Lecture -02

Paper Name:	Analog System and Application
Teacher Name:	Lalit Kumar
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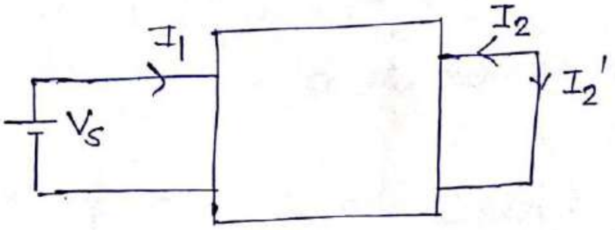
Department:	Physics

Reciprocal network.

→ A two port network is said to be reciprocal network, if the ratio of the excitation at one port to the response at other port is same, if the excitation and the response are interchange.

Condition of Reciprocity, $[h_{12} = -h_{21}]$

Proof:



So here $V_1 = V_s$, and $V_2 = 0$,

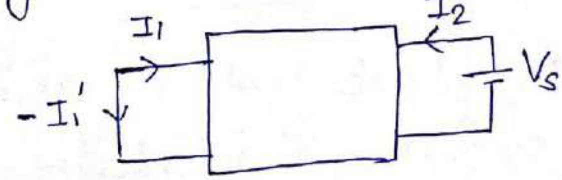
$I_2 = -I_2'$

from $V_1 = h_{11} I_1 + h_{12} V_2 \Rightarrow V_s = h_{11} I_1$

$I_2 = h_{21} I_1 + h_{22} V_2 \Rightarrow -I_2' = h_{21} I_1$

$$\Rightarrow \frac{V_s}{-I_2'} = \frac{h_{11}}{h_{21}} \Rightarrow \left[\frac{V_s}{I_2'} = -\frac{h_{11}}{h_{21}}\right] \text{——} (1)$$

Similarly, voltage Vs is applied at port 2,



so here $V_2 = V_s$, $V_1 = 0$, $I_1 = -I_1'$

Putting this value in eqn.

$$V_1 = h_{11} I_1 + h_{12} V_2 \Rightarrow 0 = -h_{11} I_1' + h_{12} V_s \Rightarrow \frac{V_s}{I_1'} = \frac{h_{11}}{h_{12}} \quad (2)$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Now if the network is reciprocal then equation (1) and eqn (2) should be equal. or $\frac{h_{11}}{h_{12}} = -\frac{h_{11}}{h_{21}} \Rightarrow [h_{21} = -h_{12}]$

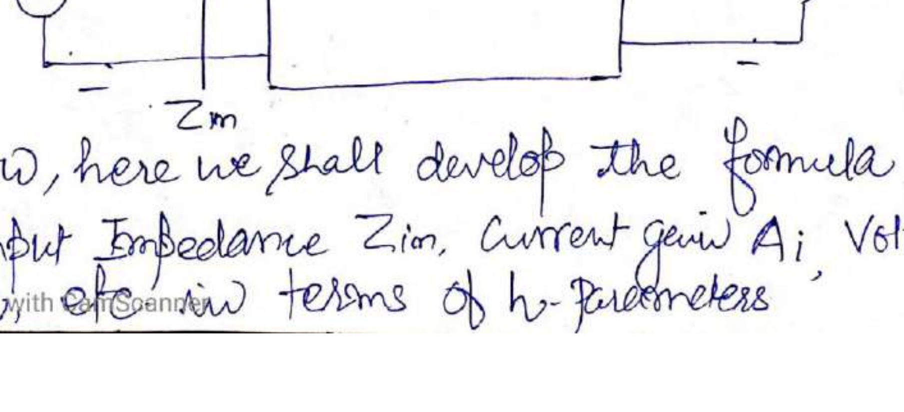
from (I) & (II)

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'} \Rightarrow \frac{h_{11}}{h_{12}} = -\frac{h_{11}}{h_{21}} \Rightarrow (h_{21} = -h_{12})$$

So the network is Reciprocal.

