Lecture - 9

The general equation is

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad F \to \text{constant}$$
$$\text{— (A)}$$

(i) if $B^2 - 4AC = 0$ then equation (A) rep parabola
(ii) if $B^2 - 4AC > 0$ then eqn (A) is hyperbola
(iii) if $B^2 - 4AC < 0$ then eqn (A) is ellipse

V.Imp Equation of conic where axis of conic is neither ∥ to x-axis nor ∥ to y-axis
In this case we rotate the axis in angle θ

Construction of rotation of axis with angle θ

Step 1 Construct (x, y) quadrant with point P(x,y) in Ist quadrant

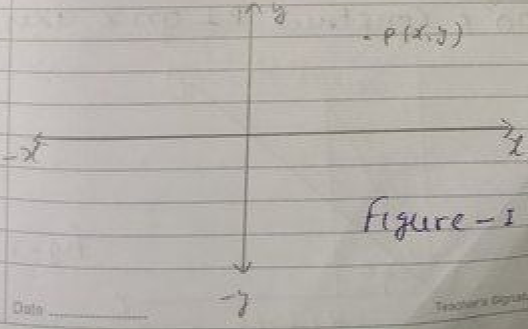


Figure - I

Step 2 Rotate (x,y) quadrant with angle θ into (x',y') quadrant & P(x,y) pt also lie (x'y') quadrant

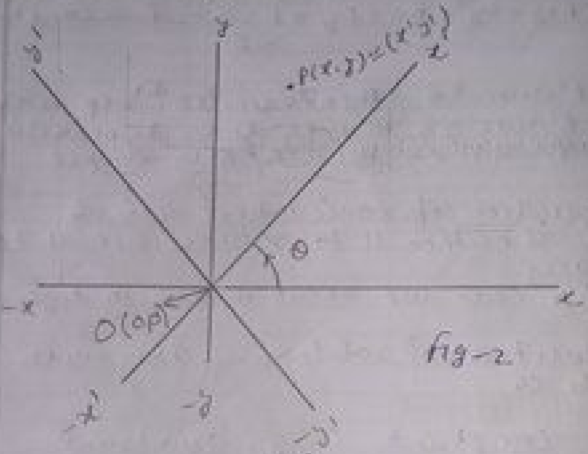


fig-2

Step 3 Draw PO = r (let) & construct P⊥ on x axis.

