Lecture-14

Thm If A is a real numbers and r is a positive rational number, then

$$\lim_{x \to +\infty} \frac{A}{x^r} = 0$$

Furthermore if r is such that x^r defined for $x<0$, then

$$\lim_{x \to -\infty} \frac{A}{x^r} = 0$$

proof To show firstly $\lim_{x \to +\infty} \frac{1}{x} = 0$

Let $\epsilon>0$ be given we must must find M s.t

$|f(x)-L|<\epsilon$ whenever $x>M$

$\Rightarrow \left|\frac{1}{x}-0\right|<\epsilon$ whenever $x>M$

$\Rightarrow \left|\frac{1}{x}\right|<\epsilon \Leftrightarrow -\epsilon<\frac{1}{x}<\epsilon$

$\Rightarrow \frac{1}{x}<\epsilon$

$\Rightarrow x>1/\epsilon$

Let $M=1/\epsilon$ (choise)

$$\Rightarrow \lim_{x \to +\infty}\left(\frac{1}{x}\right)=0$$

Now $\because$ r is rational numbers $\Rightarrow$ Let $r = p/q$

$p, q \in I$ & $q \neq 0$

$$\lim_{x \to \infty} \frac{A}{x^r} = \lim_{x \to +\infty} \frac{A}{x^{p/q}} = A \lim_{x \to +\infty} \left[\sqrt[q]{\frac{1}{x}}\right]^p$$

$$= \cancel{A\left[\sqrt[q]{\frac{1}{\infty}}\right]} = A\left[\sqrt[q]{\lim_{x \to +\infty} \frac{1}{x}}\right]^p \text{ [from ⑥ prop]}$$

$$= A\left[\sqrt[q]{0}\right]^p = A \cdot 0 = 0 \quad \left[\because \lim_{x \to +\infty} \frac{1}{x} = 0\right]$$

$$\Rightarrow \lim_{x \to +\infty} \frac{A}{x^r} = 0 \quad \text{if } r > 0$$

Similarly $\lim_{x \to -\infty} \frac{A}{x^r} = 0$ [Do yourself]

Some proper of lim $x \to \pm\infty$

If M, L, & K are real numbers and $\lim_{x \to \pm\infty} f(x) = L$ and $\lim_{x \to \pm\infty} g(x) = M$ then

1 $\lim_{x \to \pm\infty} (f(x) + g(x)) = L + M$

$\lim_{x \to \pm\infty} (f(x) - g(x)) = L - M$

3. $\lim_{x \to \pm\infty} (f(x) \cdot g(x)) = L \cdot M$

4. $\lim_{x \to \pm\infty} (K \cdot f(x)) = K \cdot L$, $\lim_{x \to \pm\infty} (K g(x)) = K M$

5. $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)} = \frac{L}{M}$ $M \neq 0$

6. Power rule: If r and s are integers with no common factor, $s \neq 0$ then

$$\lim_{x \to \pm\infty} (f(x))^{r/s} = \left(\lim_{x \to \pm\infty} (f(x))^{1/s}\right)^{r}$$
$$= L^{r/s}$$

Qn $\lim_{x \to \infty} \frac{2x^2+5x+1}{3x^3-2x+4}$ $\left[\frac{\infty}{\infty}\right]$

$$= \lim_{x \to \infty} \frac{x^2}{x^3}\left[\frac{2+5/x+1/x^2}{3-2/x^2+4/x^3}\right]$$

$$= \lim_{x \to \infty} \frac{1}{x}\left[\frac{2+5/x+1/x^2}{3-2/x^2+4/x^3}\right]$$

$$= 0 \cdot \left[\frac{2+0+0}{3-0+0}\right] = 0$$

$$\Rightarrow \lim_{x \to \infty} \frac{2x^2+5x+1}{3x^3-2x+4} = 0$$

Qn $\lim\limits_{x \to -\infty} \dfrac{2x^2-5x-3}{3x^2-x-20} \left[\dfrac{-\infty}{-\infty}\right]$

$= \lim\limits_{x \to -\infty} \dfrac{\cancel{x^2}(2-5/x-3/x^2)}{\cancel{x^2}(3-1/x-20/x^2)}$

$= \lim\limits_{x \to -\infty} \dfrac{2-\frac{5}{x}-\frac{3}{x^2}}{3-\frac{1}{x}-\frac{20}{x^2}} = \lim\limits_{x \to -\infty} \dfrac{2+\frac{5}{-x}-\frac{3}{x^2}}{3+\frac{1}{-x}-\frac{20}{x^2}}$

$= \dfrac{2+0-0}{3+0-0} = \dfrac{2}{3}$

$\Rightarrow \lim\limits_{x \to -\infty} \left(\dfrac{2x^2-5x-3}{3x^2-x-20}\right) = \dfrac{2}{3}$

Qn $\lim\limits_{x \to \infty} \left(\dfrac{2x^2+5x-1}{x+2}\right) \left(\dfrac{\infty}{\infty}\right)$

$= \lim\limits_{x \to \infty} \dfrac{x^{\cancel{2}}}{\cancel{x}} \left(\dfrac{2+5/x-1/x^2}{1+2/x}\right)$

$= \lim\limits_{x \to \infty} x \left(\dfrac{2+\frac{5}{x}-\frac{1}{x^2}}{1+\frac{2}{x}}\right)$

$= \infty \cdot \left[\dfrac{2+0-0}{1+0}\right] = \infty$

$\Rightarrow$ ~~to~~

Expt. No.

$\Rightarrow \lim_{x \to \infty} \frac{2x^2+5x-1}{x+2} = \infty$

Qn $\lim_{x \to +\infty} \sqrt{\frac{3x-5}{x-2}}$ [Do yourself]

Qn $\lim_{x \to +\infty} \left(\frac{3x-5}{x-2}\right)^3$ [Do yourself]

Qn $\lim_{x \to -\infty} \frac{95x^3+57x+30}{x^5-1000}$ [Do yourself]

Qn $\lim_{x \to \infty} e^{-x}\cos x$ [Do your self]

Qn $\lim_{x \to \infty} x^{-4/3}(2x+5)$ [Do your self]

Qn By using def^n to show

(i) $\lim_{x \to +\infty} \frac{1}{x^2} = 0$ [Do your self]

(ii) $\lim_{x \to -\infty} \frac{1}{x^2} = 0$ [Do your self]

Date

Teacher's Signature :