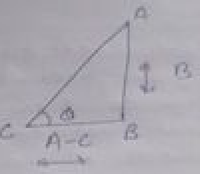
18/04/2020 Lecture - 11

$\because \cot 2\theta = \frac{A-C}{B}$

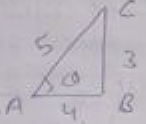


We know that

$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}}$

$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$

<u>Ex</u> $\cot 2\theta = \frac{3}{4}$ Given



$AC = 5$

$\cos 2\theta = \frac{AB}{AC} = \frac{4}{5}$

$\because \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+4/5}{2}} = \sqrt{\frac{9}{10}}$

$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-4/5}{2}} = \sqrt{\frac{1}{10}}$

the value of $\sin\theta$, $\cos\theta$ were in rotation eq n

$x = x'\cos\theta - y'\sin\theta = \frac{3x'}{\sqrt{10}} - \frac{y'}{\sqrt{10}}$

$y = x'\sin\theta + y'\cos\theta = \frac{x'}{\sqrt{10}} + \frac{3y'}{\sqrt{10}}$

$\Rightarrow x = \frac{3x'}{\sqrt{10}} - \frac{y'}{\sqrt{10}}, \; y' = \frac{x'}{\sqrt{10}} + \frac{3y'}{\sqrt{10}}$

Qn Trace the curve

$9x^2+16y^2-24xy-80x-60y+100=0$

~~Re~~ Rotating the co-ordinate axes (x,y) through an angle θ

Soln $9x^2+16y^2-24xy-80x-60y+100=0$ — ①

compare with

$Ax^2+Bxy+Cy^2+Dx+Fy+E=0$

$A=9,\ C=16\ \ B=-24 = B=-24$

$B^2-4AC = \cancel{(16)^2} - \cancel{4\times9\times24}$

$B^2-4AC = (-24)^2-4(9)(16)$

$= 576-36\times16 = 576-576=0$

$\Rightarrow B^2-4AC=0$

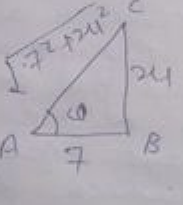
$\Rightarrow$ eqn ① rep parabola

Now find angle θ

$\cot 2\theta = \dfrac{A-C}{B} = \dfrac{9-16}{-24} = \dfrac{+7}{+24}$

$\cot 2\theta = 7/24$

By ΔABC



$AB=7,\ BC=24$

$AC=\sqrt{7^2+24^2}$

$=\sqrt{25}=5$

$\Rightarrow \cos 2\theta = \frac{AB}{AC} = \frac{7}{25}$ $\quad \sin 2\theta = \frac{BC}{AC} = \frac{24}{25}$

$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+7/25}{2}} = \sqrt{\frac{32}{50}} = \sqrt{\frac{16}{25}}$

$\cos\theta = \frac{4}{5}$

$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-7/25}{2}} = \sqrt{\frac{18}{50}} = \sqrt{\frac{9}{25}}$

$\sin\theta = \frac{3}{5}$ now $\tan\theta = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$

we have rotation eqn

$x = x'\cos\theta - y'\sin\theta$, $y = x'\sin\theta + y'\cos\theta$

$x = \frac{4x'}{5} - \frac{3y'}{5}$, $y = \frac{3x'}{5} + \frac{4y'}{5}$

~~we got~~ put the value of x & y in (1)

$\frac{9}{25}(4x'-3y')^2 + \frac{16}{25}(3x'+4y')^2 - \frac{24}{25}(4x'-3y')(3x'+4y')$
$- \frac{\cancel{80}^{16}}{\cancel{5}}(4x'-3y') - \frac{\cancel{60}^{12}}{\cancel{5}}(3x'+4y') + 100 = 0$