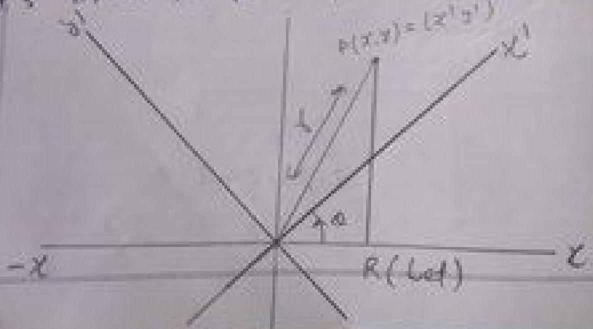


fig-3

Step 4 Draw $\perp$ from P to on x' axis and Δ POR' with angle α

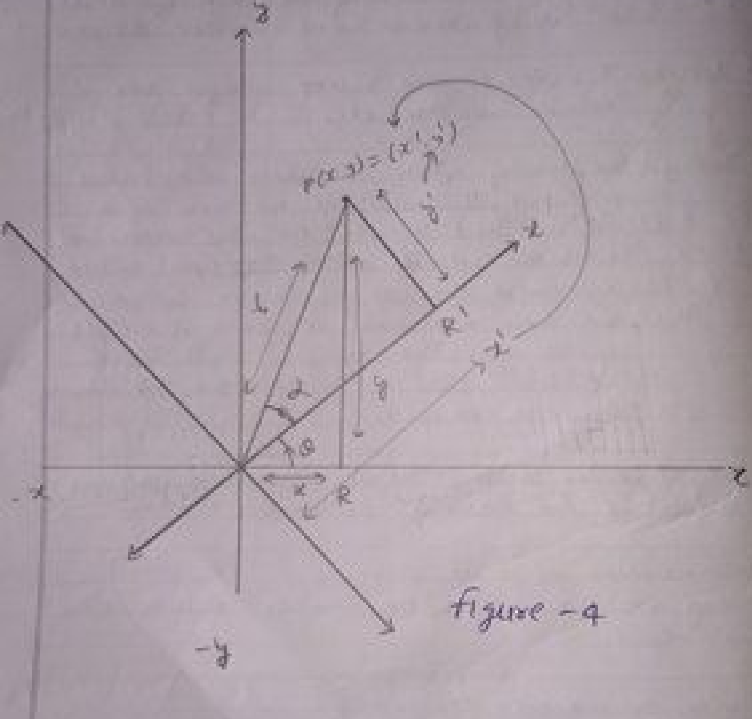


figure -4

Step 5 in $\Delta POR'$ from fig-4

fig-5

Step 6 in ΔPOR from fig-4

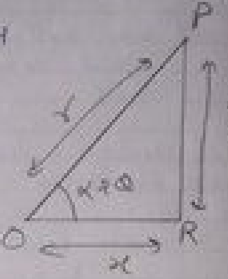


fig-

Now from fig-5 $\Delta POR'$

$$\cancel{\cos\alpha = \frac{\text{Base}}{\text{hight}} = \frac{OR'}{PR'} = x'}$$

$$\cos\alpha = \frac{OR'}{OP} = \frac{x'}{r} \Rightarrow x' = r\cos\alpha \quad \text{—(i)}$$

$$\sin\alpha = \frac{PR'}{OP} = \frac{y'}{r} \Rightarrow y' = r\sin\alpha \quad \text{—(ii)}$$

Now from fig 6 i.e in ΔPOR

$$\cos(\alpha+\theta) = \frac{OR}{OP} = \frac{x}{r} \Rightarrow x = r\cos(\alpha+\theta)$$

$$\sin(\alpha+\theta) = \frac{PR}{OP} = \frac{y}{r} \Rightarrow y = r\sin(\alpha+\theta) \text{ —(}$$

Since $x = r\cos(\alpha+\theta)$

$= r[\cos\alpha\cos\theta - \sin\alpha\sin\theta]$

[$\because \cos(x+y) = \cos x\cos y - \sin x\sin y$]

$x = \underline{r\cos\alpha}\cos\theta - \underline{r\sin\alpha}\sin\theta$

from ① & ② $x' = r\cos\alpha$, $y' = r\sin\alpha$

$\Rightarrow x = x'\cos\theta - y'\sin\theta$

$\Rightarrow \boxed{x = x'\cos\theta - y'\sin\theta}$

Since $y = r\sin(\alpha+\theta)$

$\Rightarrow y = r[\sin\alpha\cos\theta + \cos\alpha\sin\theta]$

[$\because \sin(x+y) = \sin x\cos y + \cos x\sin y$]

$y = \underline{r\sin\alpha}\cos\theta + \underline{r\cos\alpha}\sin\theta$

from ① & ② $x' = r\cos\alpha$, $y' = r\sin\alpha$

$\Rightarrow y = y'\cos\theta + x'\sin\theta = x'\sin\theta + y'\cos\theta$

$\Rightarrow \boxed{y = x'\sin\theta + y'\cos\theta}$

the eqn $x = x'\cos\theta - y'\sin\theta$

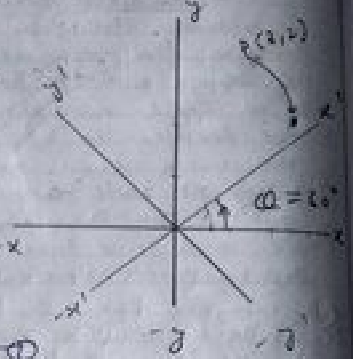
$y = x'\sin\theta + y'\cos\theta$

this relationship called the rotation eqn.

Qn find the new co-ordinate of pt (3,2) if the co-ordinate axis are rotated through an ∠ 30°

Soln $\theta = 30°$

Let (x', y') be new co-ordinate of P w.r.t (x', y')



Then By using rotation eqn

$x = x'\cos\theta - y'\sin\theta$ — (i)

$y = x'\sin\theta + y'\cos\theta$ — (ii)

