Sphere:- A sphere is the locus of a point, which moves so that its distance from a fixed point is constant
(i) The fixed point is called the centre of the plane.
(ii) The constant distance is called the radius of the sphere.

Equation of the sphere (Standard Form):-

The Equation of sphere whose centre is (0,0,0) and radius r, is

$$\boxed{x^2+y^2+z^2=r^2}$$

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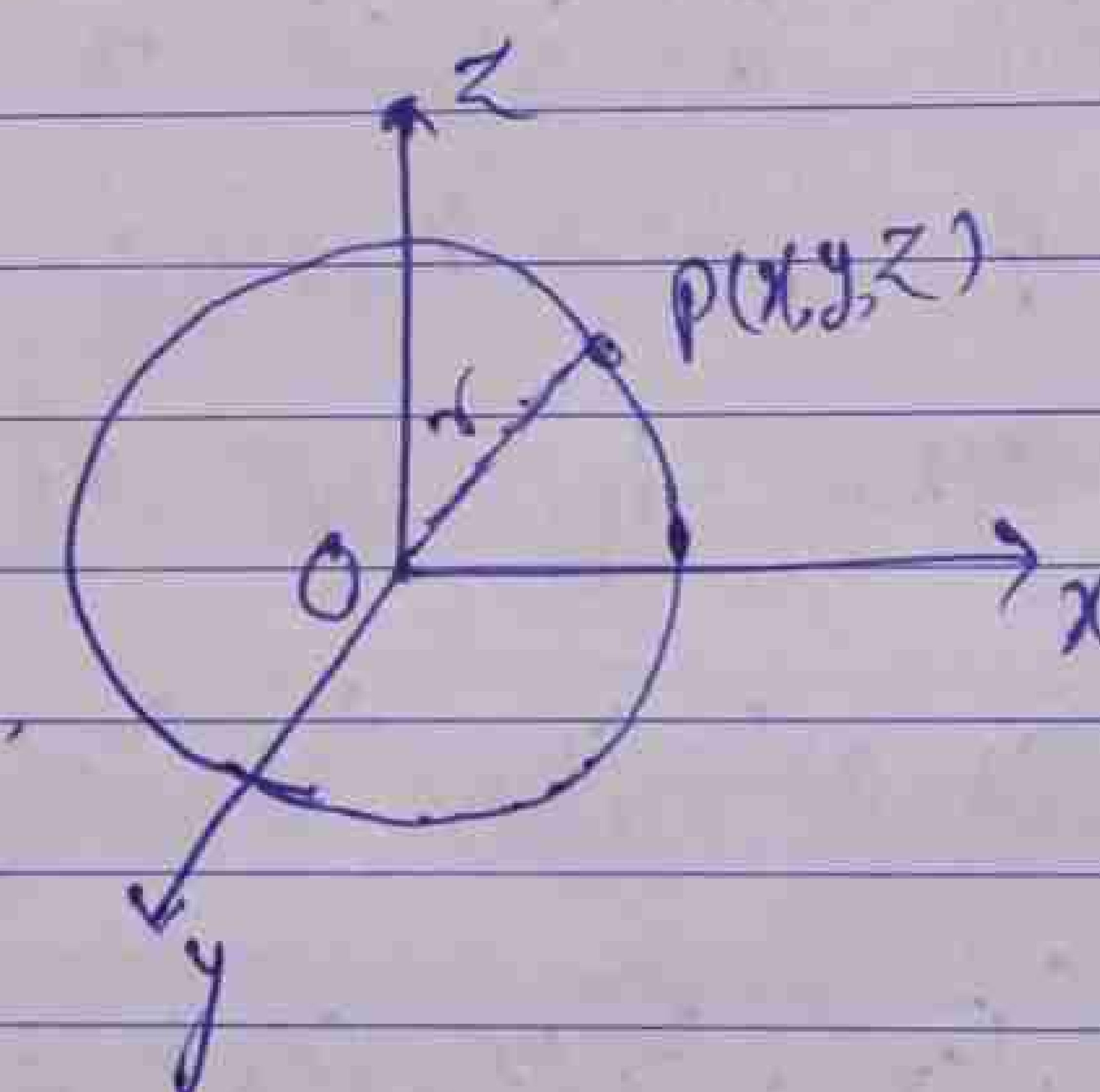


Proof: Let $P(x,y,z)$ be any point on the sphere with centre $O(0,0,0)$,

Then by definition,

$OP = r$ (radius of sphere).

$$\Rightarrow \sqrt{(x-0)^2+(y-0)^2+(z-0)^2} = r$$

$$\Rightarrow \sqrt{x^2+y^2+z^2} = r$$

$$\Rightarrow \boxed{x^2+y^2+z^2=r^2}$$

Which is the required Equation of the sphere is and called standard form.

