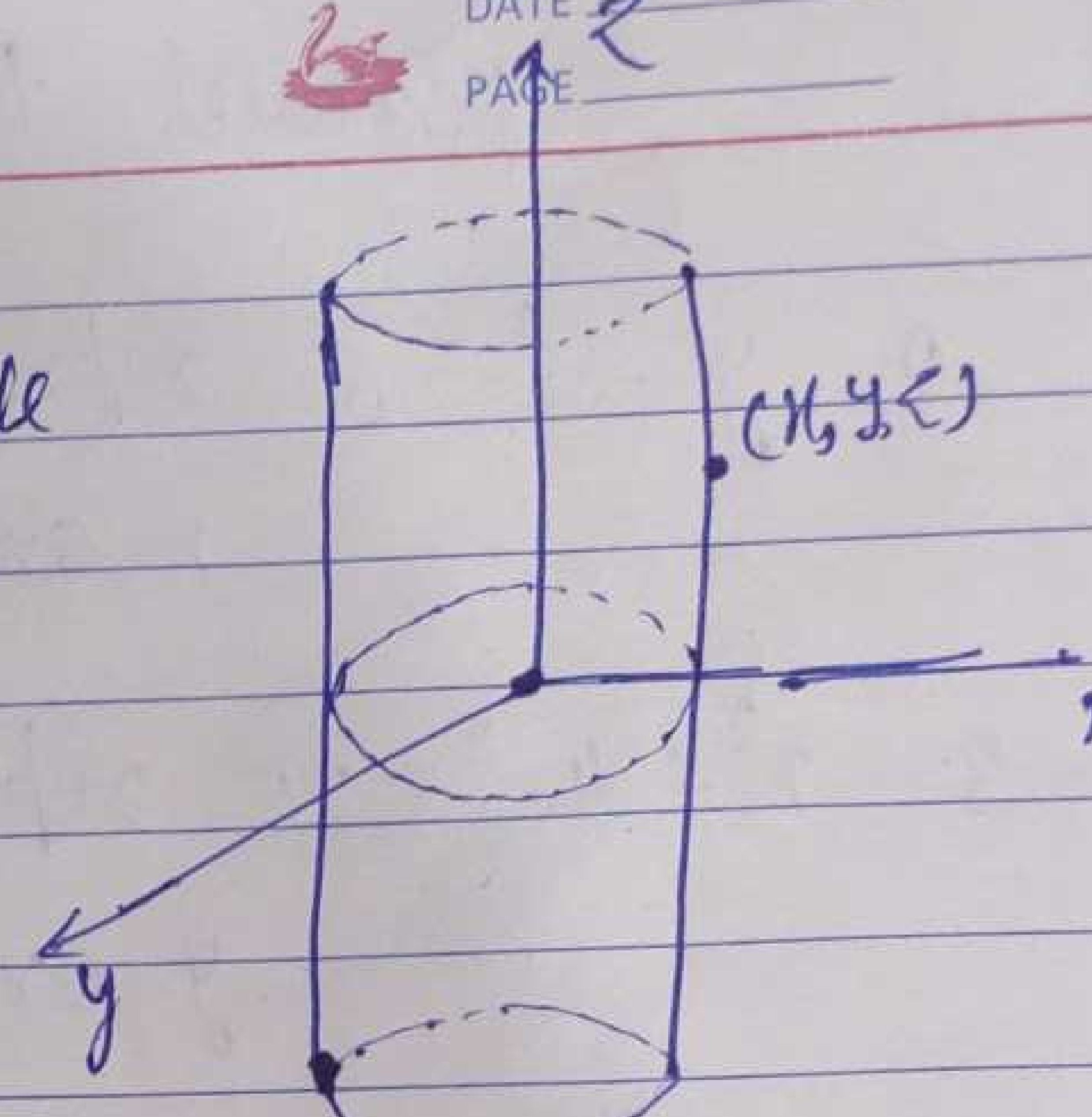


Cylinder:- In 2-D space, the graph of Equation $x^2+y^2=1$, is a circle centered at origin in x-y plane. However in 3-D space we can interpret the graph of the set

$$\{(x,y,z): x^2+y^2=1, z \text{ is arbitrary}\}$$

as a surface which is a right circular cylinder as shown in figure

Definition:- If C is a two dimension curve and L is line not parallel to the plane that contains C, then a cylinder is a three dimensional surface containing of all line passing through C that are parallel to L.
ie