§ Performance of a linear circuit in terms of h Parameters:

We have a linear two port network, has a set of h-Parameters.



Now, here we shall develop the formula for Input Impedance Z_{in}, Current gain A_i, Voltage gain A_v, etc. in terms of h-Parameters

1) Input Impedance Zin:

The input impedance Zin of a linear circuit is defined as $Z_{in} = \frac{V_1}{I_1}$ —— (1)

from the h-parameter equation we know

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—— (2)}$$

from (1) & (2) $Z_{in} = \frac{h_{11} I_1 + h_{12} V_2}{I_1} = h_{11} + \frac{h_{12} V_2}{I_1}$ —— (3)

Now, $I_2 = h_{21} I_1 + h_{22} V_2$ —— (4)

from two port network circuit (As in fig)

$$i_2 = -\frac{V_2}{R_L} \quad \text{—— (5)}$$

(-ve sign indicate that actual current is flowing opposite to the conventional current direction)

from equation (4) and (5)

$$\Rightarrow -\frac{V_2}{R_L} = h_{21} I_1 + h_{22} V_2$$

$$\Rightarrow -h_{21} I_1 = h_{22} V_2 + \frac{V_2}{R_L} = \left(h_{22} + \frac{1}{R_L}\right) V_2$$

$$\Rightarrow \frac{V_2}{I_1} = \frac{-h_{21}}{\left(h_{22} + \frac{1}{R_L}\right)} \quad \text{—— (6)}$$

Put equation (6) in equation (3)

$$Z_{in} = h_{11} - \left[\frac{h_{12} h_{21}}{h_{22} + \frac{1}{R_L}}\right]$$

This is the expression for input impedance of two port network in terms of h-parameters

(2) Current gain A_i ⇒ For the Two port network the Current gain is defined As.

$$A_i = \frac{I_2}{I_1} \quad \text{—— (7)}$$

We know for two port network.

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—— (8)}$$

$$\& \ V_2 = -I_2 R_L$$

$$\therefore \ I_2 = h_{21} I_1 - h_{22} I_2 R_L$$

$$I_2 (1 + h_{22} R_L) = h_{21} I_1$$

$$\left[A_i = \frac{I_2}{I_1} = \frac{h_{21}}{(1 + h_{22} R_L)}\right] \quad \text{—— (9)}$$

This is the expression current gain of two port network in terms of h-parameters

In actual $I_2 = -I_2$

$$\therefore \left[A_i = -\frac{h_{21}}{(1 + h_{22} R_L)}\right]$$

3) Voltage gain A_V :→

$$A_V = \frac{V_2}{V_1} \quad \text{——— (10)}$$

We know $V_1 = I_1 Z_{in}$

$$\therefore A_V = \frac{V_2}{I_1 Z_{in}} \quad \text{——— (11)}$$

from equation (6)

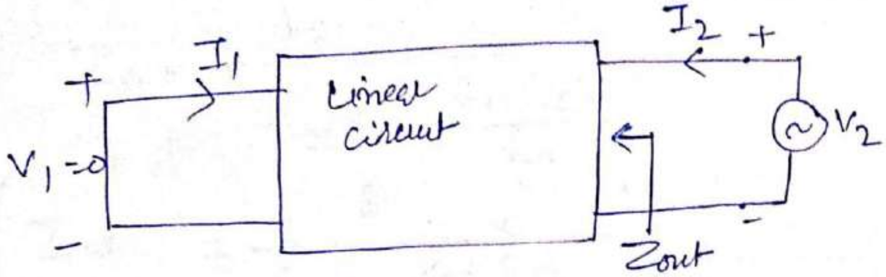
$$\frac{V_2}{I_1} = \frac{-h_{21}}{\left(h_{22} + \frac{1}{R_L}\right)} \quad \text{——— (12)}$$