Central form of Equation of the sphere:- The Equation

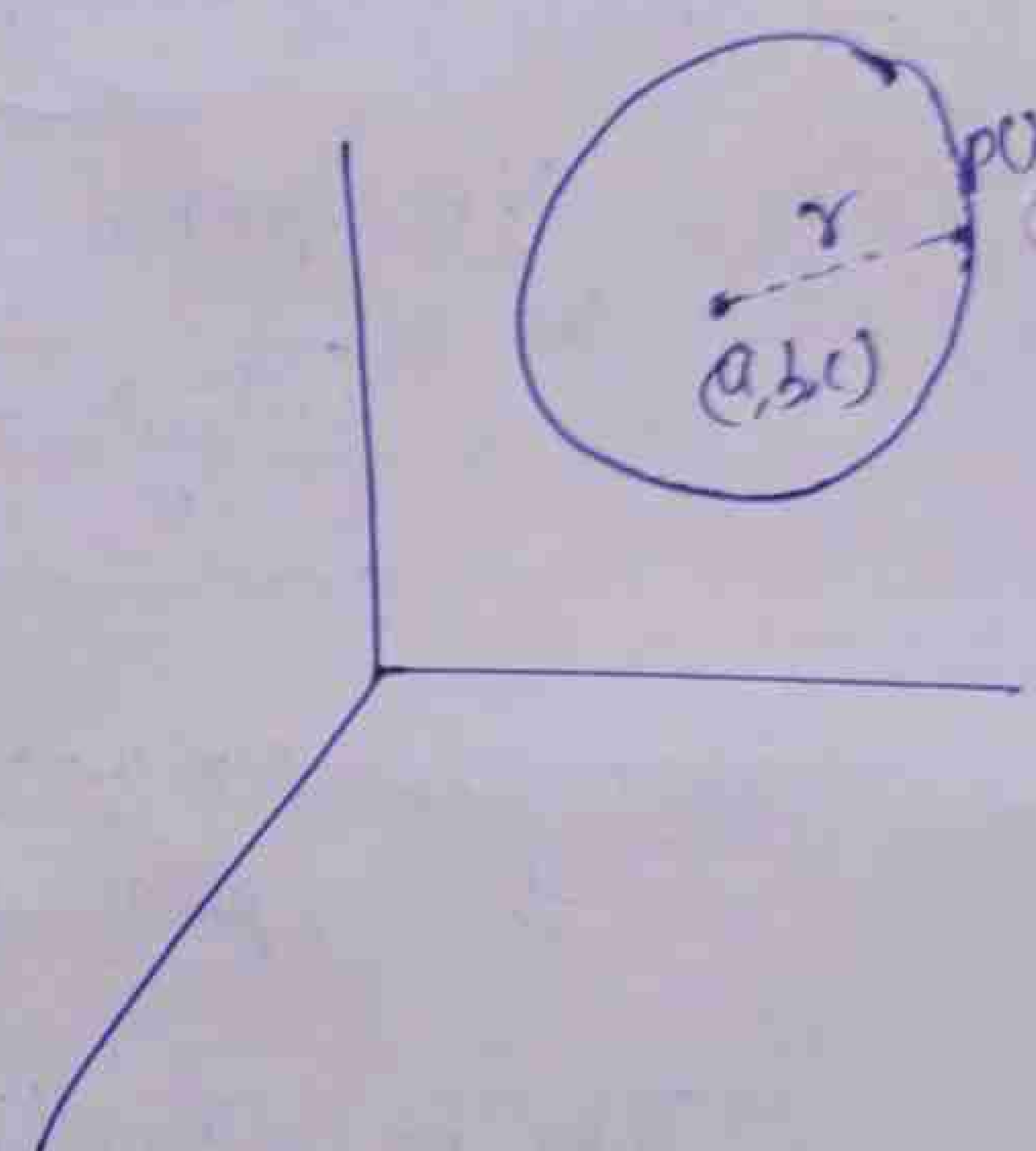
of a sphere, whose centre is (a, b, c) and radius r is

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

OR

$$x^2+y^2+z^2-2ax-2by-2cz+(a^2+b^2+c^2-r^2)=0.$$

Proof:- Let $P(x, y, z)$ be any point on the sphere,



Let $Q(a, b, c)$ be the given centre of sphere having radius r,

Then by defn of sphere.

