ob. 7.11 :- FARADAY'S LAW :-

A changing magnetic field induces an electric field,

$$\bar{\nabla} \times \bar{E} = -\frac{\partial B}{\partial t}$$

Whenever the magnetic flux through a loop changes we know an emf $= -\frac{\partial \phi}{\partial t}$ is generated

$$\mathcal{E} = \oint \bar{E}\cdot d\bar{l} = -\frac{d\phi}{dt}$$

$$= -\int \frac{\partial B}{\partial t}\cdot da$$

$$\oint \bar{E}\cdot d\bar{l} = -\int \frac{\partial B}{\partial t}\cdot da$$

$$\int_S (\bar{\nabla} \times \bar{E})\cdot da = -\int \left(\frac{\partial B}{\partial t}\right) da$$

$$\boxed{\bar{\nabla} \times \bar{E} = -\frac{\partial B}{\partial t}}$$

Ampere's law; $\nabla \times \bar{B} = \mu_0 J$.

Induced Electric Field :-

Faraday's discovery tells us that there are really two kinds of electric field; those due to electric charges and those associated with changing magnetic field and which can be found by exploiting the analogy between the faraday's law;

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

and Ampere's law:

$$\bar{\nabla}\times\bar{B} = \mu_0 \bar{J} \quad \text{~~Fara~~ Amp.} \quad (1)$$

$$\oint E\cdot dl = -\frac{d\phi}{dt} \quad \text{~~(Am~~ Faraday} \quad (2)$$

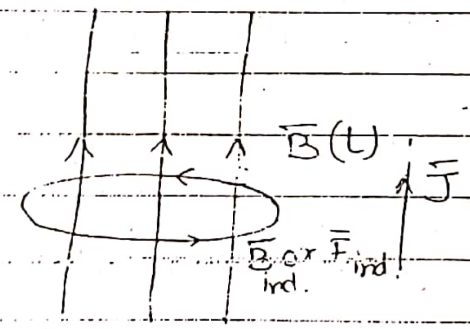
Faraday's induced electric field is determined by $-\frac{\partial \phi}{\partial t} = \left(\frac{\partial B}{\partial t}\right)A$ in exactly the same way as magnetostatic field ~~given~~ is determined by J. (in (1)).

Rate of change of magnetic flux through the amperian loop plays the role ~~for only~~ formally assigned to $\mu_0 I_{enc}$. To find the direction of induced electric field:—

$$\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t} \; ; \quad \bar{\nabla}\times\bar{B} = \mu_0 \bar{J}$$

Direction of $\bar{B}$ is direction of current I, and once the direction of current is known, ~~to~~ then we calculate the direction of magnetic field which is the direction of induced electric field.

Case (1):

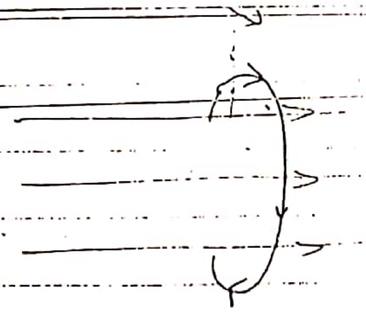


~~Case~~ Direction of $\bar{B}$ is upward so does the J vector ~~so,~~ ~~due~~ to $\bar{J}$ upward the direc^n of $\bar{B}_{ind}$ circumferential, so does $\bar{E}$ induced which is direction of induced electric field.

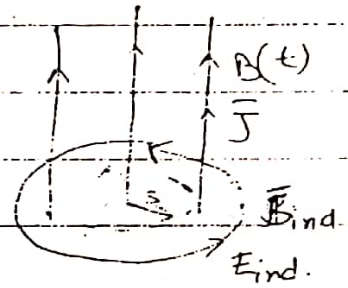
Case II

$$\bar{\nabla}\times \bar{E}_{ind} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla}\times \bar{B}_{ind} = \mu_0 \bar{J}$$



To this wire the direction of $\bar{B}$ is circumferential; so does $\bar{J}$. Now due to this configuration the direction of $\bar{E}_{ind}$ is in same direction given current



$$\oint \bar{E}\cdot d\bar{l} = -\frac{d\phi}{dt}$$

$$= -\frac{d(B\cdot A)}{dt}$$

$$E\cdot 2\pi s = -\pi s^2 \frac{dB}{dt}$$

$$\boxed{\bar{E}_{ind} = -\frac{s}{2}\frac{d\bar{B}}{dt}\,\hat{\phi}}$$

If $\bar{B}(t)$ is increasing $\bar{E}_{ind}$ runs clockwise. And suppose $\bar{B}(t)$ is continuously decreasing then direc^n. of $\bar{E}_{ind}$ is in the counterclockwise direction.