18/04/2020 Lecture - 12

Qn Trace the curve

$5x^2 - 6xy + 5y^2 - 8\sqrt{2}x + 8\sqrt{2}y - 8 = 0$ — (1)

Rotating co-ordinate axis (x, y) to (x', y') through angle θ

Compare with

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A = 5, B = -6, C = 5$

$B^2 - 4AC = 36 - 100 = -74 < 0$

$\Rightarrow$ eqn is ellipse

find angle θ $\quad \cot 2\theta = \dfrac{A - C}{B} = \dfrac{5 - 5}{-6} = 0$

$\Rightarrow \cot 2\theta = 0 \Rightarrow 2\theta = 90 \Rightarrow \theta = 45°$

put the value of $\theta = 45°$ in the rotation eqn

$x = x'\cos\theta - y'\sin\theta, \quad y = x'\sin\theta + y'\cos\theta$

$x = x'\cos 45° - y'\sin 45°, \quad y = x'\sin 45 + y'\cos 45°$

$x = \dfrac{x'}{\sqrt{2}} - \dfrac{y'}{\sqrt{2}}, \quad y = \dfrac{x'}{\sqrt{2}} + \dfrac{y'}{\sqrt{2}}$

put the value of x & y in the eqn (1)
we get

$$\Rightarrow 5\left(\frac{x'-y'}{\sqrt{2}}\right)^2 - 6\left(\frac{x'-y'}{\sqrt{2}}\right)\cdot\left(\frac{x'+y'}{\sqrt{2}}\right) + 5\left(\frac{x'+y'}{\sqrt{2}}\right)^2$$

$$+ 8\cancel{\sqrt{2}}\left(\frac{x'-y'}{\cancel{\sqrt{2}}}\right) + 8\sqrt{2}\left(\frac{x'+y'}{\cancel{\sqrt{2}}}\right) - 8 = 0$$

$$\Rightarrow \frac{5}{2}\left[(x')^2 + (y')^2 - 2x'y'\right] - \frac{6}{2}\left[(x')^2 - (y')^2\right]$$

$$+ 5\left(\frac{(x')^2 + (y')^2 + 2x'y'}{2}\right) - \cancel{8x'} + 8y' + \cancel{8x'} + 8y' - 8 = 0$$

$$\Rightarrow \frac{10(x')^2 + 10(y')^2}{2} - 3(x')^2 + 3(y')^2 + 16y' - 8 = 0$$

$$\Rightarrow 5(x')^2 + 5(y')^2 - 3(x')^2 + 3(y')^2 + 16y' - 8 = 0$$

$$\Rightarrow 2(x')^2 + 8(y')^2 + 16y' = 8$$

$$\Rightarrow (x')^2 + 4(y')^2 + 8y' = 4$$

$$\Rightarrow (x')^2 + 4\left[(y')^2 + 2y'\right] = 4$$

$$\Rightarrow (x')^2 + 4\left[(y')^2 + 2y' + 1 - 1\right] = 4$$

$$\Rightarrow (x')^2 + 4\left[(y')^2 + 2y + 1\right] - 4 = 4$$

$$\Rightarrow (x')^2 + 4(y'+1)^2 = 8$$

$$\Rightarrow \frac{(x')^2}{8} + \frac{4(y'+1)^2}{8} = 1 \quad \text{--- (2)}$$

$a^2 = 8 \quad b^2 = 2$

$a = 2\sqrt{2} \quad b = \sqrt{2}$

Centre of ellips $(0,-1)$ w.r.t (x', y')