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$$\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}=r$$

$$\Rightarrow \left(\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}\right)^2=r^2$$

$$\Rightarrow \boxed{(x-a)^2+(y-b)^2+(z-c)^2=r^2}$$

$$\Rightarrow x^2+a^2-2ax+y^2+b^2-2by+z^2+c^2-2cz=r^2$$

$$\Rightarrow \boxed{x^2+y^2+z^2-2ax-2by-2cz+(a^2+b^2+c^2-r^2)=0}$$

Proved

<u>General Form of the Equation of Sphere:-</u> The General form of Equation is,

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

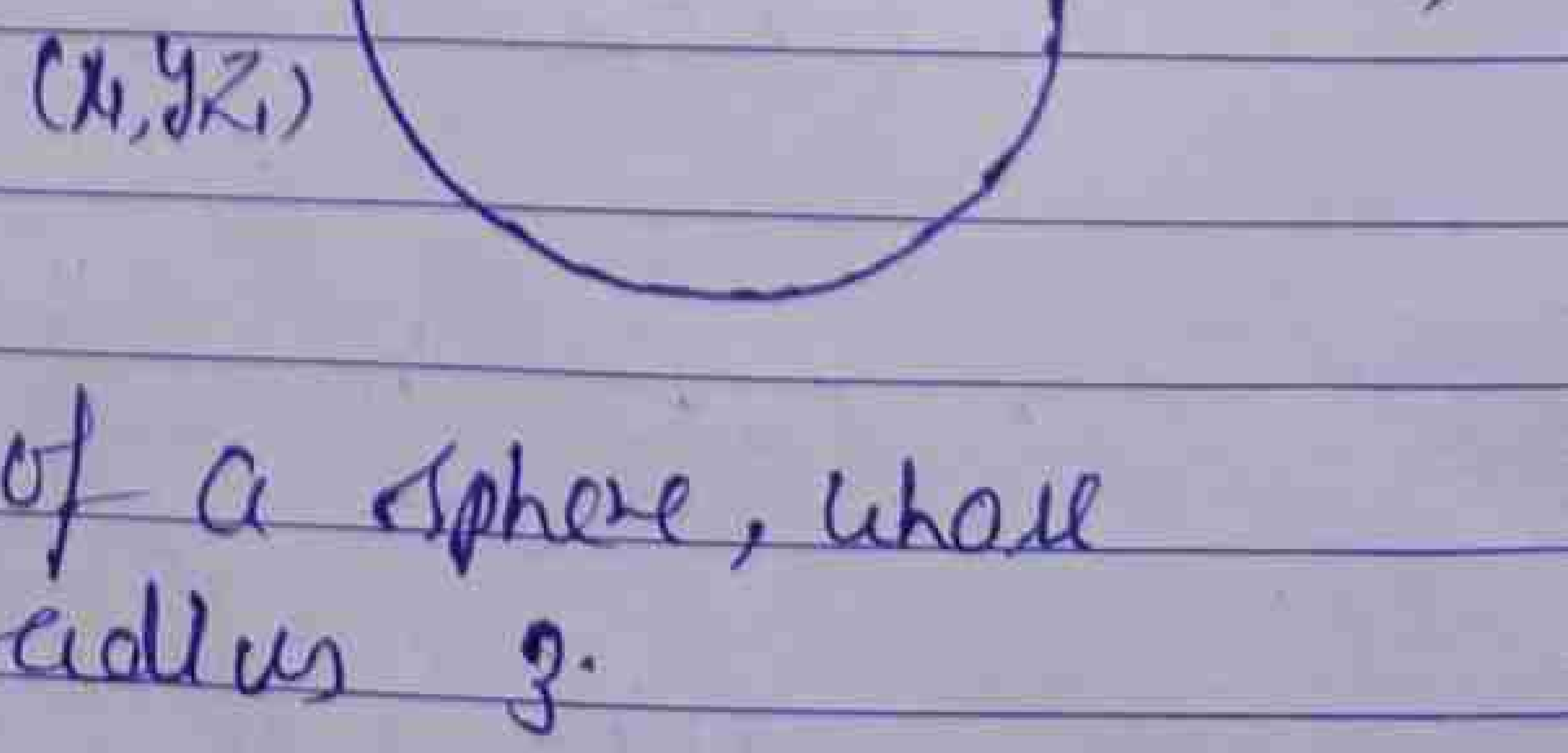
where

$$u=-a,\ v=-b,\ w=-c \text{ and } d=a^2+b^2+c^2-r^2.$$

Here, centre is $(-u,-v,-w)$
and radius $\sqrt{u^2+v^2+w^2-d} = \sqrt{u^2+v^2+w^2-d}$

<u>Diameter form of the Equation of Sphere:-</u> The Equation of the sphere on the joining of (x_1,y_1,z_1) and (x_2,y_2,z_2) as the extremities of a diameter.

$$\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0}$$



Example:- Find the Equation of a sphere, whose centre is $(2,-3,4)$ and radius 3.

