[This question paper contains 8 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : **9502A** **HC**

Unique Paper Code : 32227626

Name of the Paper : Classical Dynamics

Name of the Course : **B.Sc. (Hons.) Physics – DSE – 4**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all including question **1** which is compulsory.
3. Attempt at least **one** question from Section **A** and **B.**

1. Attempt any **five** of the following : (3×5=15)

(a) In charge particle dynamics, what is the basic role of electric field and that of magnetic field ?

(b) A particle of mass m is constrained to move on the boundary of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Identify the generalized coordinate(s) of this system.

(c) The Lagrangian of a particle of mass m moving in a plane is given by

$L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) - a(x\dot{y} - y\dot{x})$, a is some positive constant.

Determine the canonical momenta of this particle.

(d) Components of 4-vector A^{μ} are

$A^0 = 4$, $A^1 = 3$, $A^2 = 2$, $A^3 = 1$.

Determine whether this 4-vector is time-like, space-like or null.

(e) The rest length of rod is 2 m. Using 4-vectors or otherwise, determine the length of this rod when it is moving along its length with a velocity equal to $\sqrt{\pi}\, c/2$.

(f) Using four vector approach or otherwise, show that it is impossible for an isolated free electron to absorb a photon.

(g) Consider a motion of a particle in plane polar coordinates (r, θ) in the presence of a radial potential, say $V(r)$. Identify the conserved quantitie(s).

(h) What is Reynold's number and give its importance ?

SECTION A

2. (a) Using Hamilton's equations and the relation

$L = p_i \dot{q}_i - H(q_i, p_i)$, where $L = L(q_i, \dot{q}_i)$ and $i = 1, 2, \ldots, n$..

Prove that $p_i = \frac{\partial L}{\partial \dot{q}_i}$ and $\dot{p}_i = \frac{\partial L}{\partial q_i}$. (8)

(b) Two masses m_1 and m_2 (such that $m_1 < m_2$) are connected by a massless and inextensible string in Atwood's machine. Using Lagrangian approach, find the acceleration of either mass.

(Assume that pulley is massless and frictionless) (7)

3. (a) If a system with potential energy V has kinetic energy T of the form of a symmetric quadratic homogeneous function :

$$T = \sum_{ij} a_{ij} \dot{q}_i \dot{q}_j$$

show that the Hamiltonian can be written as

$$H = T + V \qquad (5)$$

(b) A particle of mass m moves under the influence of a central force whose potential is

$$V(r) = -k/r.$$

Determine the Hamiltonian and the Hamilton's equations of motion. Are the angular momentum and energy of the system are integrals of motion? (10)

4. (a) A charged particle (mass m, charge q) is moving in x-y plane. At time $t = 0$, this particle is at the origin having velocity v_0 at an angle 30^0 with the x-axis and uniform parallel electric and magnetic fields: $\vec{E} = E_0 \hat{z}$ and $\vec{B} = B_0 \hat{z}$ are switched on. Find its velocity at time $t > 0$. (9)

(b) Determine the Euler-Lagrange equations of motion of 'spring pendulum': a particle of mass m attached to an elastic massless spring of stiffness k and unstretched length L_0.

(Assume that the motion takes place in a vertical plane) (6)

5. Two blocks and three springs are configured as in figure (1) below. These blocks can execute longitudinal simple harmonic oscillations only. When the blocks are at rest, all springs are unstretched.

(a) Choosing the displacement of each block from its equilibrium position as generalized coordinates, write the Lagrangian of this system.

(b) Find the T (Kinetic energy) and V (potential energy) matrices.