~~Let~~ shifting origin to centre using transformation

$x' = x + h, \quad y' = y + k$

$x' = x \; , \; y' = y - 1$

putting these value in (2) we get

$$\frac{x^2}{8} + \frac{y^2}{2} = 1$$

major axes is along x-axes $a^2 = 8$

$b^2 = 2$

vertex w.r.t (x, y) is $(+2\sqrt{2}, 0), (-2\sqrt{2}, 0)$

vertex w.r.t $(x'y')$ is $(2\sqrt{2}, -1), (-2\sqrt{2}, -1)$

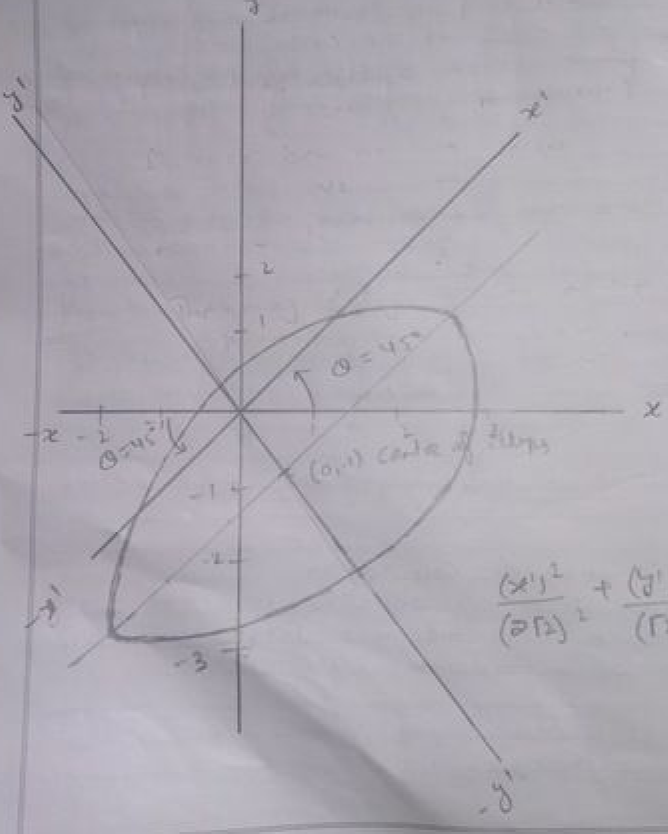
$c = a^2 - b^2 = 8 - 2 = 6 \Rightarrow c = \sqrt{6}$

focus w.r.t (x, y) $(\sqrt{6}, 0), (-\sqrt{6}, 0)$

foci w.r.t $(x'y')$ $(\sqrt{6}, -1), (-\sqrt{6}, -1)$

Length of major axes $= 2a = 4\sqrt{2}$

minor axes $= 2b = 2\sqrt{2}$



$$\frac{(x')^2}{(2\sqrt{2})^2} + \frac{(y')^2}{(\sqrt{2})^2} = 1$$

~~Location also ... of error~~

~~* fixed edge should be ... edge close to ...~~

~~* ... of both the keys should be along the small line.~~

~~* ... should be removed tip to tip.~~

point of intersection w.r.t (x, y)

put $y=0$ in ① for intersecting x-axes

$5x^2 - 8\sqrt{2}x - 8 = 0$

$$x = \frac{8\sqrt{2} \pm \sqrt{288}}{2 \times 5}$$

$x = 2.8$

$\cancel{x = -6}$ $x = -0.56$

for intersecting y-axis put $x = 0$ in ①

$5y^2 + 8\sqrt{2}y - 8 = 0$

$$y = \frac{-8\sqrt{2} \pm \sqrt{288}}{10}$$

$$y = \frac{-8\sqrt{2} + \sqrt{288}}{10} \qquad y = \frac{-8\sqrt{2} - \sqrt{288}}{10}$$

$y \approx 0.56$ $\qquad y \approx -2.8$

Qn $6x^2+24xy-y^2-12x+26y+11=0$ —(1)

compare with $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=6,\ B=24,\ C=-1$