$\frac{9}{25}\left(16(x')^2 + 9(y')^2 - 24x'y'\right) + \frac{16}{25}\left(9(x')^2 + 16(y')^2\right.$
$\left. + 24x'y'\right) - \frac{24}{25}\left(12(x')^2 + 7x'y' - 12(y')^2\right)$
$- 16(4x'-3y') - 12(3x'+4y') + 100 = 0$

$\Rightarrow \cancel{9(16x')^2}$

$\Rightarrow 9[16(x')^2 + 9(y')^2 - 24x'y') - 24(12(x')^2 + 7x'y'$

$- 12(y')^2] + 16(9(x')^2 + 16(y')^2 + 24x'y') -$

$400(4x' - 3y') - 300(3x' - 4y') + 2500 = 0$

$\Rightarrow 144(x')^2 + 81(y')^2 - 216x'y' - 288(x')^2$

$- 168x'y' + 288(y')^2 + 144(x')^2 + 256(y')^2 + 384x'y'$

$- 1600x' + 1200y' - 900x' - 1200y' + 2500 = 0$

$\Rightarrow \cancel{144}(x')^2 + 81(y')^2 - \cancel{216}x'y' - \cancel{288}(x')^2$

$- \cancel{168x'y'} + 288(y')^2 + \cancel{144}(x')^2 + 256(y')^2$

$+ \cancel{384x'y'} - 1600x' + \cancel{1200y'} - 900x' - \cancel{1200y'} + 2500 = 0$

$\Rightarrow 6\cancel{25}(y')^2 - 2\cancel{500}x' + 2\cancel{500} = 0$

$\cancel{25}(y')^2 - \cancel{100}x' + \cancel{100} = 0$

$\Rightarrow (y')^2 - 4x' + 4 = 0 \Rightarrow (y')^2 = 4x' - 4$

$\Rightarrow (y')^2 = 4(x' - 1) \quad - (1)$

~~vertex~~ ~~(1,0)~~
center $(h, k) = (1, 0)$ w.r.t (x, y)

Shifting origin to vertex V by using transformation eqn

$x' = x + h$, $y' = y + k$

from ② $y^2 = 4x$

Focus is $(a, 0) = (1, 0)$ w.r.t (x, y)

Focus w.r.t $(x', y') = (1+h, 0+k)$
$= (1+1, 0+0) = (2, 0)$

eqn of directrise w.r.t (x, y) $x + a = 0$
$\Rightarrow x + 1 = 0$
$\Rightarrow x = -1$

eqn of directrise w.r.t (x', y') is .
$x' = -1 + h$
$= -1 + 1$
$x' = 0$

length of latus rectum
$4a = 4 \times 1 = 4$

point of intersection w.r.t (x, y)

put $y = 0$ for intercept x-axis in ①

$9x^2 - 80x + 100 = 0$

$\Rightarrow y^2 - \frac{80}{9}x + \frac{100}{9} = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+\frac{80}{9} \pm \sqrt{\frac{6400}{81} - \frac{400}{9}}}{2}$