Any curve $f(x,y)=c$ in the plane (2D-plane) defines a cylinder parallel to the z-axis whose 3D-space equation is also $f(x,y)=c$.

In a similar way, any curve $g(x,z)=c$ in xz plane defines a cylinder parallel to the y-axis whose 3D-space equation is also $g(x,z)=c$ and so on.

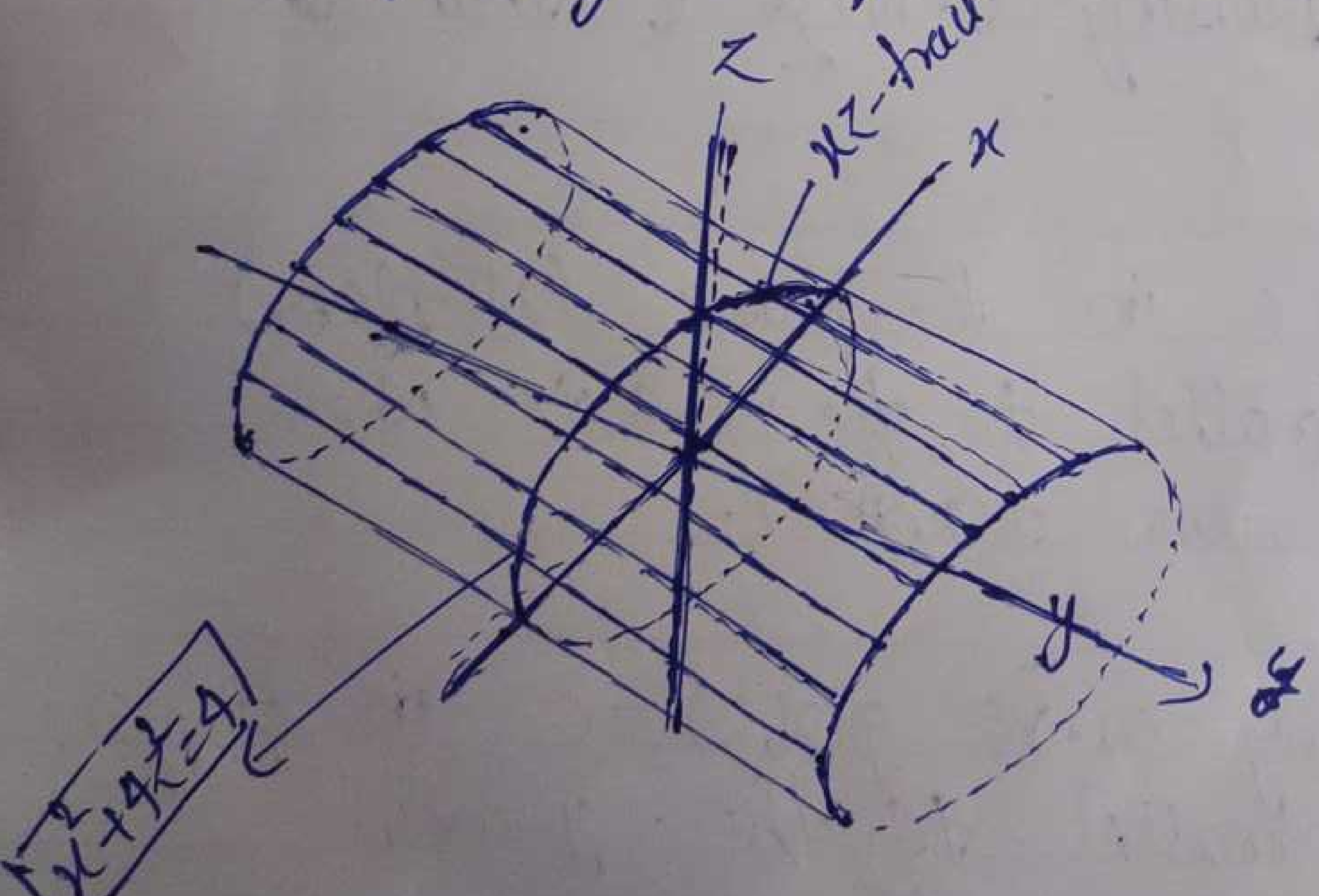
Example 1. $x^2+y^2=9$ represent a circular cylinder parallel to z-axis.