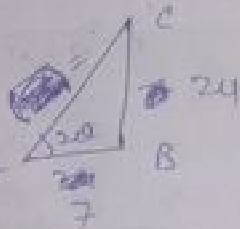
$B^2-4AC = 576+24>0$

$\Rightarrow$ eqn (1) hyperbola

find angle θ $\qquad \cot 2\theta = \frac{A-C}{B} = \frac{6+1}{24}$

$\cot 2\theta = \frac{7}{24}$

$AC = \sqrt{24^2+7^2} = 25$



$\cos 2\theta = \cancel{\frac{24}{25}} = \frac{7}{25}$

$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \cancel{\sqrt{\frac{1+\frac{24}{25}}{2}}} = $

$= \sqrt{\frac{1+7/25}{2}} = \sqrt{\frac{32}{50}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\cos\theta = 4/5$

$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-7/25}{2}} = \sqrt{\frac{18}{50}} = \frac{3}{5}$

$\sin\theta = 3/5 \qquad \cos\theta = 4/5$

$\tan\theta = \frac{3/5}{4/5} \Rightarrow \theta = \tan^{-1}(3/4)$

put the value of $\cos\theta$, $\sin\theta$ in the rotation eqn

$x = x'\cos\theta - y'\sin\theta$, $y = x'\sin\theta + y'\cos\theta$

$\Rightarrow x = \frac{4x'}{5} - \frac{3}{5}y'$, $y = \frac{3x'}{5} + \frac{4}{5}y'$

putting ~~the~~ in ① value of x & y

$$\Rightarrow 6\left(\frac{4x'-3y'}{5}\right)^2 + 24\left(\frac{4x'-3y'}{5}\right)\cdot\left(\frac{3x'+4y'}{5}\right)$$

$$+ \left(\frac{3x'+4y'}{5}\right)^2 - 12\left(\frac{4x'-3y'}{5}\right) + 26\left(\frac{3x'+4y'}{5}\right) + 11 = 0$$

$$\Rightarrow \frac{6}{25}\left(16(x')^2 + 9(y')^2 - 24x'y'\right) + 24\left(\frac{12}{25}(x')^2 + \frac{16}{25}x'y' - \frac{9}{25}x'y' - \frac{12}{25}(y')^2\right)$$

$$- \frac{1}{25}\left(9(x')^2 + 16(y')^2 + 24x'y'\right) - \frac{48x'}{5} + \frac{36y'}{5}$$

$$+ \frac{78x'}{5} + \frac{104y'}{5} + 11 = 0$$

$$\Rightarrow 375(x')^2 - 250(y')^2 + 150x' + 700y' + 275 = 0$$

divided by 25

$$15(x')^2 - 10(y')^2 + 6x' + 28y' + 11 = 0$$

$$\Rightarrow 15(x')^2 + 6x' - 10(y')^2 + 28y' + 11 = 0$$

$$\Rightarrow 15\left[(x')^2 + \frac{6}{25}x'\right] - 10\left((y')^2 + \frac{28y'}{10}\right) + 11 =$$

$\Rightarrow 15\left[\left(x'+\frac{1}{5}\right)^2-\frac{1}{25}\right]-10\left[\left(y'-7/5\right)^2-\frac{49}{25}\right]+11=0$

$\Rightarrow 15\left(x'+\frac{1}{5}\right)^2-\frac{15}{25}-10\left(y'-7/5\right)^2+\frac{\cancel{490}\,98}{\cancel{25}\,5}+11=0$

$\Rightarrow 15\left(x'+\frac{1}{5}\right)^2-\frac{3}{5}-10\left(y'-7/5\right)+\frac{98}{5}+11=0$

$\Rightarrow 15\left(x'+\frac{1}{5}\right)^2-\frac{3}{5}+\frac{98}{5}+11-10\left(y'-7/5\right)=0$

$\Rightarrow 15\left(x'+\frac{1}{5}\right)^2+\frac{95+55}{5}-10\left(y'-7/5\right)=0$