(c) Suppose $m_1 = 2m$, $m_2 = m$, $k_1 = 4k$, $k_2 = k$, $k_3 = 2k$. Find the frequencies of small oscillations. (5,5,5)

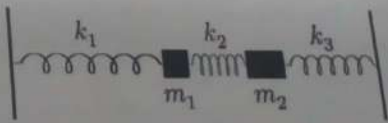

Fig. (1)

SECTION B

6. (a) Prove that 4-dimensional volume element is invariant to Lorentz transformation. (5)

(b) Axes of two inertial frames S and S' are parallel. If L(x) denotes Lorentz transformation matrix when S' moves with speed v with respect to S along x-x' axis, and L(y) be the transformation matrix when S' moves with speed v with respect to S along y-y' axis then determine whether L(x) L(y) is equal to L(y) L(x) or not. (10)

7. (a) Explain the phenomenon of length contraction using Minkowski space-time diagram. (7)

(b) Two events separated by a distance 30 m and time interval 5×10^{-8} s occur in frame S. Find the distance and time separation between the events as observed from S' moving with speed c/5 along the line of occurrence of the events. (8)

8. (a) A rod of rest length L_0 is lying parallel to x-axis in an inertial frame S. Suppose now rod starts moving with velocity 'u' along its length in frame S. Using 4-vector approach or otherwise, determine the length of this rod in frame S' (moving with velocity 'v' with respect to S along x-x' axis). (8)

(b) Two electrons are approaching each other, each with energy $\gamma m_0 c^2$ in the laboratory frame. Using 4-vector approach or otherwise, find the energy of one electron in the rest frame of the other. (7)

9. (a) An observer in S measures the proper length (L_0) of the rod making an angle θ_0 with the x-axis. Using 4-vector approach or otherwise, determine the length of the rod and the angle which it makes with the x'-axis as measured by an observer in S' moving with speed v relative to S along the positive x-axis. (8)

(b) If transformation from inertial frame S to S′ is given by L(β) and from S′ to S″ by L(β′), then prove that transformation from S to S″ is given by L(β″).

where $$L(\beta) = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$L(\beta') = \begin{pmatrix} \gamma' & -\gamma'\beta' & 0 & 0 \\ -\gamma'\beta' & \gamma' & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$L(\beta'') = \begin{pmatrix} \gamma'' & -\gamma''\beta'' & 0 & 0 \\ -\gamma''\beta'' & \gamma'' & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Also,

$$\gamma = \frac{1}{\sqrt{1-\beta^2}},\ \gamma' = \frac{1}{\sqrt{1-\beta'^2}},\ \gamma'' = \frac{1}{\sqrt{1-\beta''^2}} \text{ and } \beta'' = \frac{\beta+\beta'}{1+\beta\beta'}.$$

(7)

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S. No. of Question Paper : 2856

Unique Paper Code : 32227626 IC

Name of the Paper : **Classical Dynamics**

Name of the Course : **B.Sc. (Hons.) Physics-DSE-4**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all including Question No. 1 which is compulsory.

Attempt at least *one* question from Sections A and B each.

1. Attempt any *five* of the following : 5×3=15

(*a*) Write the Lagrangian and Hamiltonian of a free particle.

(*b*) Axes of two inertial frames S and S' are parallel. If S' moves with speed v with respect to S along y-y' axis, then write the Lorentz transformation matrix.

(*c*) Show that in units such that $c = 1$, the 4-velocity is given by $U^\mu = \gamma(1, u)$ where u is the 3-velocity.

P.T.O.

(d) A particle of mass m moves under a force $F = \frac{-A}{r^2}$. Using its equation of motion show that the total energy is conserved.

(e) Differentiate between streamline flow and laminar flow.

(f) Calculate the gyro-radius of the circular motion of a charged particle (charge q and mass m) in the presence of a uniform magnetic field, $\vec{B} = B_0 \hat{z}$.

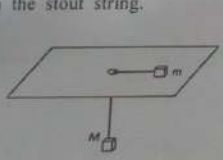
(g) In the context of the Lorentz transformation, considering two boosts along x-axis, show that the rapidity is additive.

(h) Prove that $A^{\mu}B_{\mu} = A_{\mu}B^{\mu}$, where $\mu = 0, 1, 2, 3$.

