Quantization error $e_q(n)$ is limited to

$$-\frac{\Delta}{2} \leq e_q(n) \leq \frac{\Delta}{2}$$

Quantization Process in PCM →

Consider an example of a sine wave with a peak amplitude of $\pm 3V$ varying between $+3V$ and $-3V$. A PCM code could have only ~~8~~ 3 bits which equals to only ~~2^8~~ 2^3 or 8 ~~256~~ combinations (levels)

3 BIT PCM CODE :-

	Sign	Magnitude		Decimal value	Quantization Range
Sub ranges +ve -ve	1	1	1	+3	+2.5 V to 3.5 V
	1	1	0	+2	+1.5 V to 2.5 V
	1	0	1	+1	0.5 V to 1.5 V
	1	0	0	+0	0 to 0.5 V
	0	0	0	−0	0 to −0.5 V
	0	0	1	−1	−0.5 to −1.5 V
	0	1	0	−2	−1.5 to −2.5 V
	0	1	1	−3	−2.5 to −3.5 V

Levels that have be

Quantization interval is also ~~code~~ known as the Quant and the magnitude of quantum is called as the resolution.

Resolution is equal to the voltage of the minimum step size which is equal to the voltage of the least significant bit (V_{lsb}) of PCM code. e.g. the resolution for PCM code in above table is 1 V.

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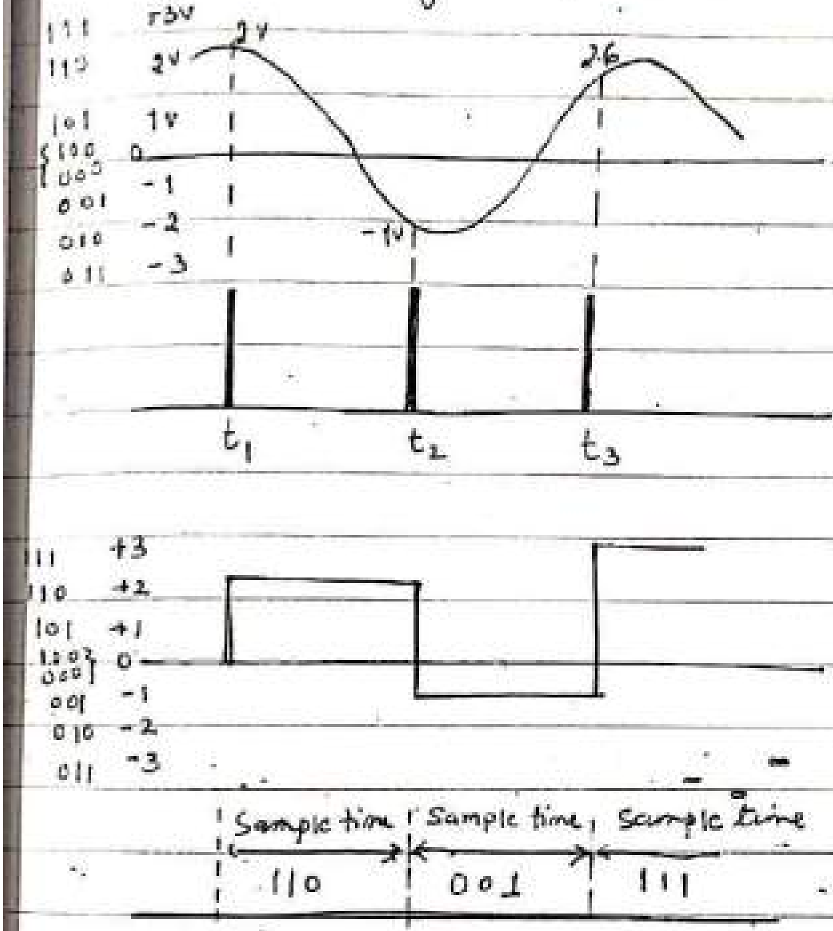
Smaller the magnitude of a quantum, better the resolution and more accurately the quantized signal will resemble the original analog sample.