2. $y^2-z^2=5$ represent a hyperbolic cylinder parallel to x-axis

3. $x^2+4z^2=25$ represents an elliptic cylinder parallel to y-axis

Example Sketch the graph of $x^2+4z^2=4$ in 3-Space.

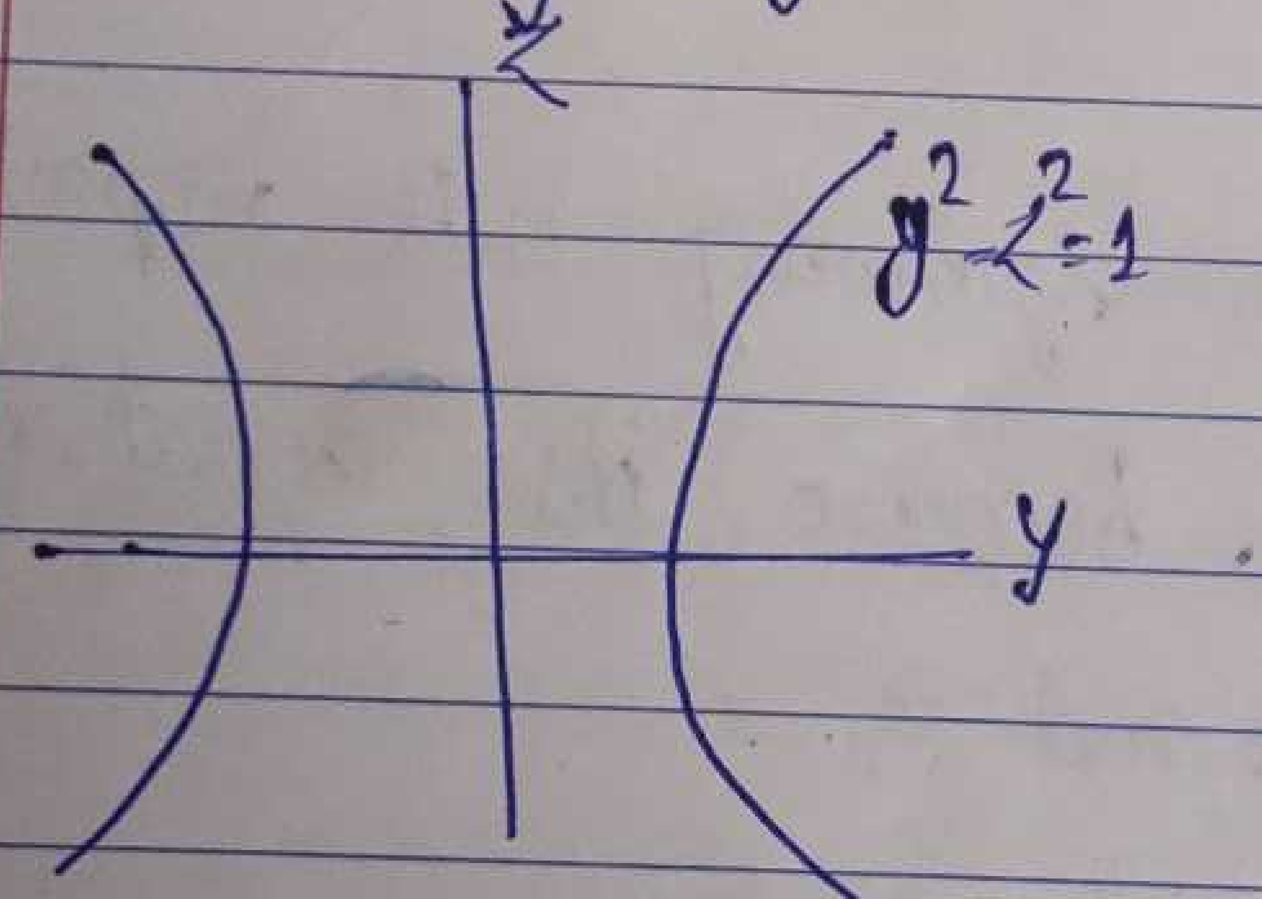
Solution- Equation of curve $x^2+4z^2=4$ in 2-D is a Ellipse. Since in the equation y-variable is missing. therefore $x^2+4z^2=4$ is elliptic cylinder made of lines parallel to the y-axis and passing through the Ellipse $x^2+4z^2=4$ in x-z plane