$\Rightarrow 15\left(x'+1/5\right)^2-10\left(y'-7/5\right)=-30$

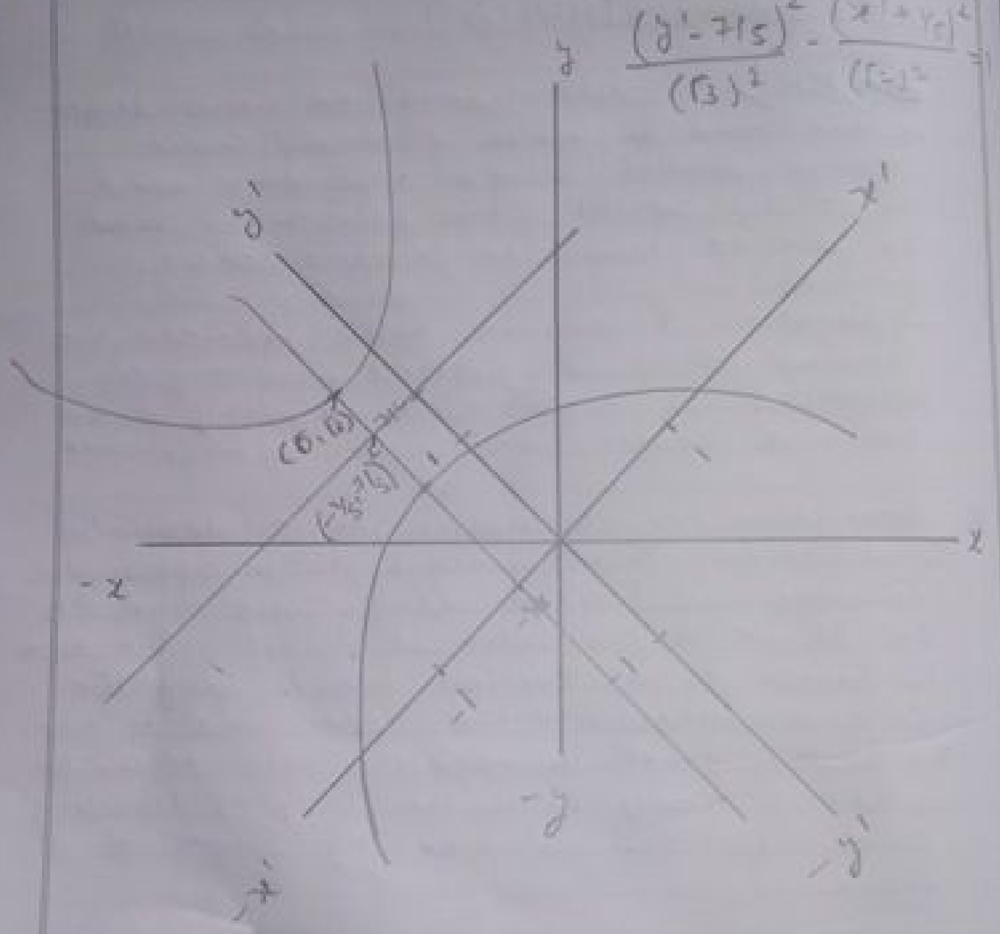
$\Rightarrow 10\left(y'-7/5\right)^2-15\left(x'+1/5\right)^2=30$

$\Rightarrow \frac{10\left(y'-7/5\right)^2}{30}-\frac{15\left(x'+1/5\right)^2}{30}=1$

$\Rightarrow \frac{\left(y'-7/5\right)^2}{3}-\frac{\left(x'+1/5\right)^2}{2}=1$

centre $(h, k) = (-1/5, 7/5)$

~~......~~ thus hyperbola is ~~...~~ conjugate hyperbola



for intersecting x-axis put y = 0 in (1)

$6x^2 - 12x + 11 = 0$ $B^2 - 4ac = 144 - 4 \times 6 \times 11 < 0$

no real roots =1 not intersect

x-axis

for y axis put x = 0 $-y^2 + 26y + 11 = 0$

$y = 26.44$ $y = -0.41$