Given P(3,2) w.r.t (x, y) & $\theta = 30°$

from (i) $3 = x'\cos 30° - y'\sin 30°$

$3 = \frac{\sqrt{3}x'}{2} - \frac{y'}{2}$

$6 = \sqrt{3}x' - y'$ — (iii)

from (ii) $2 = x'\sin 30° + y'\cos 30°$

$2 = \frac{x'}{2} + \frac{\sqrt{3}y'}{2}$

$4 = x' + \sqrt{3}y'$ — (iv)

from (11) & (12) we get

$6\sqrt{3} = 3x' - \sqrt{3}y'$

$4 = x' + \sqrt{3}y'$

$4x' = 6\sqrt{3} + 4$

$x' = \frac{6\sqrt{3}+4}{4} \Rightarrow x' = \frac{4+6\sqrt{3}}{4}$

and $y' = \frac{2\sqrt{3}-3}{2}$

Qn Suppose that the axes of an xy-co-ordinate system are rotated through an angle of $\theta = 45°$ to obtain $x'y'$-co-ordinate system. find the eq^n of the curve

$x^2 - xy + y^2 - 6 = 0$

in the (x', y') co-ordinates.

Sol^n $x^2 - xy + y^2 - 6 = 0 \quad -(1)$

Compare with the

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$\Rightarrow A = 1, \ B = -1, \ C = 1$

$B^2 - 4AC = (-1)^2 - 4 \times 1 \times 1 = -3 < 0$

$\Rightarrow$ eq^n rep ellipse

$\theta = 45^\circ$

rotation equation

$x = x'\cos\theta - y'\sin\theta \;,\; y = x'\sin\theta + y'\cos\theta$

$\Rightarrow x = x'\cos 45^\circ - y'\sin 45$

$x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$

$y = x'\sin 45^\circ + y'\cos 45^\circ$

$y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$

put in ① value of $x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}} \;,\; y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$

we get

$$\left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)^2 - \left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) + \left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)^2 = 6$$

$$\frac{(x')^2 + (y')^2 - 2x'y'}{2} - \frac{(x')^2 - (y')^2}{2} + \frac{(x')^2 + (y')^2 + 2x'y'}{2} = 6$$

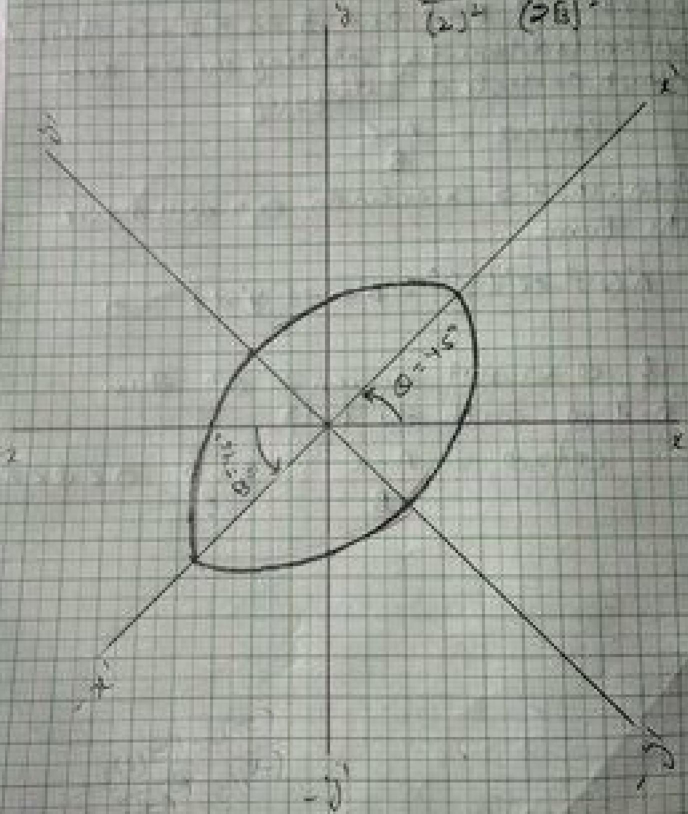
$$\frac{(x')^2 + (y')^2 - \cancel{2x'y'} - \cancel{(x')^2} + (y')^2 + \cancel{(x')^2} + (y')^2 + \cancel{2x'y'}}{2} = 6$$

$\cancel{(x')^2 + (3y')^2}$ $(x')^2 + 3(y')^2 = 6\times 2$

$$\Rightarrow \boxed{\frac{(x')^2}{12} + \frac{(y')^2}{4} = 1}$$ eqn of ellipse w.r.t (x', y')

