09/04/2020

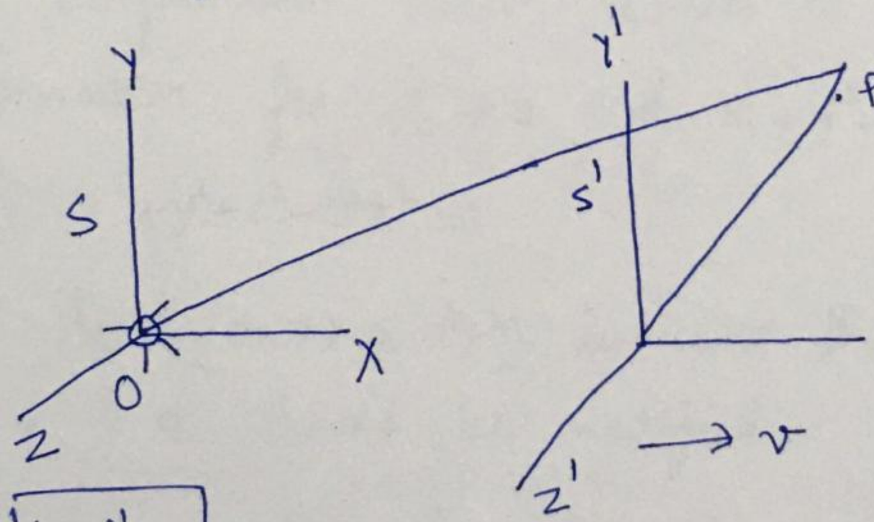
B.Sc (H) Physics
(Classical Dynamics)

Postulates of Special Theory of Relativity

1). All the laws of Physics have the same form in all inertial frames, moving with const. velocity relative to one another. (This postulate is just principle of Rele.)

2). The speed of light is const. in vacuum in every inertial frames.

Lorentz Transformation:



$x' = \gamma(x - vt)$

$y' = y \,;\; z' = z$

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$

at $\boxed{t = t' = 0}$

origins O and O' coincide

In frame S

$$t = \frac{OP}{c} = \frac{(x^2+y^2+z^2)^{1/2}}{c} \quad \text{or} \quad x^2+y^2+z^2 = c^2t^2 \quad -(1)$$

In frame S', time required to travel distance O'P by light signal is

$$t' = \frac{O'P}{c} = \frac{x'^2+y'^2+z'^2}{c} \Rightarrow x'^2+y'^2+z'^2 = c^2t'^2 \quad -(2)$$

we have from Galilean Transformation;

$$x' = x - vt$$
$$y' = y \; ; \; z' = z \text{ and } t' = t$$

using in eqn (2),

$$(x - vt)^2 + y^2 + z^2 - c^2t^2 = 0$$

$$\Rightarrow \quad x^2 - 2xvt + v^2t^2 + y^2 + z^2 - c^2t^2 = 0 \quad \text{—(3)}$$

This eqn is not agreement with eqn (1) due to extra term $\underline{(-2xvt + v^2t^2)}$

According to constancy of speed of light is valid in all frames, there should exist some tranformation which reduces to Galilean transformation for $v/c \to 0$ and $x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$ into $x^2 + y^2 + z^2 - c^2t^2 = 0$

So $(-2xvt + v^2t^2)$ indicates that transformatio in x & t should be modified.

Let $x' = \alpha(x - vt)$ for $x' = 0 \Rightarrow x = vt$

$t' = \alpha'(t + fx)$, here α, α', f are const.

(For Galilean Trans. $\alpha = \alpha' = 1$, $f = 0$)

using these parameters in eqn (2) we get

$$\alpha^2(x - vt)^2 + y^2 + z^2 = c^2\alpha'^2(t + fx)^2$$

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$\Rightarrow$

$$x^2(\alpha^2 - f^2\alpha'^2c^2) - 2xvt(\alpha^2 v + fc^2\alpha'^2) + y^2 + z^2 = \left(\alpha'^2 - \frac{\alpha^2 v^2}{c^2}\right)c^2t^2 \quad \text{--- (4)}$$

Comparing (1) & (4)

$$\alpha^2 - f^2\alpha'^2c^2 = 1 \;/\; \alpha^2 v + fc^2\alpha'^2 = 0 \;/\; \alpha^2 - \frac{\alpha^2 v^2}{c^2} = 1$$

Solving

$$\alpha = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \alpha' = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\boxed{f = -v/c^2}$$