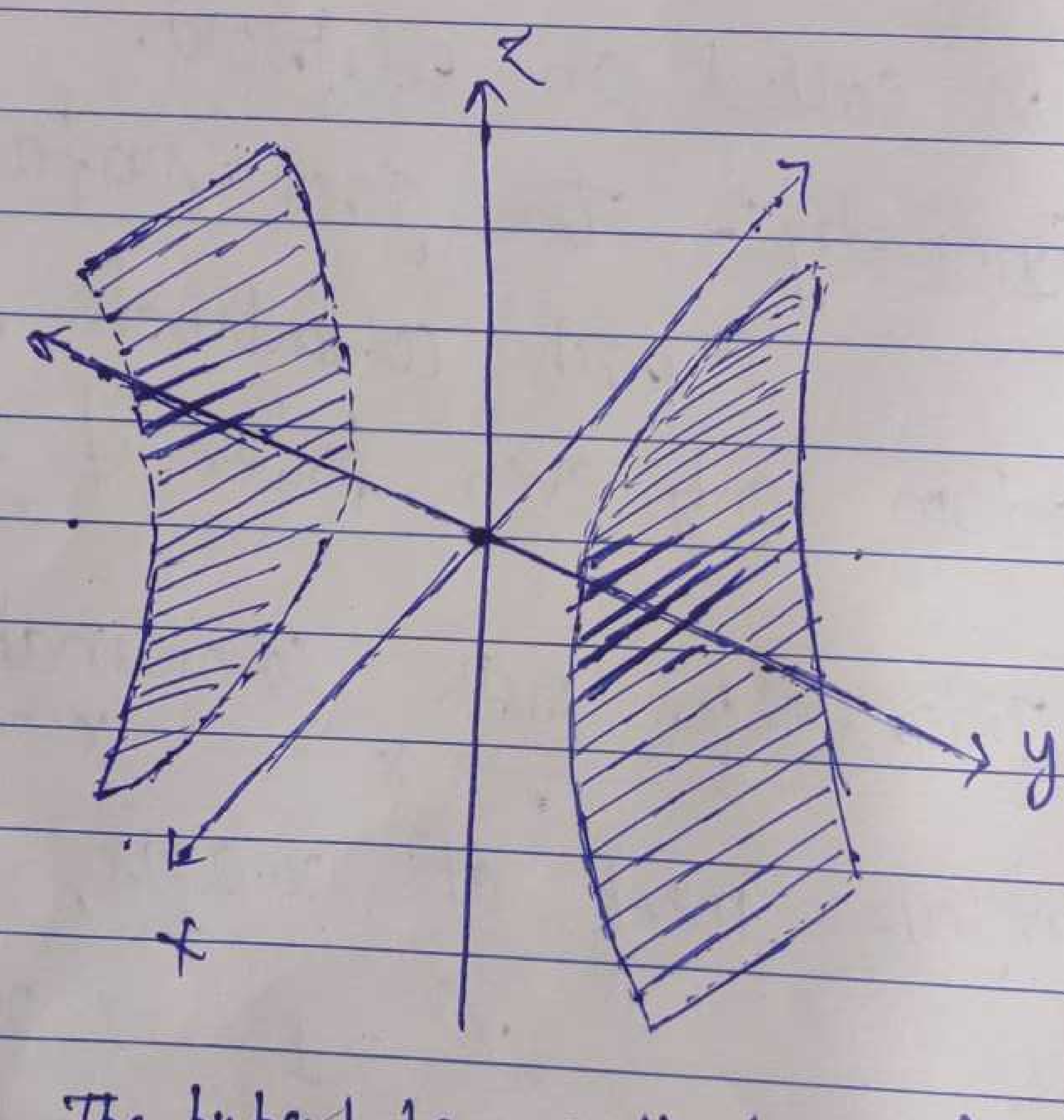
Example 2:- Sketch the graph of hyperbolic cylinder $y^2-z^2=1$