$x = \frac{80 \pm 10\sqrt{28}}{9 \times 2}$

$x = \frac{80 + 10\sqrt{28}}{9 \times 2} \approx 7.4$

$x = \frac{80 - 10\sqrt{28}}{9 \times 2} \approx 1.50$

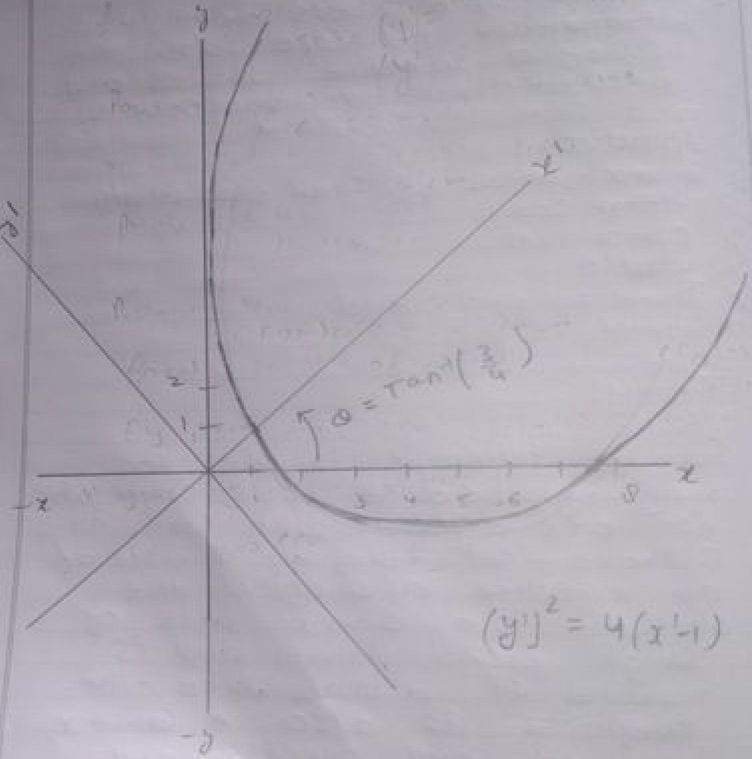
for intersect y-axes put $x = 0$ in ①

$16y^2 - 60y + 100 = 0$

$\because B^2 - 4AC = 3600 - 4 \times 16 \times 100$

$= 3600 - 6400 < 0$

$\Rightarrow$ no real roots $\Rightarrow$ curve not intersect ~~(x,y)~~ y-axes in (x, y) quadrat



$$(y')^2 = 4(x'-1)$$

A

$2x^2+xy+2y^2+x-y=0$ ---- ①

(ii) compare with $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

we get

$A=2,\ B=1,\ C=2$

$B^2-4AC = 1-16 = -15<0$

$\Rightarrow$ eqn ① is ellipse

$\because \theta$ is not given

$\Rightarrow \cot 2\theta = \dfrac{A-C}{B} = \dfrac{2-2}{1} = 0$

$\Rightarrow \cot 2\theta = 0 \Rightarrow 2\theta = 90^\circ$

$\Rightarrow \theta = \dfrac{90}{2} = 45^\circ$

rotation eqn

$x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

put $\theta = 45^\circ$

$x = \dfrac{x'}{\sqrt2} - \dfrac{y'}{\sqrt2}, \quad y = \dfrac{x'}{\sqrt2} + \dfrac{y'}{\sqrt2}$

Put the value of x & y in ①

$2\left(\dfrac{x'-y'}{\sqrt2}\right)^2 + \left(\dfrac{x'-y'}{\sqrt2}\right)\cdot\left(\dfrac{x'+y'}{\sqrt2}\right) + 2\left(\dfrac{x'+y'}{\sqrt2}\right)^2$

$+ \left(\dfrac{\cancel{x'}-y'}{\sqrt2}\right) - \left(\dfrac{\cancel{x'}+y'}{\sqrt2}\right) = 0$

$\cancel{2}\left(\dfrac{(x')^2+(y')^2-2x'y'}{\cancel{2}}\right) + \dfrac{(x')^2-(y')^2}{2} + \cancel{2}\left(\dfrac{(x')^2+(y')^2+2x'y'}{\cancel{2}}\right)$

$+ -\dfrac{2y'}{\sqrt2} = 0$

B

$(x')^2+(y')^2+\cancel{2x'y'}+\frac{(x')^2-(y')^2}{2}+\frac{(x')^2+(y')^2+2x'y'}{}$

