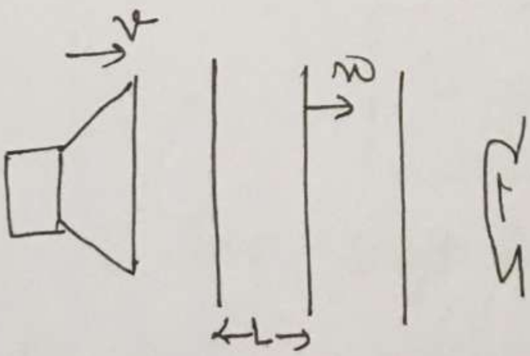
The Doppler Effect.

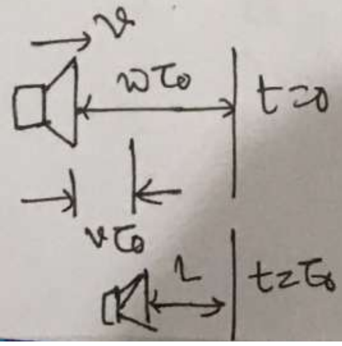
is a change in the freq^n of a wave due to motion b/w the source and observer. It provides an invaluable tool for measuring the speed of far off objects by shift in the spectral wavelength they emit.

The relativistic Doppler effect differs from the classical effect in a pleasing manner: it is simpler.

The Doppler effect in Sound:



Consider sound waves from a source moving with velocity w through the medium towards an observer at rest along the line of motion.



We will picture sound as regular series of pulses seperated by time $\tau_0 = 1/\nu_0$, where ν_0 is the number of pulses per sec. generated by source.

The distance b/w pulses is

$$\boxed{w\tau_0 = w/\nu_0 \Rightarrow \lambda}$$

If source moves toward the observer at speed v, then distance b/w successive pulses

$$\lambda_D = \lambda - v\tau_0 = \lambda - \frac{v}{\nu_0}$$

$$\Rightarrow \quad \frac{w}{\nu_0'} = \frac{w}{\nu_0} - \frac{v}{\nu_0}$$

$$\boxed{\nu_0' = \nu_0\left(\frac{1}{1 - v/w}\right)} \text{ (moving source)}$$

The shift in freq^n $(\Delta\nu)\lambda_D = \nu_0' - \nu_0$ is known as Doppler shift.

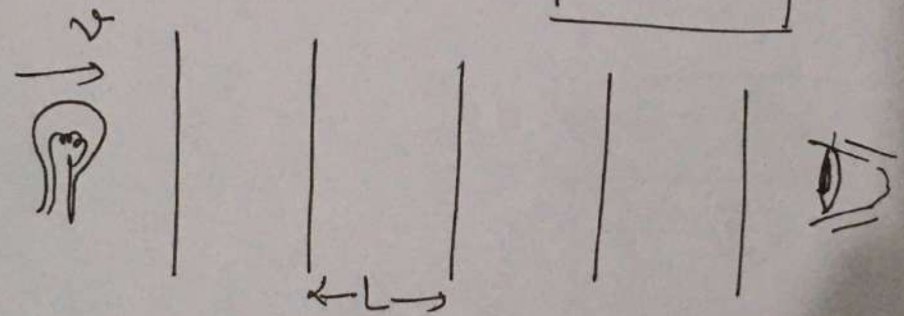
OR

$$\boxed{\nu_0' = \nu_0\left(1 + \frac{v}{w}\right)} \text{ (moving observer)}$$

If these results were valid for light waves in space, we would be able to distinguish which of two systems is at absolute rest, which is not possible. To resolve this difficulty we turn now to relativistic derivation of the Doppler effect.

