Lecture- 13

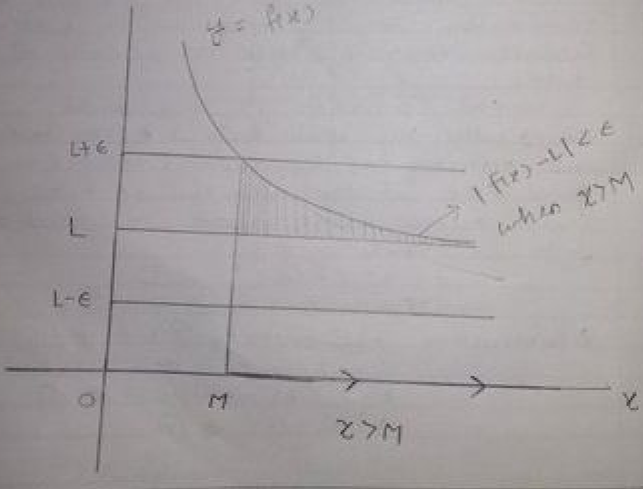
→ Def^n limit as x-approaches to ∞

Let f be a f^n defined on an interval (a, ∞). we say that $f(x)$ has the limit L as x-approaches ∞ and write

$$\lim_{x \to \infty} f(x) = L$$

If for each $\epsilon > 0$, there exist a corresponding number $M > 0$ such that

$|f(x) - L| < \epsilon$ whenever $x > M$



~~Thus we see that the expression ① for closed curve respectly therefore from the end correction team.~~

~~The velocity at 0 is given by reflection $V_0 = (V_t + 0.61 \ldots)$ in M/S~~

~~where V_t is velocity at room temperature to 0c~~

~~The sound is of maximum intensity Note the position of water level the of a set square or the scales.~~

In figure

If $|f(x) - L| < \epsilon$ when $x > M$

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eg $\lim_{x \to \infty} \frac{1}{x} = 0$ prove that

Sol: Let $\epsilon > 0$ be given we must find a number $M > 0$ such that for all x

If $x > M \Rightarrow |f(x) - L| < \epsilon$

i.e if $|\frac{1}{x} - 0| < \epsilon$

i.e if $|\frac{1}{x}| < \epsilon$

assume $x > 0$ then

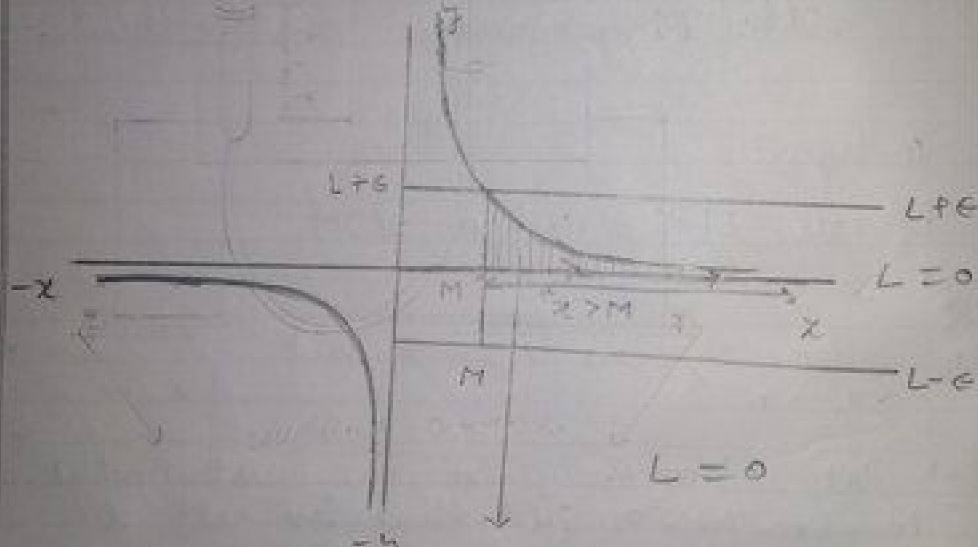
$\left|\frac{1}{x}\right| = \frac{1}{x} < \epsilon$ iff $x > \frac{1}{\epsilon}$

$\Rightarrow$ If we choose $M = \frac{1}{\epsilon}$ then for all x

$x > M \Rightarrow \left|\frac{1}{x} - 0\right| = \frac{1}{x} < \epsilon$

$\Rightarrow \lim_{x \to \infty} \frac{1}{x} = 0$

By figure



when $x > M \Rightarrow |f(x) - L| < \epsilon$

& $M = 1/\epsilon$

Defn Limit as x approaches $-\infty$

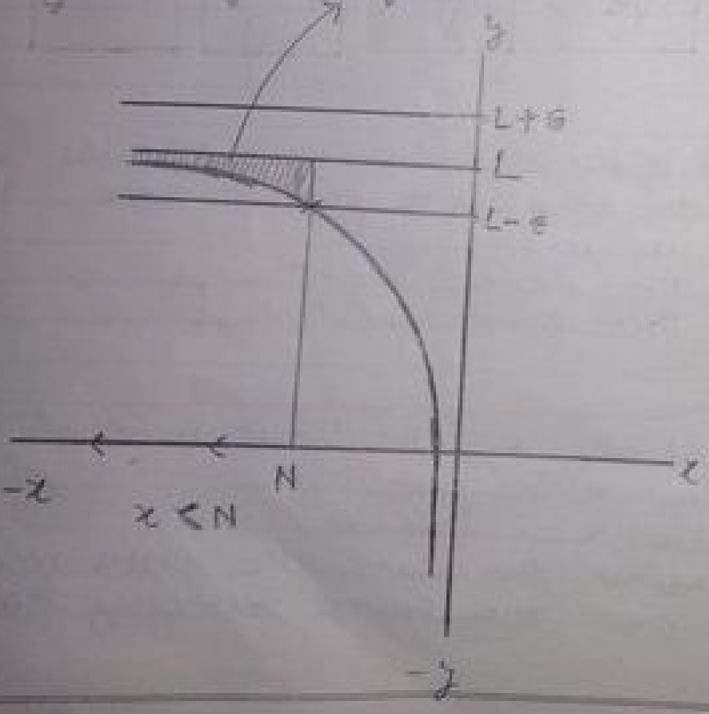
Let f be a fn defined on an interval $(-\infty, a)$, we say that $f(x)$ has the limit L as x approaches to $-\infty$, and write