$-\frac{2y'}{\sqrt{2}}=0$

$2(x')^2+2(y')^2+\frac{(x')^2-(y')^2}{2}-\frac{2y'}{\sqrt{2}}=0$

$\frac{4(x')^2+4(y')^2+(x')^2-(y')^2}{2}-\frac{2y'}{\sqrt{2}}=0$

$\frac{5(x')^2+3(y')^2}{2}-\sqrt{2}y'=0$

$5(x')^2+3(y')^2-2\sqrt{2}y'=0$

$5(x')^2+3\left[(y')^2-\frac{2\sqrt{2}}{3}y'\right]=0$

$5(x')^2+3\left[(y')^2-\frac{2\sqrt{2}}{3}y'+\left(\frac{\sqrt{2}}{3}\right)^2-\left(\frac{\sqrt{2}}{3}\right)^2\right]=0$

$5(x')^2+3\left[(y')^2-\frac{2\sqrt{2}}{3}y'+\left(\frac{\sqrt{2}}{3}\right)^2\right]-3\cdot\frac{2}{9}=0$

$5(x')^2+3\left(y'-\frac{\sqrt{2}}{3}\right)^2=\frac{2}{3}$

$\frac{5(x')^2}{2/3}+\frac{3\left(y'-\frac{\sqrt{2}}{3}\right)^2}{2/3}=1$

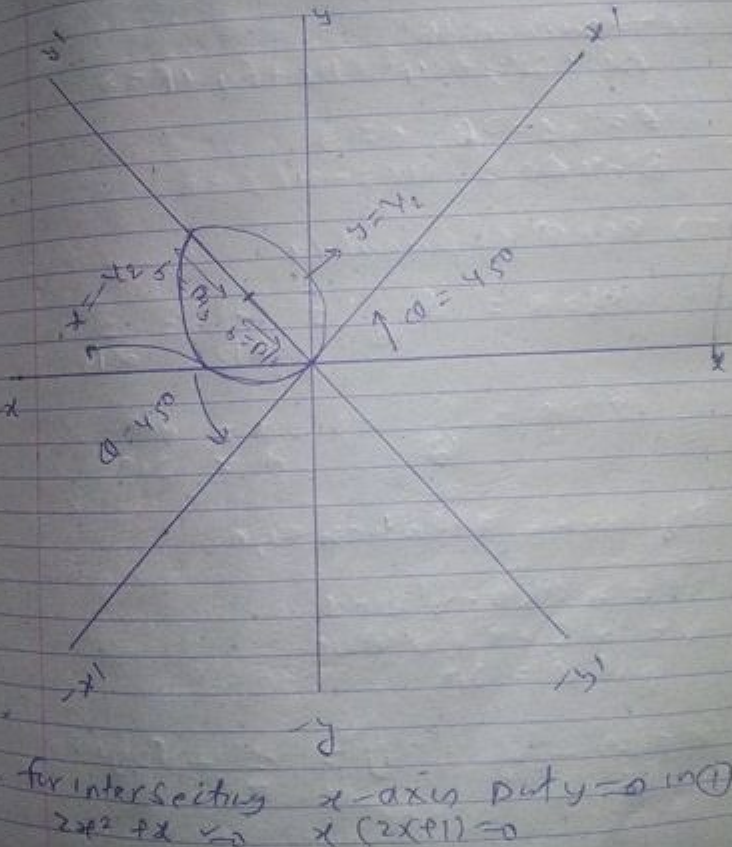
$\frac{(x')^2}{\frac{2}{15}}+\frac{\left(y'-\frac{\sqrt{2}}{3}\right)^2}{2/9}=1$

$\cancel{x'}$

C

$$\frac{(x')^2}{\left(\frac{\sqrt{2}}{\sqrt{15}}\right)^2} + \frac{\left(y' - \sqrt{2}/3\right)^2}{\left(\sqrt{2}/3\right)^2} = 1 \qquad a = \frac{\sqrt{2}}{\sqrt{15}} \quad b = \sqrt{2}/3$$

this is shifting w.r.t. origin $(0, \sqrt{2}/2)$



for intersecting x-axis put $y=0$ in (1)

$2x^2 + x = 0 \qquad x(2x+1) = 0$

$x = -1/2$