Solution:- Since in the given Equation of hyperbolic cylinder, $y^2-z^2=1$ x-variable is missing, therefore the hyperbolic cylinder $y^2-z^2=1$ is made of line of parallel to x-axis and passing through the hyperbola. $y^2-z^2=1$ in yz plane.

Sketch the cross-section or traces of the cylinder in parallel plane on either side of the generating hyperbola as shown in following figure



(i) yz-trace is hyperbola



The hyperbola cylinder $y^2-z^2=1$

Quadric Surface :- A quadric surface is the graph of a second degree equation in three variable. The general form of such an equation is,

$$Ax^2+By^2+Cz^2+Dxy+Eyz+Fxz+Gx+Hy+Iz+J=0$$

where A, B, C -- J are constant

There are six type of quadric surface

To trace a Ellipsoid :- The graph of an equation of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $a,b,c>0$

is called an ellipsoid.

(i) Symmetry :- The given surface is symmetry with respect to each co-ordinate plane. because its equation contain only even power of x, y. and z.

(ii) Intersection with co-ordinate axis :- Putting $y=0$, $z=0$, the surface meet at x-axis, where

$$\frac{x^2}{a^2}=1 \Rightarrow x^2=a^2 \Rightarrow x=\pm a$$

i.e at the point $(\pm a, 0, 0)$

Similary it meets y-axis at point $(0, \pm b, 0)$ and z-axis at point $(0, 0, \pm c)$

(III) <u>Trace in the co-ordinate planes</u>:- The trace of the ellipsoid in xy-plane, obtained by putting $z=0$, is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$ which is ellipse in that plane.

