

$$K_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1h)$$

Ralston's Method:- The Ralston method is given as

$$y_{i+1} = y_i + \left(\frac{1}{3}K_1 + \frac{2}{3}K_2\right)h$$

where $K_1 = f(x_i, y_i)$

$$K_2 = f\left(x_i + \frac{3}{4}h, y_i + \frac{3}{4}K_1h\right)$$

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Question: Given the equation $\frac{dy}{dx} = \frac{2y}{x}$, with $y(1) = 2$
Estimate $y(2)$ using Midpoint
Ralston Method with $h = 0.25$.

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Solution: Given $f(x, y) = \frac{2y}{x}$, $x_0 = 1$, $y_0 = 2$, $h = 0.25$

Iteration 1:- $K_1 = h f(x_0, y_0) = (0.25) f(1, 2) = 1$

$$K_2 = f\left(x_0 + \frac{3h}{4}, y_0 + \frac{3}{4}K_1\right)$$

$$= f(1.1875, 2.75) = 4.632$$

$$\text{Now } y_1 = y_0 + \frac{h}{3} (K_1 + 2K_2)$$

$$= 2 + \frac{(0.25)}{3} (1 + 2(4.632)) = 2.855$$

Iteration 2:- $x_1 = 1.25, y_1 = 2.855$

$$K_1 = h f(x_1, y_1) = (0.25) f(1.25, 2.855) = 1.142$$

$$K_2 = f(x_1 + \frac{3}{4}h, y_1 + \frac{3}{4}K_1)$$

$$= f(1.4375, 3.7115) = 5.1638$$

$$\Rightarrow y_2 = y_1 + (\frac{1}{3}K_1 + \frac{2}{3}K_2)h = 3.8108$$

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Iteration 3:- $x_2 = 1.5, y_2 = 3.8108$

$$K_1 = h f(x_2, y_2) = (0.25) \cdot f(1.5, 3.8108) = 1.273$$

$$K_2 = f(x_2 + \frac{3}{4}h, y_2 + \frac{3}{4}K_1)$$

$$= f(1.6875, 4.7655) = 5.648$$

$$y_3 = y_2 + (\frac{1}{3}K_1 + \frac{2}{3}K_2)h$$

$$\boxed{y_3 = 4.858}$$

Similarly $\boxed{y_4 = y(2) = 6.0893}$