Analog input Signal

Sampled Pulse

PAM Signal

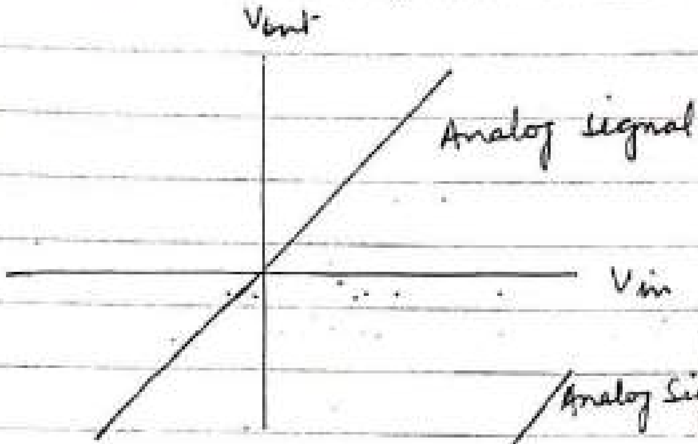
PCM Code

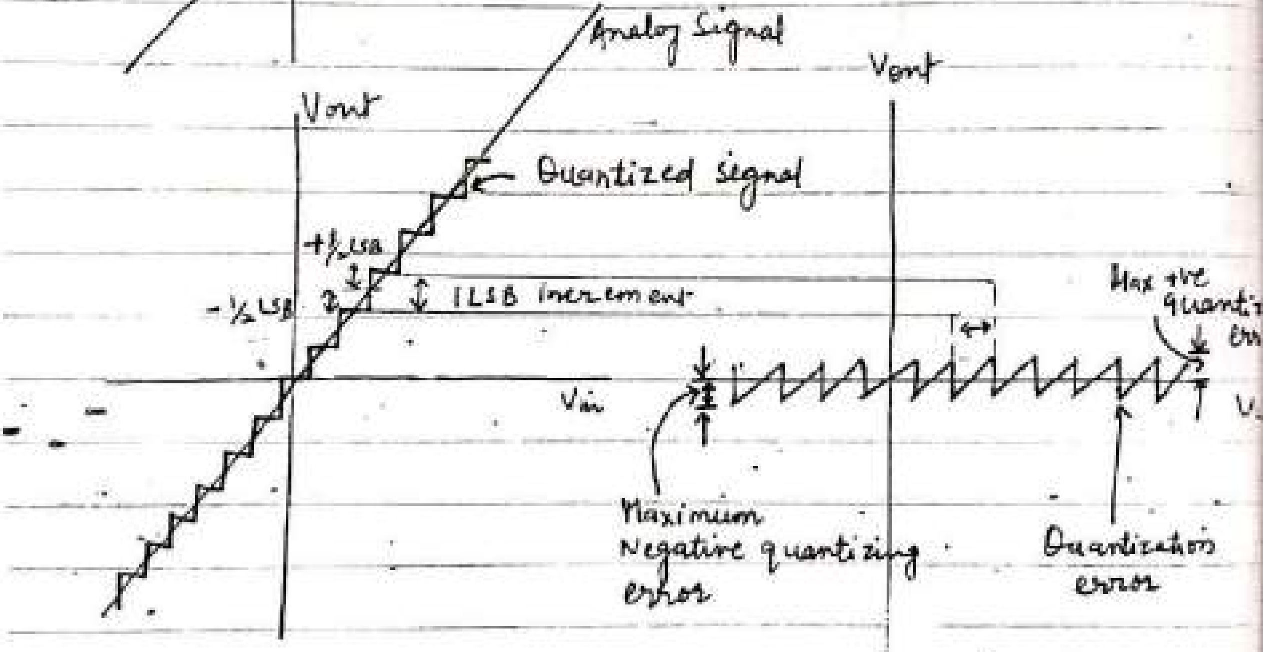
First sample at time t_1 :— input voltage = +2 V
Corresponding PCM code :- 110
(There is no quantization error)