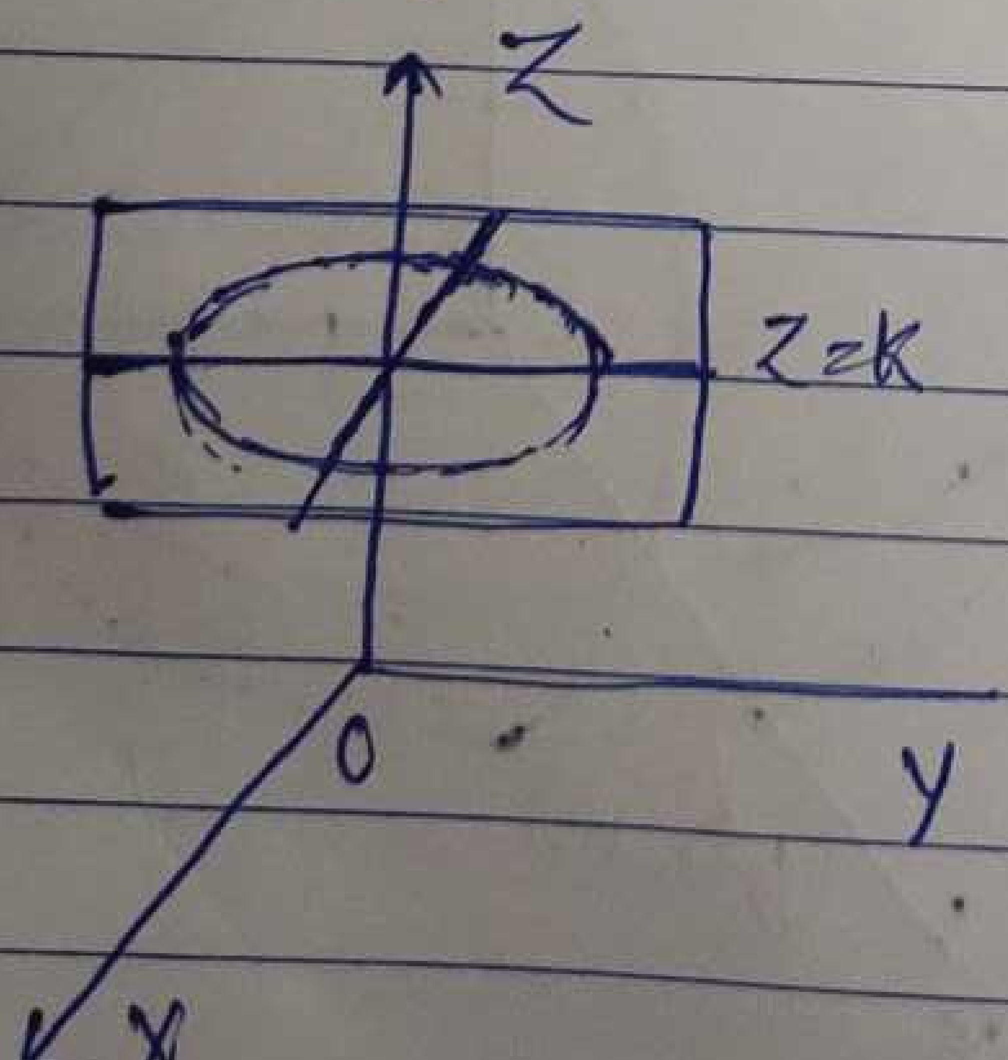
Similarly its yz and xz trace are the Ellipses