Put eqn (12) in eqn (11) ——— (13)

$$A_V = \frac{-h_{21}}{Z_{in}\left[\left(h_{22} + \frac{1}{R_L}\right)\right]}$$

Voltage gain of two Port network in Terms of h- Parameters

(4) Output Impedance Z_{out} :→ To find out the Z_{out}, we require to set $V_1 = 0$, and connect a current generator of voltage V_2 at output terminal. the new modified circuit will be



then $Z_{out} = \frac{V_2}{I_2}$ ——— (14)

from h-parameter equations for two port network

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{——— (15)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{——— (16)}$$

with $V_1 = 0$ (input) and applying KVL to the input ckt

from (15) $0 = h_{11} I_1 + h_{12} V_2$

$$I_1 = -\frac{h_{12}}{h_{11}} V_2 \quad \text{——— (17)}$$

from (16) $I_2 = h_{21} I_1 + h_{22} V_2$

Put I_1 from (17) $I_2 = -h_{21}\left(\frac{h_{12}}{h_{11}}\right) V_2 + h_{22} V_2$

$$I_2 = -\frac{h_{21} h_{12} V_2}{h_{11}} + h_{22} V_2 \quad \text{——— (18)}$$

Put. I_2 from eqn (18) into (14)

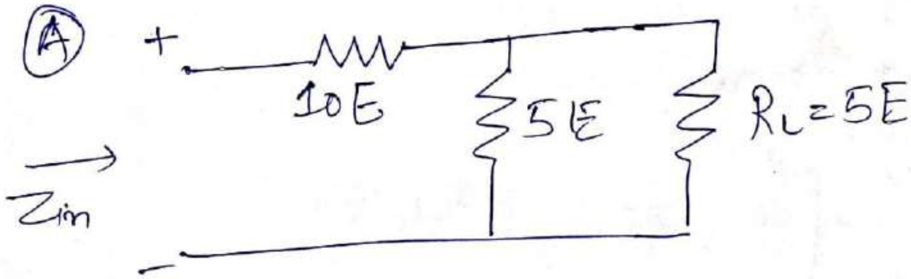
$$Z_{out} = \frac{V_2}{h_{22}V_2 - \frac{h_{21}h_{12}V_2}{h_{11}}}$$

$$\left[Z_{out} = \frac{1}{h_{22} - \frac{h_{21}h_{12}}{h_{11}}} \right] \quad \text{——— (19)}$$

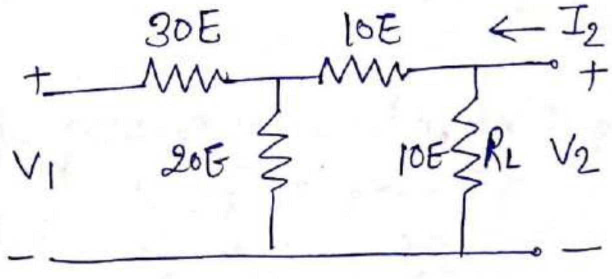
* Numerical Problems *

Find the (1) Input Impedance
(2) Voltage gain for the circuits

(A)

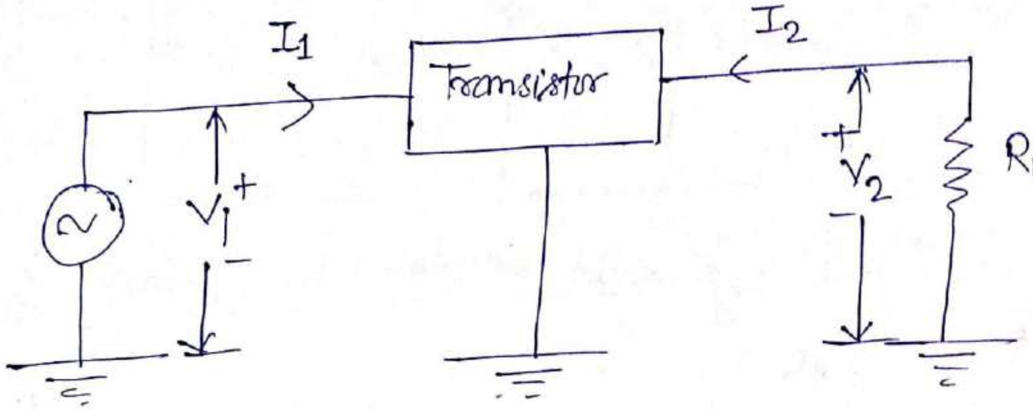


(B)