Second sample at time t_2 :- input voltage = −1 V
Corresponding PCM Code = 001
(No quantization Error)

Third sample at time t_3 :- input Voltage = +2.6 V
Corresponding PCM code :- 111 (As 3V)
(This introduces quantization error of 0.4V)

The quality of the PAM signal can be improved by using a PCM code with more bits, reducing the magnitude of a quantum and improving the resolution.





e_q = Quantizing error = $\pm \frac{1}{2}$ LSB

Dynamic Range → (DR) → It is the ratio of the largest possible magnitude to the smallest possible magnitude (other than 0V) that can be decoded by the Digital to Analog convertor in the receiver.

$$DR = \frac{V_{max}}{V_{min}} \quad \text{— (1)}$$

V_{min} ← Resolution

$$\Rightarrow \quad DR = \frac{V_{max}}{Resolution} \quad \text{— (2)}$$

In decibel

$$DR = 20 \log_{10} \frac{V_{max}}{V_{min}} \quad \text{— (3)}$$

The no. of bits used for a PCM code depends on the dynamic range. The relation between DR and no of bits (n) is

$$(2^n - 1) \geq DR$$

Minimum no of bits $= 2^n - 1 = DR$

$$\Rightarrow \quad 2^n = DR + 1$$

taking log on both side

$$\log 2^n = \log (DR + 1)$$

$$n \log 2 = \log (DR + 1)$$

$$\boxed{n = \frac{\log (DR+1)}{\log 2}} \quad \text{— (4)}$$

Relation between total no of bits n and DR.

DR can be expressed in dB as

$$DR_{(dB)} = 20 \log \frac{V_{max}}{V_{min}} = 20 \log (2^n - 1) \quad \text{— (5)}$$

Coding Efficiency → It is a numerical indication of how efficiently a PCM code is utilized. It is the ratio of minimum no. of bits required to achieve a certain dynamic range to the actual no. of PCM bits used.

$$\text{coding efficiency} = \frac{\text{Min no of bits (including sign bit)}}{\text{Actual no of bits (including sign bit)}} \times 100\% \quad \text{——(6)}$$

Problem 1:- For the PCM coding scheme for 3 bit, determine the quantized voltage, quantization error (e_q) and PCM code for the analog ~~with~~ sample voltage of +1.07 V.

Soln Data 3 bit.

Min quantized voltage = +1V = Resolution

⇒ Quantized level for +1.07V = $\frac{1.07}{1}$ = 1.07 = 1

Quantization error = e_q = (1.07 − 1) V = 0.07 V

PCM Code = 101

Problem 2: For a minimum line speed with an 8-bit PCM for speech signal ranging upto 1V

(a) Calculate the Resolution and quantization error

(b) Calculate the coding efficiency for a resolution of 0.01 V with 8 bit PCM.

Solution :- (a) $V_{max} = 1V$

$$\Delta = \text{Resolution} = \frac{V_{max}}{2^n - 1} = \frac{1}{2^8 - 1} = 0.003922V$$

$$e_q = \frac{\Delta}{2} = 0.00196\ V$$

(b) Coding efficiency = ?

$$DR = 20 \log \frac{V_{max}}{V_{min}}$$

$$= 20 \log \frac{1}{0.01} \ dB = 40\ dB \quad \text{or } 100$$

Min. no of bits 'n' to achieve the dynamic range is

$$n = \frac{\log(DR+1)}{\log 2} = 6.6 = 7.0$$

$$\text{Coding efficiency} = \frac{\text{Min. number of bits}}{\text{actual number of bits}} \times 100$$

$$= \frac{7}{8} \times 100 = 87.5\%$$

Signal-to-Quantization Noise Ratio →

Signal voltage-to-quantization noise voltage ratio (SQR) occurs when the input signal is at its minimum amplitude (101 or 001).