Solution:- we know equation of sphere whose centre is (a,b,c) and radius r is

$$(x-a)^2+(x-b)^2+(y-c)^2=r^2$$

$\therefore$ Equation of sphere whose centre is $(2,-3,4)$ and radius 3 is

$$(x-2)^2+(y+3)^2+(z-4)^2=9$$

$$\Rightarrow x^2+4-4x+y^2+9+6y+z^2+16-8z=9$$

$$\Rightarrow \boxed{x^2+y^2+z^2-4x-6y-8z+20=0}$$

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Example: Find the centre and radius of the sphere given by,

(i) $x^2+y^2+z^2-2x-4y-6z+4=0$ (ii) $2x^2+2y^2+2z^2-2x+4y+2z+3=0$

Solution:- (ii) $2x^2+2y^2+2z^2-2x+4y+2z+2=0$

i.e $\Rightarrow x^2+y^2+z^2-x+2y+z+\frac{2}{2}=0.$

Comparing it with $x^2+y^2+z^2+2ux+2vy+2wz+d=0.$

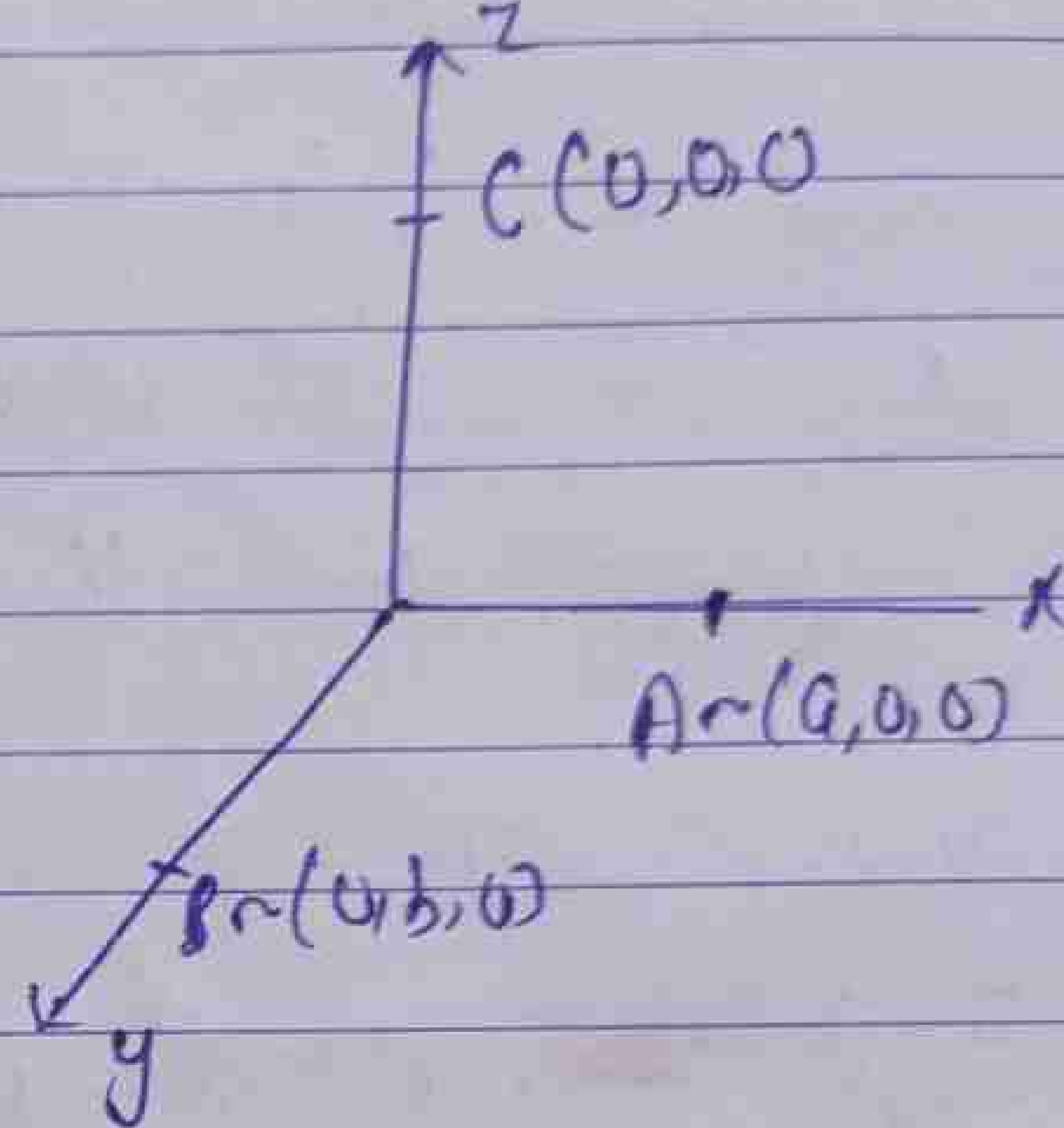
we have $u=-\frac{1}{2}$, $v=1$, $w=\frac{1}{2}$, $d=2/2=1$