$\Rightarrow$ $x' = \alpha(x - vt)$ $\quad y' = y \quad z' = z$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

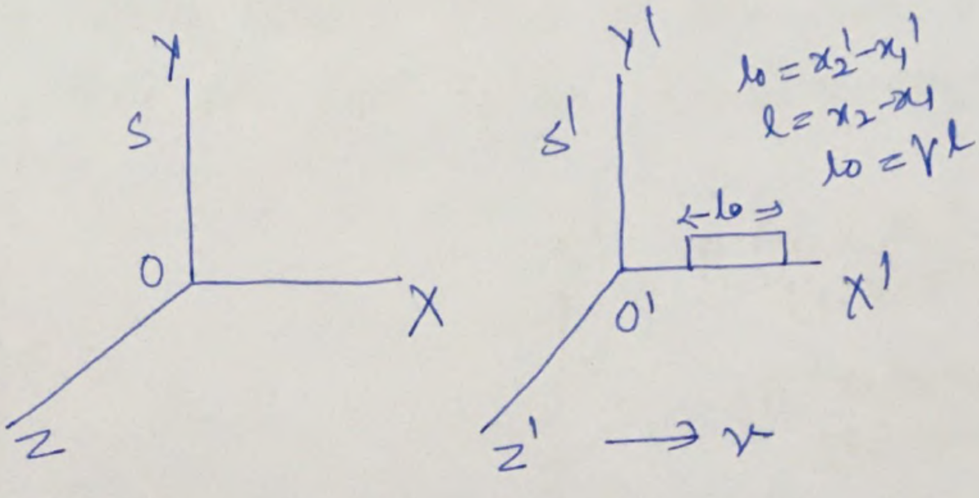
These eq^{n}s are called Lorentz Transformation.

If $\frac{v}{c} << 1$ i.e. $\frac{v}{c} \to 0$, we get Galilean Transformation.

$$x = \gamma(x' + vt') \,;\; y \equiv y' \,;\; z' \equiv z \,,\; t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$ (Inverse Lorentz Transformation.)

1) Length Contraction.



$l_0 = x_2' - x_1'$, is called proper length of rod measured by stationary observer relative to the rod.

If X- co-ordinates of the end points of rod in frame S are measured to be x_1 & x_2 at time t. Then,

$$l = x_2 - x_1$$

Using L.T.

