$$\Rightarrow x = \frac{1}{\sqrt{1+x}} = \phi(x)$$

$$\Rightarrow \phi'(x) = \frac{-1}{2(1+x)^{3/2}}$$

$$\Rightarrow |\phi'(1/2)| = c < 1$$

$\because c < a,\ c < b$

$$\Rightarrow \phi(x) = \frac{1}{\sqrt{1+x}}$$

$$\Rightarrow x_n = \phi(x_{n-1})$$

$$\Rightarrow x_n = \frac{1}{\sqrt{1+x_{n-1}}}$$

$x_1 = 0.81649$

$x_2 = 0.74196$

$x_3 = 0.75767$

11th/Jan/2020

Lagrange's Interpolation :-

Let $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ be three points passing through the polynomial

$$y(x) = a_0(x - x_1)(x - x_2) + a_1(x - x_0)(x - x_2) + a_2(x - x_0)(x - x_1) \quad \text{--- (1)}$$

where a_i are Lagrange's constant.

$\because (x_0, y_0)$ satisfy eqn (1)

$$\Rightarrow f(x_0) = y_0 = y(x_0) = a_0(x_0 - x_1)(x_0 - x_2)$$

$$\Rightarrow a_0 = \frac{y_0}{(x_0 - x_1)(x_0 - x_2)}$$

Similarly $\Rightarrow$ $$a_1 = \frac{y_1}{(x_1 - x_0)(x_1 - x_2)}$$

$$a_2 = \frac{y_2}{(x_2 - x_0)(x_2 - x_1)}$$

$$\Rightarrow y(x) = y_0\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + y_1\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_0)}$$

Ques. From the given table find the f^n $y(x) = f(x)$ by Lagrange's Method and find $f(3) = ?$

x	0	2	5
y	6	0	6

Soln

$$y(x) = y_0\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + y_1\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$\Rightarrow y(x) = 6\times\frac{(x-2)(x-5)}{(-2)(-5)} + 0 + 6\frac{(x-0)(x-2)}{5\times 3}$$

$$\Rightarrow y(x) = \frac{3}{5}(x-2)(x-5) + \frac{2}{5}x(x-2)$$

$$\Rightarrow y(3) = \frac{3}{5}\times 1\times(-2) + \frac{2}{5}\times 3\times 1$$

$$\Rightarrow y(3) = -\frac{6}{5} + \frac{6}{5} = 0 \qquad \Rightarrow y(3) = 0$$

Ques.

x	0	1	2	3
y	0	2	8	27

$f(2.5) = ?$

Soln

$$y(x) = y_0\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} + y_2\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + y_3\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$\Rightarrow y(x) = 0 + \frac{2x(x-2)(x-3)}{1\times(-1)(-2)} + \frac{\overset{4}{\cancel{8}}\times x(x-1)(x-3)}{2\times 1\times(-1)} + \frac{\overset{9}{\cancel{27}}x(x-1)(x-2)}{\cancel{3}\times 2\times 1}$$

$$\Rightarrow y(x) = x(x-2)(x-3) + \frac{4x(x-1)(x-3)}{-1} + \frac{9}{2}x(x-1)(x-2)$$

$\frac{25}{10} = \frac{5}{2}$

$$\Rightarrow y(x) = \left(\tfrac{5}{2}\right)\left(\tfrac{5}{2}-2\right)\left(\tfrac{5}{2}-3\right) - 4\left(\tfrac{5}{2}\right)\left(\tfrac{5}{2}-1\right)\left(\tfrac{5}{2}-3\right) + \frac{9}{2}\times\frac{5}{2}\left(\tfrac{5}{2}-1\right)\left(\tfrac{5}{2}-2\right)$$

$$\Rightarrow y\left(\tfrac{5}{2}\right) = \frac{5}{2}\times\frac{1}{2}\times\left(-\frac{1}{2}\right) - \left(4\times\frac{5}{2}\times\frac{3}{2}\times\left(-\frac{1}{2}\right)\right) + \frac{9}{2}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}$$

$$\Rightarrow y\left(\tfrac{5}{2}\right) = -\frac{5}{8} + \frac{60}{8} + \frac{135}{16} = \frac{55}{8} + \frac{135}{16}$$

$15 \times 9 = 135$

$$\Rightarrow y\left(\tfrac{5}{2}\right) = \frac{110+135}{16} = \frac{245}{16}$$

Equally Spaced Points :-

Let $x_0, x_1, x_2 \ldots x_n$ be the points in (a, b) s.t $x_{i+1} - x_i = h$, $i = 1$ to n. Then x_i's are called equally spaced points.

Forward Difference Operator :-

The forward operator is denoted by Δ and defined by

$$\Delta f(x_i) = f(x_{i+1}) - f(x_i)$$

or $\Delta y_i = y_{i+1} - y_i$

i.e.

$\Delta y_0 = y_1 - y_0$ $\qquad \Delta^2 y_0 = \Delta(\Delta y_0) = \Delta(y_1 - y_0) = \Delta y_1 - \Delta y_0$

$\Delta y_1 = y_2 - y_1$ $\qquad \Delta^2 y_1 = \Delta y_2 - \Delta y_1$

$\vdots$

$\Delta y_{n-1} = y_n - y_{n-1}$ $\qquad \Delta^2 y_{n-2} = \Delta y_{n-1} - \Delta y_{n-2}$

$\Delta y_n = y_{n+1} - y_n$ (X) $\qquad \Delta^2 y_{n-1} = \Delta y_n - \Delta y_{n-1}$ (X)

No value of y_{n+1} $\qquad$ No Δy_n

They are first order diff. operator $\qquad$ They are second order diff-operator