$\therefore$ centre is $\left(\frac{1}{2}, -1, -\frac{1}{2}\right)$ and

radius $=\sqrt{\frac{1}{4}+1+\frac{1}{4}-1} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Ans

Example:- Find the Equation of the sphere which passes through the origin and meets the axes in $A\sim(a,0,0)$, $B\sim(0,b,0)$ and $C\sim(0,0,c)$.



Solution:- Let the Equation of curve sphere.

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0.$$

Since it is passes through the origin $(0,0,0)$.

$$\Rightarrow \boxed{d=0}$$

Also it is passes through $A\sim(a,0,0)$.

$$\Rightarrow a^2+2ua+d=0 \qquad \Rightarrow a^2+2ua=0$$

$$\Rightarrow u=-a/2$$

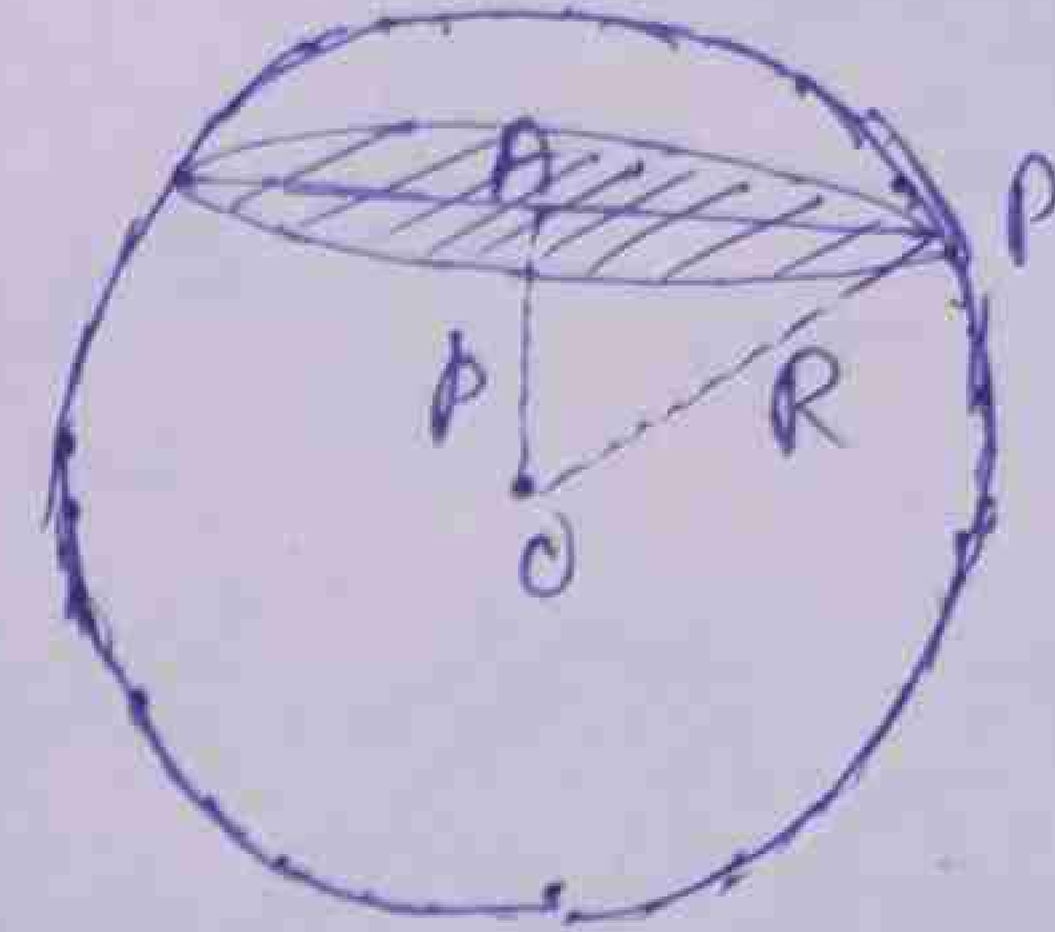
Similarly it is passes through B and C

$$\Rightarrow v=-b/2, \quad w=-c/2.$$

Hence, required Equation of sphere

$$\boxed{x^2+y^2+z^2-ax-by-cz+0=0}$$

Plane Section of a Sphere:– Suppose that the sphere and the plane intersect and have common points. The set of points ~~common~~ common to the sphere and the plane (when they intersect) is called the plane section of a sphere and it is ~~cir~~ a circle.



Now, $AP^2 = OP^2 - OA^2$

$\Rightarrow AP^2 = R^2 - p^2$ ie $\boxed{AP = \sqrt{R^2 - p^2}}$

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Hence the locus of P is a circle, whose centre is the point A and radius $= \sqrt{R^2 - p^2}$.

Equation of a circle:– We have shown that the section of a sphere by a plane is a circle.

Thus two Equation, one of a sphere and other of a plane, together represent a circle.

ie $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ (sphere) —(1)

$Ax + By + Cz + D = 0$ [plane] —(2)

These two Equation (1) and (2) together represent a circle.

(a) The centre of circle is the foot of the perpendicular from the centre of the sphere on the plane of the circle ie point A in figure.

(b) The radius of the circle $r_1 = \sqrt{R^2 - p^2}$
where