$$SQR = \frac{\text{Resolution}}{e_q} = \frac{V_{lsb}}{V_{lsb}/2} = 2$$

$\Rightarrow$ Min. SQR is,

(example) $\Rightarrow$ $SQR_{(min)} = \frac{1}{0.5} = 2 = 20 \log(2)$

$= 6\ dB.$

Max. SQR is,

$$SQR_{(max)} = \frac{V_{max}}{e_q}$$

(example) $\Rightarrow$ $SQR_{(max)} = \frac{3}{0.5/2} = 6$

$= 15.6\ dB.$

Signal power-to-quantizing noise power ratio (or signal-to-distortion ratio or signal-to-noise ratio) is

$$SQR_{(dB)} = 10 \log\left[\frac{v^2/R}{(q^2/12)/R}\right] \quad \text{—} \quad (7)$$

Where, $R \rightarrow$ resistance in Ω.

$v \rightarrow$ rms signal voltage (volts)

$\Delta = q \rightarrow$ quantization interval (volts)

$v^2/R \rightarrow$ average signal power (watts)

$(q^2/12)/R \rightarrow$ average quantization noise power (watts).

$$= 10 \log\left[\frac{12 v^2}{q^2}\right] = 20 \log\left[\frac{12 \cdot v}{q}\right]$$

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Quantization

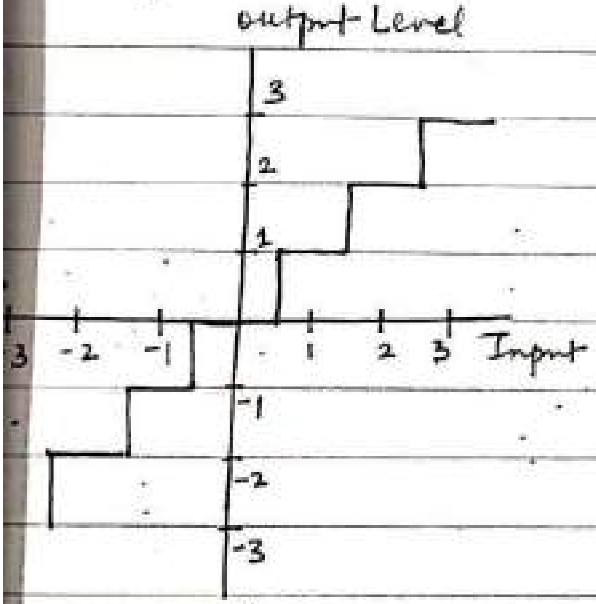
↓

Uniform Quantization — Non-Uniform Quantization

Uniform Quantization ↓

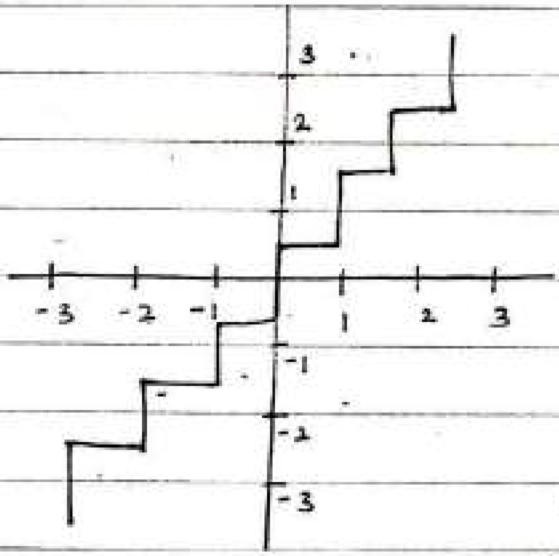
Midtread type — Midrise type

Uniform Quantization → Type of quantization in which the step size remains same througout the input range.



