution law, the number of particles in the n^{th} oscillating system can be written as

$$N_n = N_0 e^{-n\varepsilon/KT} \text{ ---- } (5)$$

$$N_n = N_0 \, e^{-n\bar{\varepsilon}/kT}$$

where ε is the average energy.
K is Boltzmann costant.

$$N = N_0 + N_0 e^{-\bar{\varepsilon}/kT} + N_0 e^{-2\bar{\varepsilon}/kT} + N_0 e^{-3\bar{\varepsilon}/kT} + \cdots \quad [n = 0, 1, 2 \cdots]$$

$$N = N_0 \left[1 + e^{-\bar{\varepsilon}/kT} + e^{-2\bar{\varepsilon}/kT} + e^{-3\bar{\varepsilon}/kT} + \cdots \right]$$

$$N = \frac{N_0}{(1 - e^{-\bar{\varepsilon}/kT})} \quad \text{------} \; (6)$$

$$1 + x + x^2 + \cdots = \frac{1}{(1-x)}$$

Similarly total energy of the body combe writen as

$$E = 0 + \bar{\varepsilon} \cdot N_0 e^{-\bar{\varepsilon}/kT} + 2\bar{\varepsilon} N_0 e^{-\bar{\varepsilon}/kT} + \cdots \quad [\text{From eq}^n \; (4)]$$

$$E = N_0 \bar{\varepsilon} e^{-\bar{\varepsilon}/kT} \left[1 + 2e^{-\bar{\varepsilon}/kT} + 3e^{-\bar{\varepsilon}/kT} + \cdots \right]$$

$$E = \frac{N_0 \bar{\varepsilon} e^{-\bar{\varepsilon}/kT}}{(1 - e^{-\bar{\varepsilon}/kT})} \quad (7)$$

$$1 + 2x + 3x + \cdots = \frac{1}{(1-x)^2}$$

So From eq^n (4) [These value i.e. eq^n (7) and eq^n (6) ~~sub~~ Substituting in eq^n (4)]

We have got expressions for ~~total energy~~. average energy.

$$\bar{\varepsilon}_T = \frac{\dfrac{N_0 \bar{\varepsilon} e^{-\bar{\varepsilon}/kT}}{(1 - e^{-\bar{\varepsilon}/kT})}}{\dfrac{N_0}{(1 - e^{-\bar{\varepsilon}/kT})}} = \frac{\bar{\varepsilon} e^{-\varepsilon/kT}}{(1 - e^{-\varepsilon/kT})}$$

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$$\bar{\varepsilon} = \frac{\varepsilon e^{-\varepsilon/kT}}{(1 - e^{-\varepsilon/kT})}$$

$$\bar{\varepsilon} = \frac{\varepsilon}{(e^{\varepsilon/kT} - 1)}$$

$$\boxed{\bar{\varepsilon} = \frac{h\nu}{(e^{h\nu/kT} - 1)}} \quad [\because \varepsilon = h\nu] \qquad (8)$$

this equation gives the average energy of oscillator.

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