$$\lim_{x \to -\infty} f(x) = L$$

If for each $\epsilon > 0$, there exist a ~~....~~ corresponding number $N < 0$ such that

$$|f(x) - L| < \epsilon \quad \text{whenever} \quad x < N$$

If $x < N \Rightarrow |f(x) - L| < \epsilon$



Qn Prove that

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$

Soln Let $\epsilon > 0$ be given, we must find a number $N < 0$ such that for all x

If $x < N \Rightarrow |f(x) - L| < \epsilon$ [By defn]

$$\Rightarrow |f(x) - 0| < \epsilon$$

$$\Rightarrow \left|\frac{1}{x}\right| < \epsilon$$

Let $x < 0$ then

$$\left|\frac{1}{x}\right| = -\frac{1}{x} < \epsilon \Leftrightarrow +x < -\frac{1}{\epsilon}$$

$$\Rightarrow x < -\frac{1}{\epsilon} \Rightarrow x < -\frac{1}{\epsilon}$$

Thus if we choose $N = -\frac{1}{\epsilon}$ then for all x

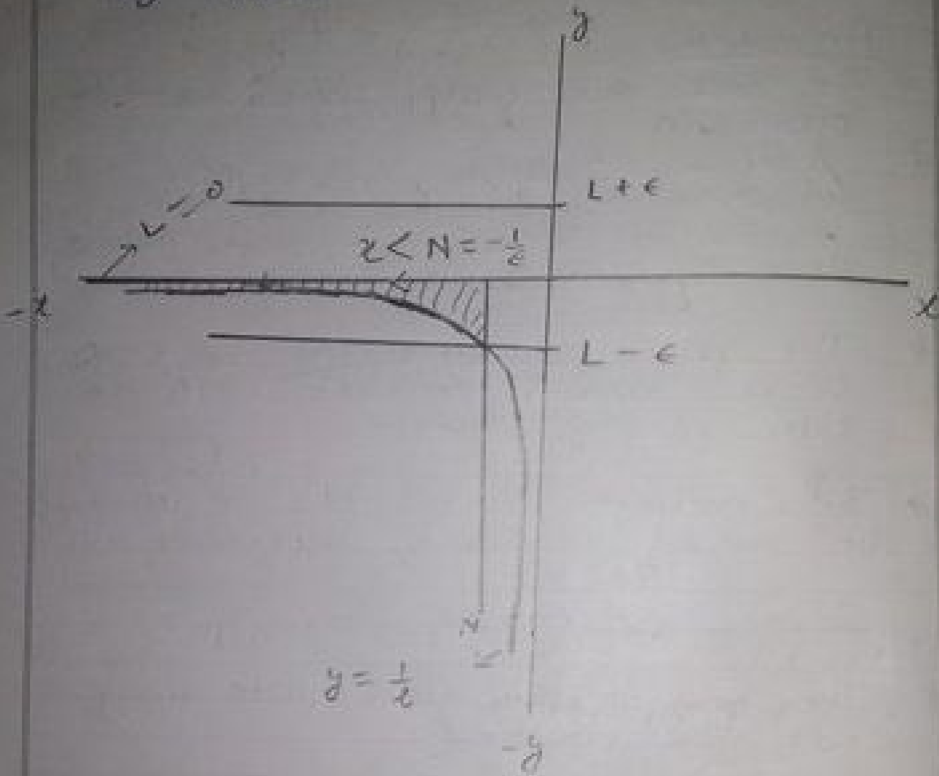
$$x < N \Rightarrow \left|\frac{1}{x} - 0\right| = \left|\frac{1}{x}\right| = -\frac{1}{x} < \epsilon$$

$\Rightarrow$ If $x < -\frac{1}{\epsilon}$ then $|f(x) - L| < \epsilon$

$$\Rightarrow \lim_{x \to -\infty} f(x) = 0$$

Hence $\lim_{x \to -\infty} \frac{1}{x} = 0$

By figure.



if $x < N = -\frac{1}{\epsilon}$ then $|f(x) - L| < \epsilon$

i.e. if $x < -\frac{1}{\epsilon}$ then $\left|\frac{1}{x} - 0\right| < \epsilon$

$\Rightarrow \lim\limits_{x \to -\infty} \frac{1}{x} = 0$

→ Defⁿ limit as x-approaches to ∞

Let f be a fⁿ defined on an interval (a, ∞). we say that $f(x)$ has the limit L as x-approaches ∞ and write

$$\lim_{x \to \infty} f(x) = L$$

If for each $\epsilon > 0$, there exist a corresponding number $M > 0$ such that $|f(x) - L| < \epsilon$ whenever $x > M$