Midtread

(Origin lies in the middle of a tread of the staircase like graph)



Midrise

(Origin lies in the middle of a rising part of the staircase like graph)

ENCODING :- Encoding process translates the discrete set of sample values to a more appropriate form of signal. The plan for representing each of this discrete set of val as a particular arrangement of discrete events is called a Code. One of the discrete events in a code is called a Code Element or Symbol.

A particular arrangement of symbols used in a Code to represent a single value of the discrete set is called a Code Word or Character.

Code Word / character
↓
Code Element / Symbol
↓
Code

Different types of Code Systems :-

Binary Code :- 0 / 1 ← Presence of Pulse } generally
↳ Absence of Pulse

Ternary Code :- −1 / 0 / +1

In Binary code, several Coding are used to represent signal between 0 and 1

1. ON - OFF Signaling → 1 → ON state (Pulse of Constant Am
0 → OFF state (Pulse of zero Ampli

OR

UNIPOLAR NON Return to Zero (NRZ) Signaling →
1 → Pulse of Amplitude A
0 → Switching of Pulse to Amplitude zero

Signal is zero only when binary data is zero and 1 when binary data is 1.

2. Polar non return - to - zero (NRZ) signaling →

1 :- Pulse of Amplitude +A

0 :- Pulse of Amplitude − A

3. Unipolar return-to-zero (RZ) signaling →

1 :- Pulse of Amplitude A of half width

0 :- No Pulse

If Binary data is 1, a Pulse of Amplitude A will appear for half the time period

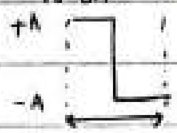
4. Bipolar return - to - zero (BRZ) signaling →

1 :- Positive Pulse followed by a negative Pulse of same Amplitude for a ~~Conseqn~~ consecutive OR Alternate positive negative pulses.

0 :- NO Pulse.

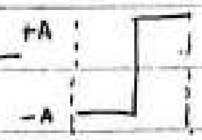
5. Split Phase (Manchester) Code :-

1 :- Positive Pulse of Amplitude A followed by a negative Pulse of Amplitude − A with both Pulses of half symbol width