$$l_0 = \gamma(x_2 - vt) - \gamma(x_1 - vt)$$

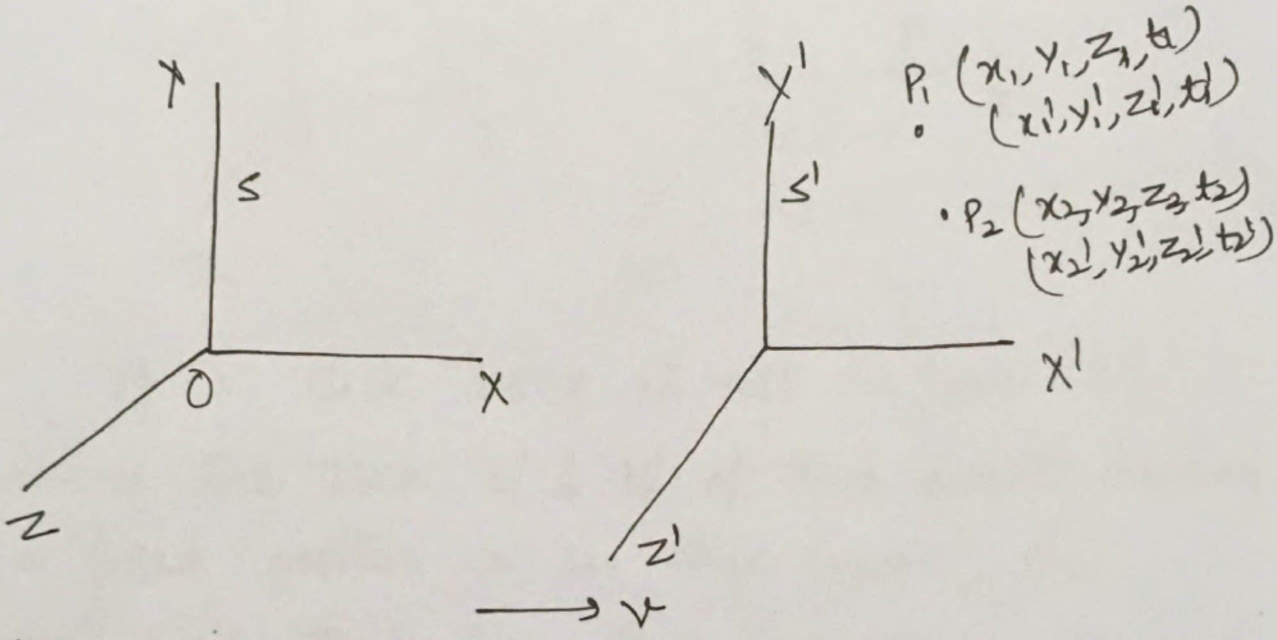
$$= \gamma(x_2 - x_1)$$

$$l_0 = \gamma l \Rightarrow \boxed{l = l_0\sqrt{1 - \frac{v^2}{c^2}}}$$

$\Rightarrow l < l_0$. This is called Lorentz Fitzgerald Contradiction.

2) <u>Simultaneity:</u>

If two events occur at the same time in a frame, they are said to be simultaneous.



Let two events occur simultaneously in frame S at points P_1 and P_2, respectively as measured by observer O of the frame S. It means $t_1 = t_2$. If t_1' and t_2' are the corresponding times of the same two events are measured by O' of frame S', then

$$t_1' = \gamma\left(t_1 - \frac{v x_1}{c^2}\right) \qquad t_2' = \gamma\left(t_2 - \frac{v x_2}{c^2}\right)$$

$$\Rightarrow \quad t_2' - t_1' = \gamma\left(t_2 - t_1\right) - \frac{v}{c^2}\gamma\left(x_2 - x_1\right)$$

as $t_2 = t_1$ $\quad \boxed{t_2' - t_1' = -\frac{v}{c^2}\gamma\left(x_2 - x_1\right)} \Rightarrow \boxed{t_2' \neq t_1'}$

$\Rightarrow$ Simultaneity is not absolute but relative.

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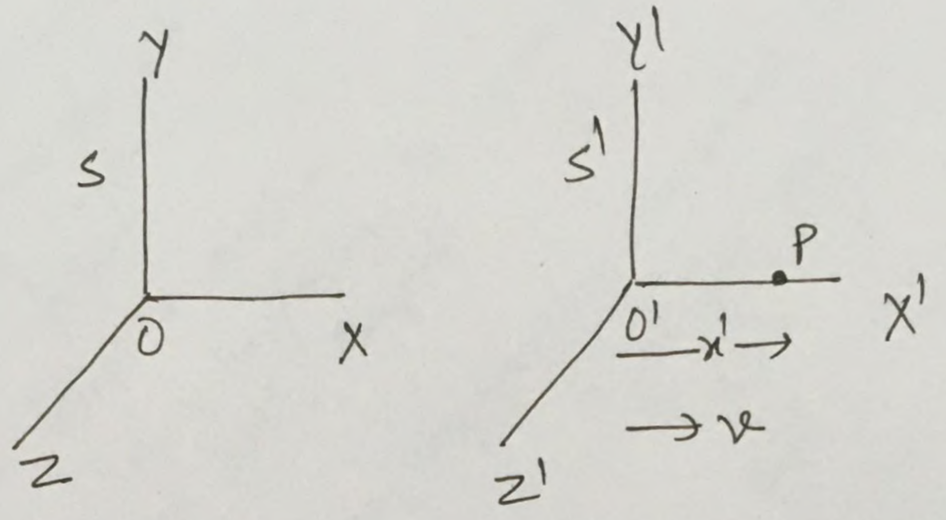
Time Dilation:



If a clock being at rest in frame S', measures the time t_1' & t_2' of two events occuring at a fixed position x' in this frame, then interval of time b/w these events is;

$$\Delta t' = t_2' - t_1' = \Delta t_0 \text{ (say)}$$

According to L.T.

$$t_1 = \gamma\left(t_1' + vx'/c^2\right) \quad t_2 = \gamma\left(t_2' + vx'/c^2\right)$$

$$t_2 - t_1 = \gamma\left(t_2' - t_1'\right)$$

$$\Rightarrow \boxed{\Delta t = \gamma \Delta t_0}$$

$$\Rightarrow \Delta t > \Delta t_0$$

Thus the time interval measured in the frame S is larger than time interval in frame S'. This effect is called time dilation.