SECTION A

2. (a) Starting with the Lagrangian of a conservative system show that $\Sigma_j p_j \dot{q}_j - L$ is a conserved quantity. 5

(b) Two masses m and M are connected by a massless string of total length l. The mass m is free to rotate on a frictionless plane as shown. The string from mass m leads through a hole in the plane to the other mass M which hangs from the stout string.



Determine the Lagrangian and the Euler-Lagrange equations of the system. 4+6

3. (a) If two capillaries of radii r_1, r_2 and length L_1, L_2 respectively are joined in series, then derive the expression for the rate of flow of liquid through this arrangement using Poiseuille's formula. 5

(b) A particle of mass m is constrained to move on the surface of a cylinder of radius R. The particle is subjected to a force directed towards the origin and proportional to the distance of the particle from the origin $\vec{F} = -k\vec{r}$. Determine the Hamiltonian and the Hamilton's equations of motion. 6,4

4. (a) Give any *two* advantages of Lagrangian mechanics over Newtonian mechanics. 3

(b) Consider a particle of mass m moving in a central field of force with potential V(r). Determine the Lagrangian and the Euler-Lagrange equations. Show that the motion of the particle in the radial direction is in an effective potential of $l^2/2mr^2 + V(r)$ where l is a constant of motion. 3,5,4

5. (*a*) A particle of mass m is in a potential $V(x) = [-(1/2)ax^2 + (1/4)bx^4]$ where a and b are positive constants. Obtain the equilibrium position and the frequency of small oscillations. 3,3

(*b*) A charged particle (mass m, charge q) is moving in xy plane. At time $t = 0$, this particle is at the origin having velocity v_0 at an angle $60°$ with the x-axis and uniform crossed electric and magnetic fields : $\vec{E} = E_0\hat{x}$ and $\vec{B} = B_0\hat{z}$ are switched on. Find its velocity at time $t > 0$. 9

SECTION B

6. (*a*) Two rods having the same length L_0 move lengthwise towards each other parallel to a common axis with the same velocity 'v' relative to the Laboratory frame. Using 4-vectors or otherwise show that the length of one rod with respect to the other rod is

$$L = L_0 (1 - \beta^2)/(1 + \beta^2),\ \beta = v/c.$$ 8

(*b*) A particle of rest mass m_0 describes a circular path with parametric equations

$$x = a \cos t,\ y = a \sin t,\ z = 0$$

in an inertial frame S. Find the four velocity components. Also find the norm of the four-velocity. 7

7. (*a*) Starting from 4-displacement (x^μ), show that the four-momentum (p^μ) is orthogonal to the four-force (F^μ) vector. 7

(*b*) Find the norm of the 4-force vector ($F^\mu F_\mu$) if the angle between 3-velocity and 3-force is θ. 8

8. (*a*) With the help of space-time diagram determine the length of a metre stick measured by an observer at rest, when the stick is moving along its length with a velocity equal to $\sqrt{3}c/2$. 7

(*b*) If A^μ is a time-like 4-vector and $A^\mu B_\mu = 0$, then prove that B^μ is space-like. 5

(*c*) Using 4-vector approach or otherwise, show that $\frac{dt}{d\tau} = \gamma$, where the symbols have their usual meanings. 3

9. (*a*) Consider two-body decay of an unstable particle of mass M initially at rest. If the daughter particles (of rest masses m_1 and m_2) have energies E_1 and E_2 and back-to-back momentum magnitude p in the rest frame of the mother particle, then in units such that $c = 1$, show that (using 4-vectors or otherwise).

$p = (1/2M)\,[(M^2 - (m_1 + m_2)^2)\,(M^2 - (m_1 - m_2)^2]^{1/2}$,

$E_1 = (1/2M)\,[M^2 + m_1^2 - m_2^2]$,

$E_2 = (1/2M)\,[M^2 + m_2^2 - m_1^2]$ 9

(*b*) Two spaceships A and B are leaving the earth along perpendicular directions. If A is observed by a stationary earth observer to have velocity $u_y = 0.9c$ and B to have velocity $u_x = 0.8c$, using 4-vectors or otherwise, determine the speed of ship A with respect to ship B. 6