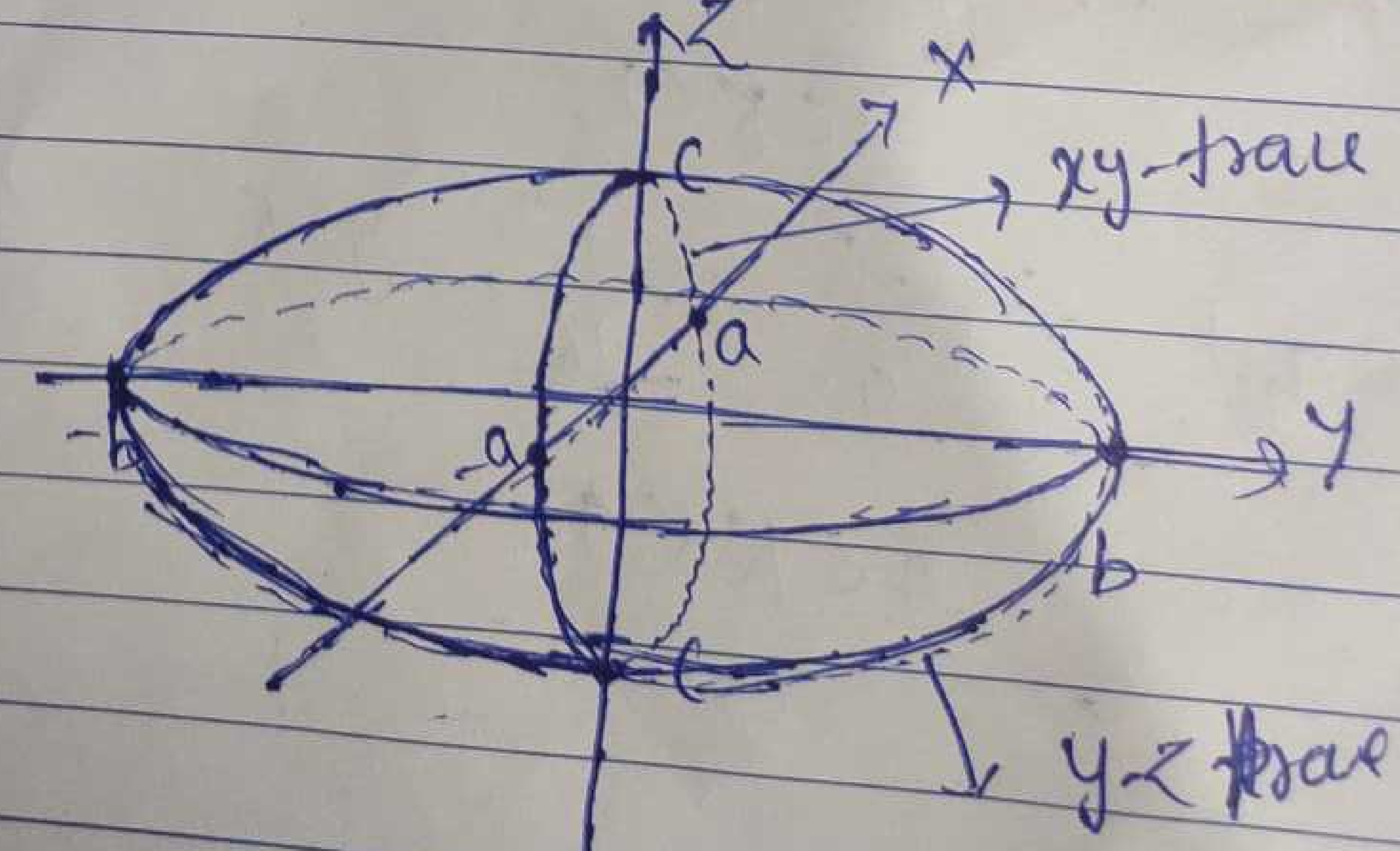
$$\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ and $$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1.$$ respectively.

(IV) <u>Trace in planes parallel to co-ordinate planes</u>:- The trace of the surface in the horizontal plane $z=k$ where $-c<k<c$ is a variable ellipse.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1-\frac{k^2}{c^2}$$

Similary, its trace in the plane $x=k\ (-a<k<a)$ and $y=k\ (-b<k<b)$ are variable ellipse.





Example Sketch the Ellipsoid $x^2+\frac{y^2}{16}+\frac{z^2}{9}=1$

Solution The Given Equation is $x^2+\frac{y^2}{16}+\frac{z^2}{9}=1.$

which represents an ellipsoid.

Put $y=0, z=0,$ $\Rightarrow x^2=1 \Rightarrow \boxed{x=\pm 1}$

Put $x=0, y=0,$ i.e. $z^2=9 \Rightarrow \boxed{z=\pm 3}$

Put $x=0, z=0$ i.e. $y^2=16 \Rightarrow \boxed{y=\pm 4}$

The trace in x-y plane $(z=0)$ is $x^2+\frac{y^2}{16}=1$, which is Ellipse

Similarly trace in xz and yz-plane are also Ellipses.

The horizontal trace in the plane $z=k$ (parallel to xy-plane) is a variable ellipse.

$\Rightarrow \quad x^2+\frac{y^2}{16}=1-\frac{k^2}{9}, \quad (z=k)$

such that $k^2<9$, i.e. $3<k<3$.

Similarly, $\frac{y^2}{16}+\frac{z^2}{9}=1-k^2, \; x=k$

$-1<k<1$

and $x^2+\frac{z^2}{9}=1-\frac{k^2}{16}, \; y=k$

$4<k<4$