Qn $\lim_{x \to \infty} e^{-x} \cos x = ?$

Soln $\because$ $x \to \infty$ Given

Let $x = \frac{1}{t}$ $\because$ $x \to \infty \Rightarrow \frac{1}{t} \to \infty$

$\Rightarrow t \to 0$

Now

$$\lim_{x \to \infty} e^{-x} \cos x = \lim_{t \to 0} e^{-1/t} \cos \frac{1}{t}$$

Now $\lim_{t \to 0} \cos\left(\frac{1}{t}\right) e^{-1/t} = ?$

By using Result

If $h(x) = f(x).g(x)$ be a fn with either one of $f(x)$ or $g(x)$ is bdd and one of $\lim_{x \to 0} f(x) = 0$ or $\lim_{x \to 0} g(x) = 0$

then $\lim_{x \to 0} f(x).g(x) = 0$

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$\Rightarrow -\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$ — (I)

By using Squeeze theorem

let $g(x) \le f(x) < h(x)$ — (II)

and $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = l$

$\Rightarrow \lim_{x \to a} f(x) = l$

from (I) & (II) we get

$g(x) = -1/x \quad f(x) = \frac{\sin x}{x}$ and $h(x) = \frac{1}{x}$

$\therefore \lim_{x \to \infty} g(x) = -\lim_{x \to \infty} \frac{1}{x} = -0 = 0$

$\therefore \lim_{x \to \infty} h(x) = \lim_{x \to \infty} \frac{1}{x} = 0$

$\Rightarrow \lim_{x \to \infty} g(x) = \lim_{x \to \infty} h(x) = 0$

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The Sandwich Theorem (Squeeze theorem)

If $g(x) \le f(x) \le h(x)$ be a fn & $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = l$ then $\lim_{x \to a} f(x) = l$.

this is also valid for infinit limit

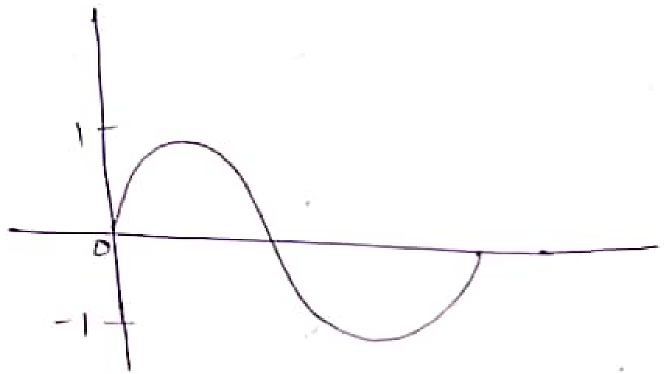
Q $\lim_{x \to \infty} \frac{\sin x}{x}$

$\because$ $\sin x$ is bdd fn & $-1 \le \sin x \le 1$

but $\lim_{x \to \infty} \sin x =$ does not exist

$\lim_{x \to a} f(x) =$ exist [exist mean limit ma be finit or infinite]

$\lim_{x \to a} f(x) =$ doesn't exist it mean limit is not unique.



$\Rightarrow \lim_{x \to \infty} \sin x =$ not defined [Oscilating finitly]

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$\because -\frac{1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}$

$\Rightarrow \lim_{x \to \infty} \frac{\cos x}{x} = 0$ $\left[\because \lim_{x \to \infty} -\frac{1}{x} = \lim_{x \to \infty} \frac{1}{x} = 0 \right]$

Now

$$\lim_{x \to \infty} \left[\frac{2 + \sin x / x}{1 + \cos x / x} \right] = \frac{2 + \lim_{x \to \infty} \frac{\sin x}{x}}{1 + \lim_{x \to \infty} \frac{\cos x}{x}}$$

$$= \frac{2+0}{1+0} = 2$$

Hence $\lim_{x \to \infty} \frac{2x + \sin x}{x + \cos x} = 2$

Qn $\lim_{x \to \pi/4} \frac{\sec x}{\tan x - 1}$ [Do yourself]

Qn $\lim_{x \to +\infty} \frac{\tan^{-1} x}{e^x}$ [Do yourself]

Qn $\lim_{x \to 0} x^2 \sin \frac{1}{x}$ [Do yourself]

Qn $\lim_{x \to 0} x^p \cos \frac{1}{x^q}$ = exist finitely for what value of p & q

[Do yourself]

$\Rightarrow$ By squeeze theorem

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{\sin x}{x} = 0$$

Qn $\lim_{x \to \infty} x \sin \frac{1}{x} = ?$

let $\frac{1}{x} = t \Rightarrow x = \frac{1}{t}$

$\because x \to \infty \Rightarrow \frac{1}{x} \to 0 \Rightarrow t \to 0$

$$\Rightarrow \lim_{x \to \infty} x \sin \frac{1}{x} = \lim_{t \to 0} \frac{\sin t}{t}$$

Now $\lim_{t \to 0} \frac{\sin t}{t}$ $\left[\frac{0}{0} \text{ form}\right]$

Applying L-Hopital rule

$$\lim_{t \to 0} \frac{\cos t}{1} = \cos 0 = 1$$

$$\Rightarrow \lim_{t \to 0} \frac{\sin t}{t} = 1$$

Hence $\lim_{x \to \infty} x \sin \frac{1}{x} = \lim_{t \to 0} \frac{\sin t}{t} = 1$

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Qn $\lim_{x \to \infty} \frac{2x + \sin x}{x + \cos x} = \lim_{x \to \infty} \frac{x}{x} \left[\frac{2 + \sin x / x}{1 + \cos x / x} \right]$

$= \lim_{x \to \infty} \left[\frac{2 + \frac{\sin x}{x}}{1 + \frac{\cos x}{x}} \right] = ?$

$\because \lim_{x \to \infty} \frac{\sin x}{x} = \infty$ [From last Qn]

Now $\lim_{x \to \infty} \frac{\cos x}{x} = ?$

$\because -1 \leq \cos x \leq 1$

If $x \neq 0$ $\quad -\frac{1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x}$

Let $g(x) = -\frac{1}{x}$ $\quad f(x) = \frac{\cos x}{x}$ and $h(x) = \frac{1}{x}$

$\therefore \lim_{x \to \infty} g(x) = \lim_{x \to \infty} -\frac{1}{x} = 0$

$\lim_{x \to \infty} h(x) = \lim_{x \to \infty} \frac{1}{x} = 0$

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$$\lim_{t \to 0} \cos\left(\frac{1}{t}\right) e^{-1/t} \quad \text{—(1)}$$

$\because$ $\cos x$ is bdd $\forall x$

$\Rightarrow$ $\cos \frac{1}{x}$ is also bdd $\forall x$

$\Rightarrow$ $\cos \frac{1}{t}$ is bdd $\forall t$

Now $\lim_{t \to 0} e^{-1/t} = e^{-\infty} = 0$

$\Rightarrow$ $\lim_{t \to 0} e^{-1/t} = 0$

By using last Result

$\cos \frac{1}{t}$ is bdd & $\lim_{t \to 0} e^{-1/t} = 0$

$\Rightarrow$ $\lim_{t \to 0} \cos\left(\frac{1}{t}\right) e^{-1/t} = 0$

$\Rightarrow$ $\lim_{x \to \infty} e^{-x} \cos x = 0$

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