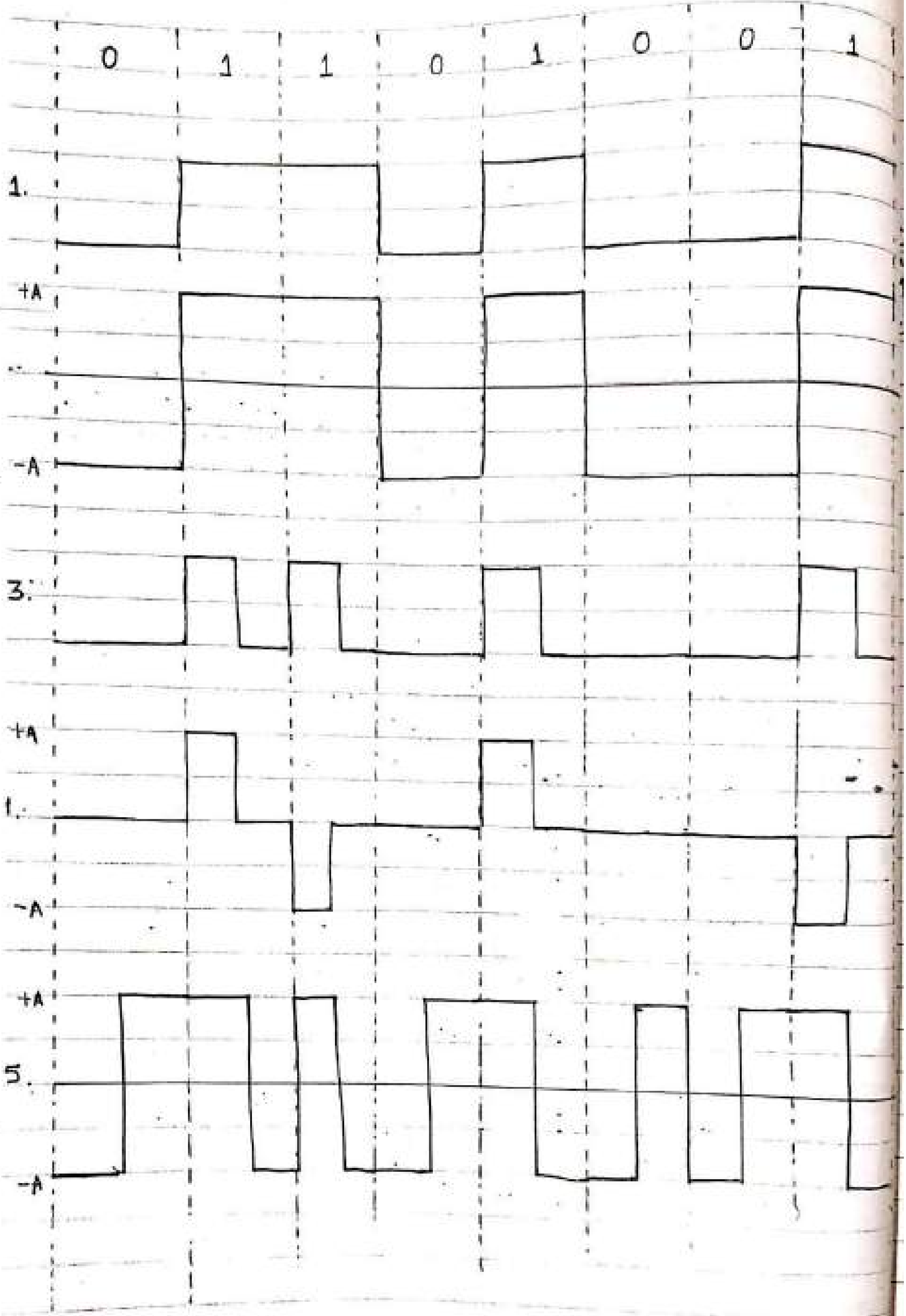


Time Period

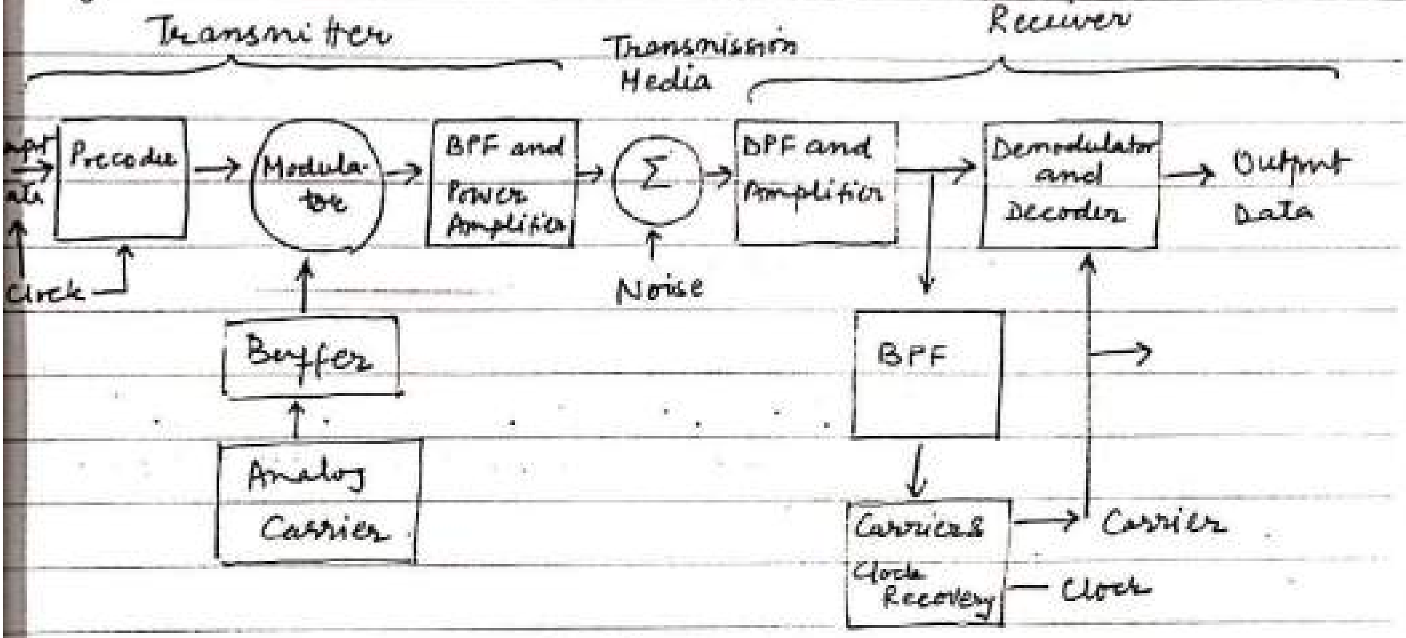
0 :-



Polarities of two Pulses are reversed.


0 1 1 0 1 0 0 1
1.
+A
-A
3.
+A
1.
-A
+A
5.
-A

Digital carrier Modulation Technique →



Digital Radio System

BPF - Band pass filter

In the transmitter, Precoder performs level conversion and then encodes the incoming data into groups of bits that modulate an analog carrier. Modulated carrier is shaped (filtered), amplified and then transmitted through the transmission medium to the receiver.

Transmission medium can be a metallic cable, optical fibre cable, earth's atmosphere or a combination of two or more types of transmission systems.

In the receiver, the incoming signals are filtered, amplified and then applied to the demodulator and decoder circuits which extracts the original source information from the modulated carrier.

Clock and carrier recovery circuits recover the analog carrier and digital timing clock signals from the incoming modulated wave since they are necessary to perform the demodulation operation

Baud and Minimum Bandwidth →

Baud :- Rate of change of a signal on the transmission medium after encoding and modulation have occurred. It is a unit of transmission rate, modulation rate or symbol rate.

OR

