Lecture : 05 (Multipole Expansion)

Dr. Rajveer Singh

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Course	B.Sc (Hons.) Physics
Semester	II
Section	A
Paper Name	Electricity and Magnetism
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Teacher's Name	Dr. Rajveer Singh
Google classroom code	xn2a5q.

Dear students,

I hope you are well at your home. I am happy to know that you are very serious for attending my class on Google meet. If you have any difficulty in understanding my study materials, contact me at my mail, Google class room, Google meeting and Whats App. In addition you have to join my class on Google meeting as per schedule. Attendance and Assignment, test will be recorded on Google class room and the grade will be given on the same.

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We have discussed in the last lecture 04 (Google Class room) about the

1. Method of Images and
2. Electric potential due to Multipole

We derived the formula for induced charge, electric potential for infinite sheet and spherical sphere. In addition, We derive the formula for electric potential (monopole, dipole, quadrupole and octopole). We have noticed that electric potential vary with $1/r^{n+1}$. We also solved some numerical problems based on the electric potential and method of images.

In today lecture, I am going to explain electric field for multipole and Dielectric polarization. In the end of lecture an assignment of 4 question based on today lecture will be given to you. You have to solve the problems after the class over and submit it on the Google classroom.

The electric potential for multipole expansion is given as

$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')^n \rho(r') d\tau'$$

(I)Monopole (n=0) $V(r) \propto \frac{1}{r}$ same as point charge

(II) Dipole (n = 1) $V(r) \propto \frac{1}{r^2}$

(III) Quadrupole (n =2) $V(r) \propto \frac{1}{r^3}$

(IV) Octopole (n =3) $V(r) \propto \frac{1}{r^4}$

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A. Monopole and Dipole Terms

$$V_{mono}(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Where $Q = \int \rho d\tau$ is the total charge. This is the potential of a point charge at the origin. It represent the exact potential.

If the total charge is zero, the dominant terms in the potential will be the dipole.

$$V_{dipole}(r) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\int r' \cos\theta' \rho(r')d\tau'$$

Here θ' is the angle between r' and r as shown in fig.

$$r' \cos\theta' = \hat{r}.r'$$

$$V_{dipole}(r) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\int \hat{r}.r' \rho(r')d\tau'$$

$$V_{dipole}(r) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{\boldsymbol{r}}.\int r' \rho(r')d\tau'$$

$$V_{dipole}(r) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{\boldsymbol{r}}.\boldsymbol{P}$$

Where $\mathbf{P} = \int r' \rho(r')d\tau'$ is called the dipole moment of the distribution.

Hence dipole contribution to the potential is given as

$$V_{dipole}(r) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\boldsymbol{P}.\hat{\boldsymbol{r}}.$$

The dipole moment depends on geometry (size, shape and density) of the charge distribution.

The dipole moment of collection of point charge

$$\mathbf{P} = \sum_{i=1}^{n} q_i \, \mathbf{r'_i}$$

For physical dipole having equal and opposite charges ∓q: $\mathbf{P} = q\mathbf{r'}_+ - q\mathbf{r'}_- = q\mathbf{d}$

Dipole moments are vector quantity and they can be added as vectorically. If there are two dipoles P_1 and P_2 in the same direction then the total dipole moment will be $P_1 + P_2$.

Reference books :

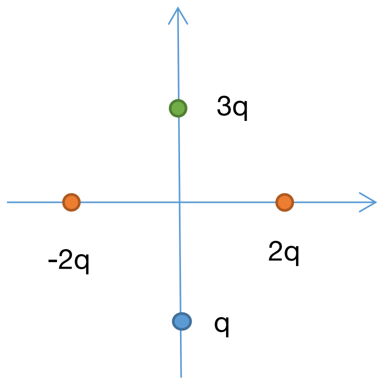
1. David J. Griffith, Introduction to electrodynamics, IIIrd ed, @ 1999 Pearson Education Inc., Upper Saddle River, New Jersey 07458, U.S.A

2. Matthew N. O. Sadiku, S. V. Kulkarni, Principle of Electromagnetics 6th ed Oxford University Press, Ground Floor 2/11, Ansari Road, Daryaganj, New Delhi, 110002, India.

PROBLEMS

1. A sphere of radius R, centered at the origin, carries charge density: $\rho\,(r,\theta) = k\,\frac{R}{r^2}(R-2r)\,sin\,\theta$, where k is constant, r and θ spherical coordinates. Find the approximate potential on the z axis, far from the sphere.

2. Four particles (one of charge q, one of charge 3q, and two of charge -2q) are placed as shown in the figure, each a distance a from the origin. Find a simple approximation formula for the potential, valid at points far from the origin. (Express your answer in spherical coordinates).



3. A specified charge density $\sigma(\theta) = k\cos\theta$ is glued over the surface of a spherical shell of radius R. The resulting **exact** potential inside and outside the sphere are obtained as

$$V(r,\theta) = \frac{k}{3\varepsilon_0} r\cos\theta \quad (r \le R)$$

$$V(r,\theta) = \frac{kR^3}{3\varepsilon_0}\frac{1}{r^2}\cos\theta \quad (r \ge R)$$

(a) calculate the dipole moment of this charge distribution.

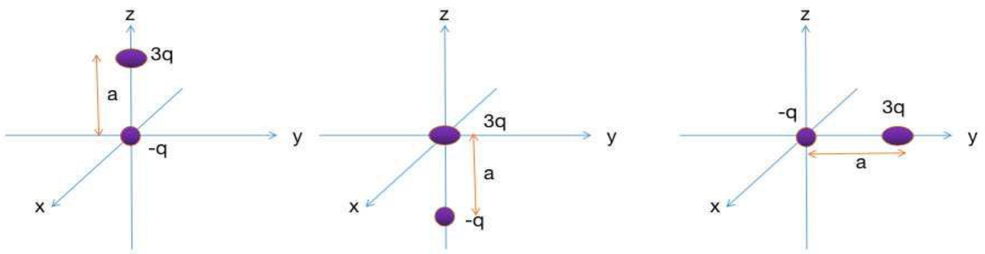
(b) Find the approximate potential, at points far from the sphere, and compare the exact answer. What can you conclude about the higher multipoles.

4. Two point charges,3q and - q, are separated by a distance a. For each of the arrangements in fig (given below). Find

(i) the monopole moment,

(ii) the dipole moment, and

(iii) the approximate potential (in spherical coordinates) at large r (include both the monopole and dipole contributions).



5. Two point charge Q are placed a distance d away from a grounded conducting plane (as shown in the figure). The distance between the point charge is 2 d. find the net force on the point charges located at the point (d,d).

6. A pure dipole p is situated at the origin, pointing in the z-direction.

(a) What is the force on a point charge q (a, 0, 0) ? (Cartesian coordinates)

(b) What is the force on a point charge q (0, 0, a)?

(c) How much work does it take to move q from (a, 0, 0) to (0, 0, a)?

7. Three point charges are located as shown in the fig. each a distance a from the origin. Find thee approximation electric field at points far from the origin. Express your answer in spherical coordinates, and include the two lowest order in the multipole expansion.

8. Show that the electric field of a "pure" dipole in coordinate free form is

$$E_{dipole}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left[3(\boldsymbol{p}.\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{p}\right]$$