V.IMP

$$\frac{(x')^2}{(2)^2}+\frac{(y')^2}{(26)^2}=1$$



If the equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad -(1)$$

is such that $B \neq 0$ and if an $x'y'$-co-ordinate system is obtained by rotating the xy-axes through an angle θ satisfying

$$\cot 2\theta = \frac{A-C}{B}$$

then, in $x'y'$ co-ordinates, eqn will have the form

$$A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$$

If θ is not given in Qn then find θ with the help

$$\cot 2\theta = \frac{A-C}{B} \quad B \neq 0 \quad 0 < \theta < \pi/2$$

Q. Find new co-ordinate of the pt $(2,4)$ if the co-ordinate axes are ~~rotad~~ rotated through an angle of $\theta = 30°$ [Do yourself]

Q. Let $\theta = 60°$. Find the eqn of the curve $\sqrt{3}xy + y^2 = 6$ in the $x'y'$-co-ordinate

[Do yourself]

Q. If $\theta = 45°$ find the $x'y'$ co-ordinates of the pt whose xy-co-ordinates are $(3, -\sqrt{3})$

[Do yourself]

17/04/2020

Lecture-10

Qn Identify and sketch the curve $xy=1$ also find the new eqn

Soln since we have

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

Compare with we get $A=0, B=1$
$C=0$

Now $B^2-4AC = 1-4\times0\times0$
$=1>0$

$\because B^2-4AC>0 \Rightarrow xy=1$ is the hyperbola

$\because \theta$ is not given

so for finding angle $=\theta$

we have $\cot 2\theta = \frac{A-C}{B} = \frac{0-0}{1} = 0$

$\Rightarrow \cot 2\theta = 0$

$\Rightarrow 2\theta = 90 = \pi/2$

$\Rightarrow \theta = \pi/4 = 45^\circ$

we have rotation eqn

$x = x'\cos\theta - y'\sin\theta,$

$y = x'\sin\theta + y'\cos\theta$

$\therefore \; x = x'\cos\alpha - y'\sin\alpha \quad \alpha = \pi/4$

$x = x'\cos\pi/4 - y'\sin\pi/4$

$x = \dfrac{x'}{\sqrt{2}} - \dfrac{y'}{\sqrt{2}}$

$y = x'\sin\alpha + y'\cos\alpha \;, \; \alpha = \pi/4$

$= x'\sin\pi/4 + y'\cos\pi/4$

$y = \dfrac{x'}{\sqrt{2}} + \dfrac{y'}{\sqrt{2}}$

put the value x & y in $xy = 1$

we get

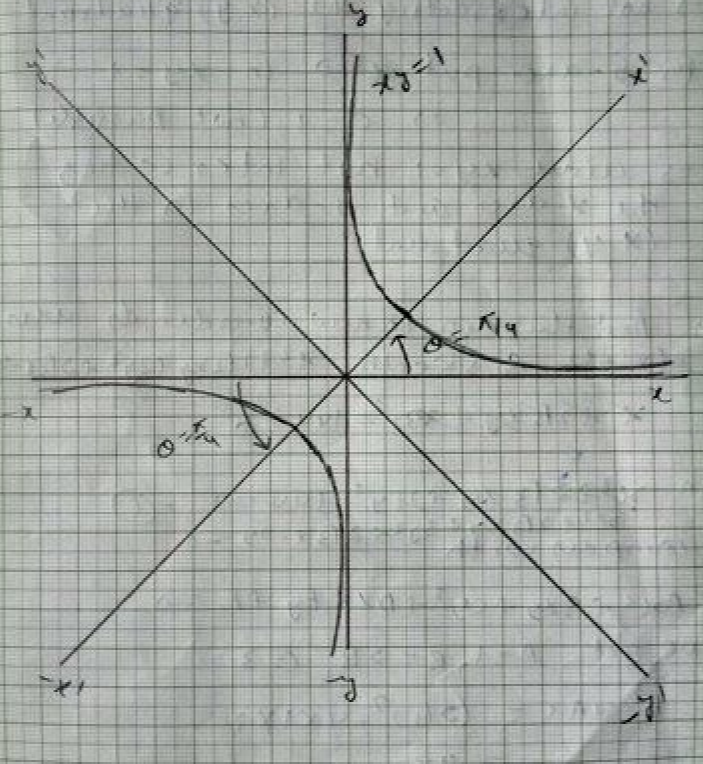
$\left(\dfrac{x'}{\sqrt{2}} - \dfrac{y'}{\sqrt{2}}\right)\left(\dfrac{x'}{\sqrt{2}} - \dfrac{y'}{\sqrt{2}}\right) = 1$