Transistor as a two port network

§ C.E amplifier:



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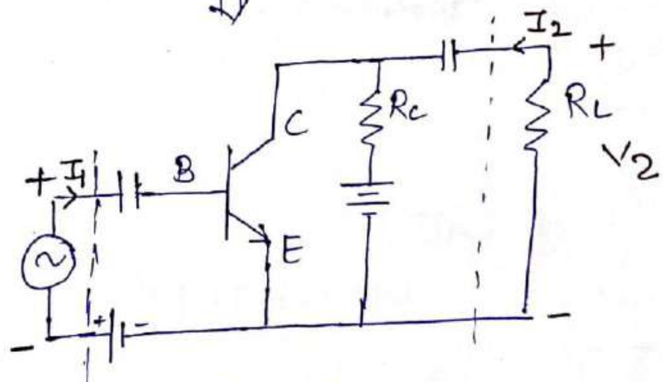


The h-parameters equation for two port network (Transistor)

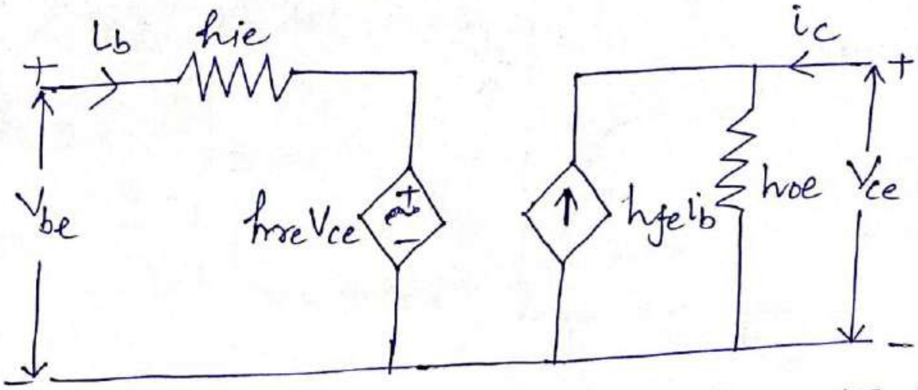
$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2 .$$

⇒ Nomenclature for transistor h-parameters (in CE)

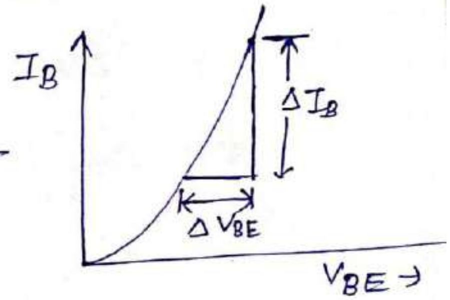
$h_{11} = h_{ie}$, $h_{12} = h_{re}$, $h_{21} = h_{fe}$, $h_{22} = h_{oe}$

⇒ Common emitter equivalent circuit in h-parameters



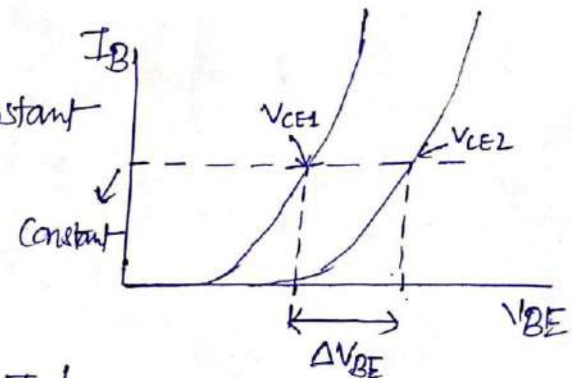
§ Graphical determination of H-Parameters (For CE)