Baud is the reciprocal of the time of one output signaling element expressed as

$$\text{Baud} = \frac{1}{t_s}$$

t_s = time of one signaling element (s)

Baud - symbols per sec

Relation between channel capacity and bandwidth

$$\boxed{f_b = 2B \log_2 M}$$

f_b = channel capacity

B = Minimum Nyquist bandwidth in Hz

M = No of discrete signals or voltage levels

$$\boxed{\text{Baud} = 2/\log_2 \text{ Nyquist criteria}}$$

$\Rightarrow$ Baud = $\frac{f_b}{\log_2 M}$ Nyquist Criteria = 2B

$\Rightarrow$ $\boxed{\text{Baud} = \frac{f_b}{N}}$

where $N = \log_2 M$

AMPLITUDE SHIFT KEYING (ASK) →

Here a binary information signal directly modulates the amplitude of an analog carrier. It is also known as digital amplitude modulation (DAM).

ASK modulated wave is represented as

$$V_{ASK}(t) = [1 + V_m(t)]\left[\frac{A}{2} \cos w_c t\right] \quad \text{—— (1)}$$

where $V_{ASK}(t)$ = amplitude shift keying wave

$V_m(t)$ = Digital information or modulating signal

$A/2$ = Unmodulated carrier amplitude

w_c = Analog carrier radian freq. = $2\pi f_c$

Case I :- Binary input = Logic 1

$$V_m(t) = +1\ V \quad \text{(reference)}$$

eqn. (1) becomes,

$$V_{ASK}(t) = [1+1]\cdot\left[\frac{A}{2} \cos w_c t\right]$$

$\Rightarrow V_{ASK}(t) = A \cos w_c(t)$

$\Rightarrow$ Carrier wave of freq. f_c and amp $= A$ when binary input $= 1$.