$\Rightarrow \dfrac{(x')^2}{2} - \dfrac{(y')^2}{2} = 1 \quad - (1)$

this is the required eqn of hyperbola w.r.t new co-ordinate axes (x', y')

$\therefore$ hyperbola rotated an angle $\alpha = \pi/4$ in the co-ordinate (x, y)

$$\frac{(x')^2}{(\sqrt{2})^2} - \frac{(y')^2}{(\sqrt{2})^2}$$



For intersecting (x, y) co-ordinates
for x-axes put $y = 0$ in $xy = 1$
$0 = 1$ not possible
$\Rightarrow$ not intersecting the (x, y) quadrant

For y-axes put $x = 0$ in $xy = 1$
$\Rightarrow 0 = 1$ (not possible)
$\Rightarrow$ curve $xy = 1$ not intersecting the x-axes and y-axes in the (x, y) quadrant

Qn find the eqn in new co-ordinate axes $(x' y')$ & sketch $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y$

$\cancel{x^2 + 2\sqrt{3}xy + 3y^2 - 14y = 0}$

Soln $\cancel{x^2 + 2\sqrt{3}xy + 2y^2 - 14y = 0}$ — ①

$x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$

compare with

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

we get $A = 1, B = 2\sqrt{3}, C = 3$

$B^2 - 4AC = (2\sqrt{3})^2 - 4 \times 1 \times 3$
$= 4 \times 3 - 12 = 0$

$\Rightarrow B^2 - 4AC = 0 \Rightarrow$ eqn ① is parabola to find angle θ

$$\cot 2\theta = \frac{A-C}{2B} = \frac{3-1}{2\sqrt{3}} = \frac{\cancel{2}}{\cancel{2}\sqrt{3}}$$

$$\cot 2\theta = \frac{1}{\sqrt{3}} \Rightarrow 2\theta = 60^\circ$$

$$\Rightarrow \theta = 30^\circ = \pi/6$$

we have rotating angle $\theta = \pi/6$

By using rotation eq^n

$x = x'\cos\theta - y'\sin\theta$, $y = x'\sin\theta + y'\cos\theta$

$x = x'\cos\frac{\pi}{6} - y'\sin\frac{\pi}{6}$, $y = x'\sin\pi/6 + y'\cos\pi/6$

$x = \frac{\sqrt{3}x'}{2} - \frac{y'}{2}$, $y = \frac{x'}{2} + \frac{\sqrt{3}y'}{2}$

put the value of x, y in eq^n

$$x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$$

$$\left(\frac{\sqrt{3}x'}{2} - \frac{y'}{2}\right)^2 + 2\sqrt{3}\left(\frac{\sqrt{3}x'}{2} - \frac{y'}{2}\right)\left(\frac{x'}{2} + \frac{\sqrt{3}}{2}y'\right)$$

$$+ 3\left(\frac{x'}{2} + \frac{\sqrt{3}}{2}y'\right)^2 + \cancel{2}\sqrt{3}\left(\frac{\sqrt{3}}{\cancel{2}}x' - \frac{y'}{\cancel{2}}\right) - \cancel{2}\left(\frac{x'}{\cancel{2}} + \frac{\sqrt{3}y'}{\cancel{2}}\right) = 0$$

$$\frac{3(x')^2}{4} + \frac{(y')^2}{4} - \frac{\cancel{2}\sqrt{3}x'y'}{\cancel{4}} + \cancel{2}\sqrt{3}\left(\frac{\sqrt{3}(x')^2 + 3x'y' - y'x' - \sqrt{3}(y')^2}{\cancel{4}}\right)$$

$$+ 3\left(\frac{(x')^2}{4} + \frac{3(y')^2}{4} + \frac{2\sqrt{3}x'y'}{2}\right) + 3x' - \sqrt{3}y' - x' - \sqrt{3}y' = 0$$

(8) (9)

$$\frac{3(x')^2}{4} + \frac{(y')^2}{4} - \cancel{\sqrt{3}x'y'} + 3(x')^2 + 2\sqrt{3}x'y' - 3(y')^2$$

$$+ \frac{3(x')^2}{4} + \frac{3(y')^2}{4} + \cancel{\sqrt{3}x'y'} + 2x' - 2\sqrt{3}y' = 0$$

$$\cancel{6\frac{(x')^2}{4} + 4\frac{(y')^2}{4}}$$

After solving we get

$$y' = (x')^2 \quad (1)$$

For intersecting x-axes in (x, y) quadrant
put $y = 0$ in $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$