The Relativistic Doppler Effect:

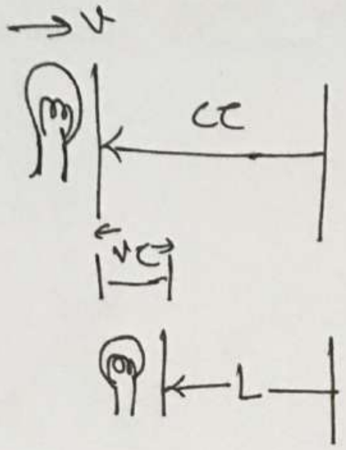
$$\boxed{\tau = \gamma \tau_0}$$



A light source flashes with period $\tau_0 = 1/\nu_0$ in its rest frame. The source is moving toward an observer with velocity v. Due to time dilation the period in the observer's rest frame is

$$\tau = \gamma \tau_0$$

Scanned with CamScanner



If λ_D is the distance b/w pulses in the observer's rest frame, the freqn of the pulses is $\nu_D = c/\lambda_D$, where the wavelength λ_D is the distance b/w pulses in observer's frame. Because the source is moving toward observer this distance is

$$\lambda_D = c\tau - v\tau = (c-v)\tau$$

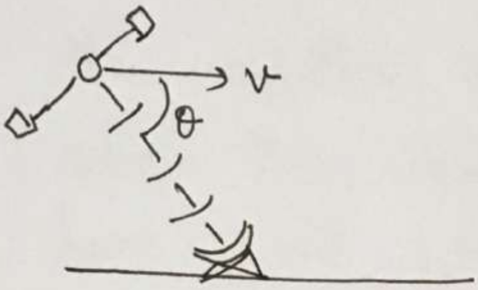
$$\nu_D = \frac{c}{(c-v)\tau} = \frac{1}{\left(1-\frac{v}{c}\right)} \cdot \frac{1}{\gamma \tau_0}$$

$$\nu_D = \frac{\nu_0 \sqrt{1 - v^2/c^2}}{\left(1-\frac{v}{c}\right)}$$

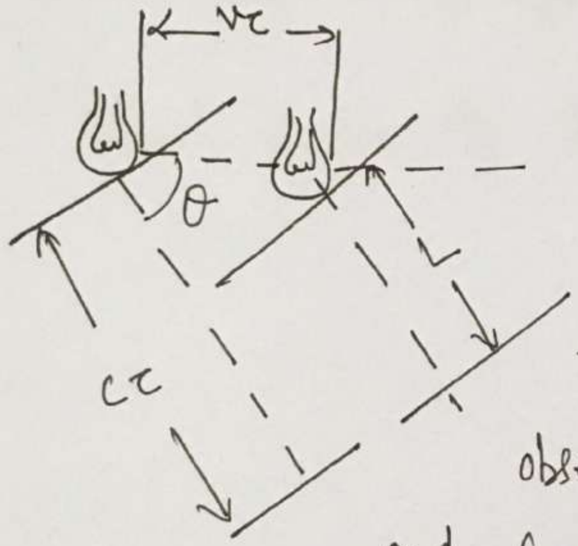
$$\Rightarrow \quad \nu_D = \nu_0 \sqrt{\frac{1+v/c}{\left(1-\frac{v}{c}\right)}}$$

ν_D is the freqn in observer's rest frame and v is the relative speed of source and observer.

The Doppler Effect off the Line of Motion:



Consider a Satellite broad-casting a radio beacon signal to a ground tracking station that monitors the Doppler-shifted freqn.



To find the Doppler effect for an observer in the direction at angle θ from the line of motion.

The period of flashes in the observer's rest frame is $\tau = \gamma\tau_0$ and freqn seen by observer is c/λ_D.

The Source moves a distance $v\tau$ b/w flashes and it is apparent from the sketch that

$$\lambda_D = c\tau - v\tau\cos\theta$$

$$= (c - v\cos\theta)\tau$$

$$\nu_D = \frac{c}{\lambda_D} = \frac{c}{(c - v\cos\theta)\tau_0\gamma}$$

$$\nu_D = \frac{\nu_0\sqrt{1 - v^2/c^2}}{\left(1 - \frac{v}{c}\cos\theta\right)}$$

where $T_0 = \frac{1}{\nu_0}$. Here θ is the angle measured in the rest frame of the observer. Along the line $\theta = 0$ and we recover our previous result. At $\theta = \frac{\pi}{2}$ the relative velocity b/w source and observer is zero. The classical Doppler effect would vanish here, but relativistically, there is a shift in freqn: ν_D differs from ν_0 by the factor $\sqrt{1-\frac{v^2}{c^2}}$ which is due to time dilation.