1) $h_{11} = h_{ie} = \frac{\Delta V_{BE}}{\Delta I_B}\Big|_{V_{CE}=\text{constant}}$



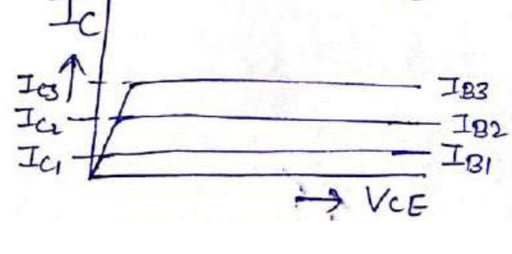
2) $h_{12} = h_{re} = \frac{\Delta V_{BE}}{\Delta V_{CE}}\Big|_{I_B=\text{Constant}}$

$\Delta V_{CE} = V_{CE2} - V_{CE1}$



3) $h_{21} = h_{fe} = \frac{I_2}{I_1}\Big|_{V_2=0} = (\beta)$

$= \frac{\Delta I_C}{\Delta I_B}\Big|_{V_{CE}=\text{Const}}$

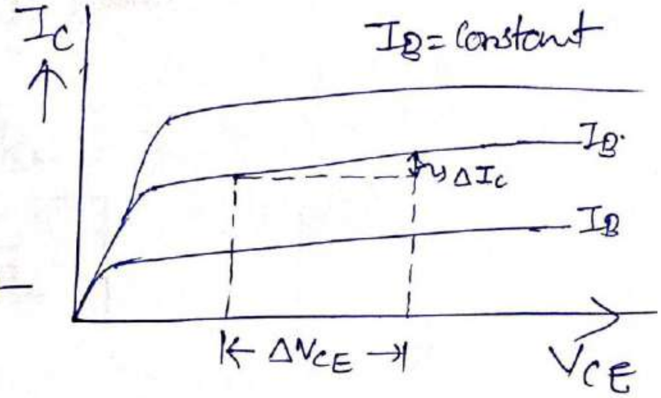


$\Delta I_C = I_{C2} - I_{C1}$

$\Delta I_B = I_{B2} - I_{B1}$

4) $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$

$h_{22} = \left.\frac{\Delta I_C}{\Delta V_{CE}}\right|_{I_B = \text{constant}}$



§ Transistor circuit performance in h-parameter

1) Input Impedance: The general expression for input impedance

$$Z_{in} = h_{11} - \frac{h_{12} h_{21}}{h_{22} + \frac{1}{R_L}}$$

by using nomenclature of h parameter for Transistor

$$\left[Z_{in} = h_{ie} - \frac{h_{re} h_{fe}}{h_{oe} + \frac{1}{R_L}} \right]$$

* Proof: H.W

2) <u>Current gain</u>: General expression for current gain

$$A_i = \frac{h_{21}}{1 + h_{22} R_L}$$

$$\Rightarrow \left[A_i = \frac{h_{fe}}{1 + h_{oe} R_L} \right]$$

<u>Proof</u>: <u>H.W.</u>

3) <u>Voltage gain</u> A_v: General expression

$$A_v = \frac{-h_{21}}{Z_{in}\left(h_{oe} + \frac{1}{R_L}\right)}$$

$$\Rightarrow A_v = \frac{-h_{fe}}{Z_{in}\left(h_{oe} + \frac{1}{R_L}\right)}$$

* <u>Proof</u> : <u>H.W</u>

4) <u>Output Impedance</u>: Z_{out} :→ General expression

$$Z_{out} = \frac{1}{h_{22} - \frac{h_{21} h_{12}}{h_{11}}}$$

$$\left[Z_{out} = \frac{1}{h_{oe} - \frac{h_{fe} h_{re}}{h_{ie}}} \right]$$

<u>Proof</u>: <u>H.W</u>