# Electricity and Magnetism
## Study Material -3.

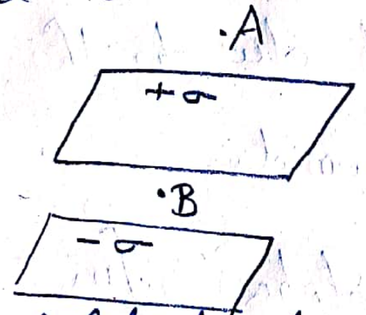
**Problem** - A large parallel plate capacitor with uniform surface charge density $\sigma$ on the upper plate and $-\sigma$ on the lower plate is moving with a constant speed $v$.

(a) Find the magnetic field between the plates and also above and below them.

(b) Find the magnetic force per unit area on the upper plate, including its direction.

(c) At what speed $v$ would the magnetic force balance the electric force.

Sol$^n$ (a)




Magnetic field produced by the top tuely charged plate $B = \frac{\mu_0 K}{2}$ where $K = \sigma v$

$B = \frac{\mu_0 \sigma v}{2}$    **Direction** Out of the page on A
Into the page at B

Similarly B due to $-\sigma$    Into the page at B

$B = \frac{-\mu_0 \sigma v}{2}$    Direction out of page at C

Therefore

$$B = \begin{cases} 0 & \text{at } A \\ \mu_0 \sigma v & \text{at } B \\ 0 & \text{at } C \end{cases}$$

2) The force per unit area on $+\sigma$ plate

$$f_m = \sigma v B_{avg} = \sigma v \frac{\mu_0 \sigma v}{2} = \frac{\mu_0 \sigma^2 v^2}{2}$$

Direction upwards.

Hint - Lorentz force $\vec{F} = \int (\vec{K} \times \vec{B})\, da$

Force per unit area $F = \vec{K} \times \vec{B}$

$= \sigma v B$

(c) The electric field due to the lower plate is

$\vec{E} = \frac{\sigma}{2\epsilon_0}$ downwards

For balancing condition $f_m = f_e$

$$\frac{\mu_0 \sigma^2 v^2}{2} = \frac{\sigma}{2\epsilon_0}$$

$$\mu_0 v^2 = \frac{1}{\epsilon_0}$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3 \times 10^8 \quad \therefore \text{ speed of light}$$

Problem: An infinite solenoid ( n turns per unit length with current I ) is filled with a linear material of susceptibility $\chi_m$.

Find the following

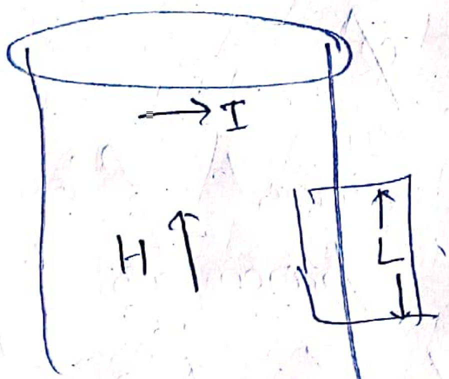
a). The H field inside & outside of solenoid

b) The total magnetic field "

c) The magnetization inside solenoid.

d) The bound surface current density

e) " " volume " "




a) Because of the symmetry

$\vec{\nabla}\cdot\vec{H} = 0$ everywhere

$\Rightarrow$ $\vec{H}$ field can be obtained by amperian loop as shown above

$$\oint_{line} \vec{H}\cdot d\vec{l} = HL$$

& $\vec{H} = NI\hat{k}$ → direction of current

L no. of turns

Therefore magnetic field inside the solenoid

$$\vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu_0(1+\chi_m)NI\hat{k}$$

The magnetisation of the material

$$\vec{M} = \chi_m H = \chi_m N I \hat{k} \quad \text{(uniform)}$$

i.e. there will be no bound volume current in the material.

& The bound surface current is equal to

$$\vec{K}_b = M \times \hat{n} = \chi_m (\vec{H} \times \hat{n})$$

$$= \chi_m N I \hat{\phi}$$

& Bound surface current flows in the same direction for paramagnetic material & in opposite direction for diamagnetic materials.