Case II :- Binary input : Logic 0

$$V_m(t) = -1 \text{ V} \quad (\text{suppose})$$

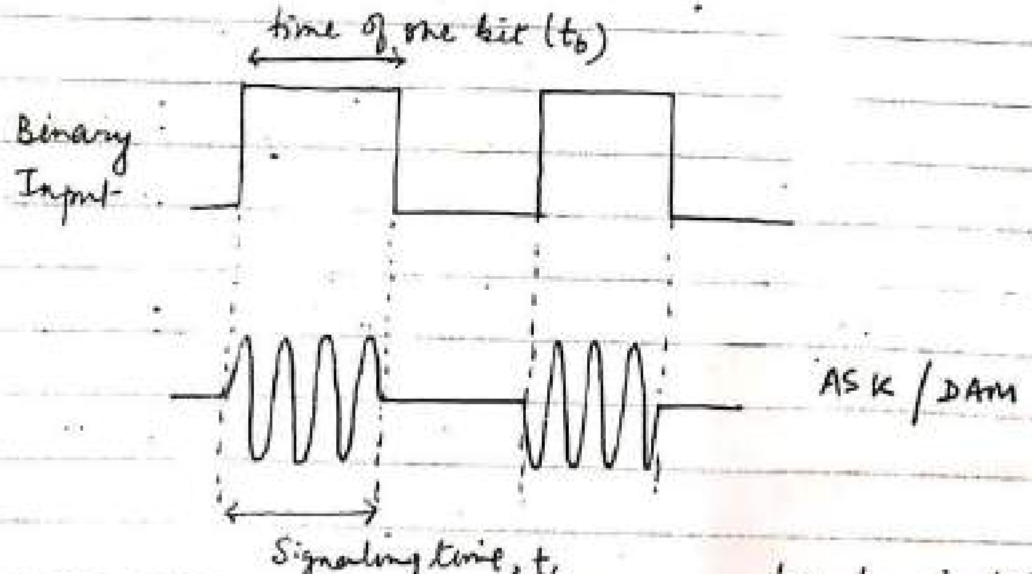
equ (1) becomes

$$V_{ASK}(t) = (1 - 1)\left(\frac{A}{2} \cos w_c t\right)$$

$$= 0$$

$\Rightarrow$ No wave when input is logic 0.

Carrier is either ON or OFF for logic 1 and 0 respectively. Hence, ASK is also referred as ON - OFF KEYING (OOK)



$t_s = t_b$ in this case

For every change in the input binary data, there is one change in the ASK waveform and the time of one bit (t_b) equals the time of one analog signaling element (t_s)

Baud is the rate of change of the ASK waveform or rate of change of the binary input (bps)

∴ In ASK,

$$(\text{Bandwidth}) \quad B = \frac{f_b}{1} \qquad\qquad B = \frac{f_b}{2\log_2 M}$$

└ Signaling rate = 1

$$\Rightarrow B = f_b$$

$t_s = t_b \Rightarrow$ Signaling rate = 1

$$\text{Baud} = \frac{f_b}{1} = f_b = B$$

Problem 1 :- Determine the signal to noise ratio required to transmit a multiplexed telephone signal at 8.448 Mbps through a channel of 2.048 MHz Bandwidth.

Solution → using eqn.

$$(\text{Channel Capacity}) \quad I = 3.32\, B \log_{10}(1 + S/N)$$

$$I = 8.448 \text{ Mbps} \quad (\text{given})$$

$$= 8.448 \times 10^6 \text{ bps}$$

$$B = 2.048 \times 10^6 \text{ Hz} \quad (\text{Bandwidth})$$

$$S/N = ?$$

$$= 16.447 = 12.17 \text{ dB}$$