R = radius of sphere
p = OA (perpendicular from O on the plane).

<u>Example</u> Find the centre and radius of the section of the sphere $x^2+y^2+z^2 = 25$ by the plane $2x+y+2z=9$

<u>Solution</u>: The Given sphere is $x^2+y^2+z^2 = 25$
its centre is $O(0,0,0)$ and radius $= 5$ — (1)

The Equation of plane is $2x+y+2z=9$
$\Rightarrow 2x+y+2z-9=0$ — (2)

Equation (1) and (2) taken together represent a circle, whose centre is the foot A of the perpendicular drawn from O (centre of the sphere) to the plane (2).

Now Equation of perpendicular (OA) are
$$\frac{x-0}{2} = \frac{y-0}{1} = \frac{z-0}{2} = r \text{ (say)}$$
$$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{z}{2} = r \Rightarrow x = 2r,\ y = r,\ z = 2r$$
$\therefore$ Any point on this line is $(2r, r, 2r)$ which is A.

But A lies on the sphere (2).
$\Rightarrow 2(2r)+r+2(2r) = 9 \Rightarrow \boxed{r=1}$

Thus the point A is $(2, 1, 2)$, which is the centre of the circle.

Now $p = OA =$ ~~per~~ perpendicular from $O(0,0,0)$ on the plane (2)

$$= \frac{0+0+0-9}{\sqrt{4+1+4}} = \frac{-9}{3} = -3 = 3 \text{ (Numerically)}$$

$\therefore$ Radius of circle $= AP = \sqrt{R^2-p^2} = \sqrt{25-9} = 4$

Hence, the centre of circle is $(2,1,2)$ and radius is 4.

<u>Intersection of two sphere</u>:- Let the two sphere be

$$x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0 \quad \text{— (1)}$$

$$x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0 \quad \text{— (2)}$$

The points common to (1) and (2) will also lie on the surface

$$\Rightarrow \cancel{x^2}+\cancel{y^2}+\cancel{z^2}+2u_1x+2v_1y+2w_1z+d_1-(\cancel{x^2}+\cancel{y^2}+\cancel{z^2}+2u_2x+2v_2y+2w_2z+d_2)=0$$

$$\Rightarrow \boxed{2(u_1-u_2)x+2(v_1-v_2)y+2(w_1-w_2)z+(d_1-d_2)=0} \quad \text{— (3)}$$

which is a plane

Thus the common point of (1) and (2) are also the common points (1) and (3) [or (2) and (3)] and hence they lie on a circle.