$\Rightarrow x^2 + 2\sqrt{3}x = 0 \quad x(x + 2\sqrt{3}) = 0$

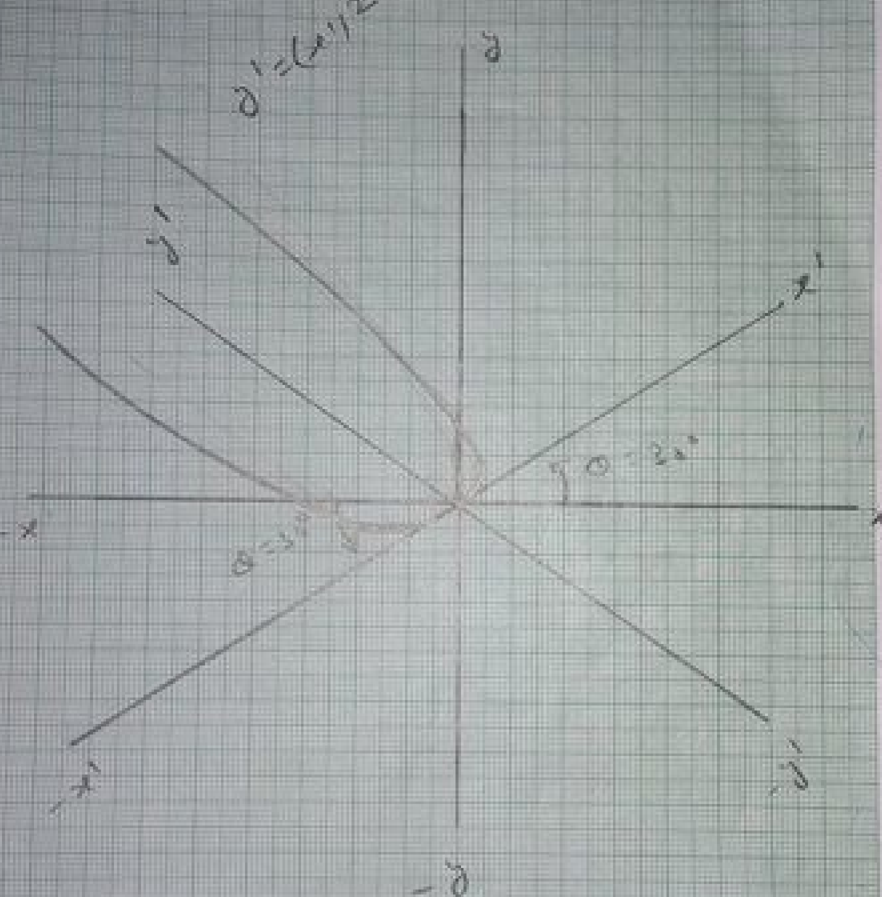
$x = 0, \quad x = -2\sqrt{3}$

$\Rightarrow$ eqn (1) intersect (x y) quadrant at x-axes at $x = -2\sqrt{3}$

For intersecting y-axes in (x, y) quadrant
put $x = 0$ in $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$

$3y^2 - 2y = 0 \quad y(3y - 2) = 0 \quad y = 2/3$

$\Rightarrow$ eqn (1) intersect (x, y) quadrant at y axes at $y = 2/3$


y' = (x')^2
y
y'
x'
-x
x
-x'
-y'
-y

Qn 1 find angle θ & rotation curve ~~Sketch~~ Sketch the curve

(1) $2x^2+xy+2y^2+x-y=0$

(2) $x^2+2\sqrt{3}xy+3y^2-2x+y=1$

(3) $3x^2+\sqrt{3}xy+2y^2+y=0$

[Do your self]

Qn 2 Let $x'y'$-Co-ordinate system be obtained by rotating an xy-co-ordinate system through an angle of $45°$ find an equation of the curve $3(x')^2+(y')^2=10$ in x-y co-ordinates also ~~Sketch~~ draw the curve.

[Do your self]

Qn 3 $5x^2-6xy+5y^2-8\sqrt{2}x+8\sqrt{2}y=8$

Draw the curve

[Do your self]