Vertices $(\pm 2, 0)$

Foci $(\pm c, 0)$

$c = \sqrt{a^2+b^2} = \sqrt{4+9} = \sqrt{13}$

$c = \sqrt{13}$

Foci $(\pm\sqrt{13}, 0)$

~~to find the asymptote~~

Step(2) Draw a box $a=2$ units from center $(0,0)$ to focal axis (x-axis) and $b=3$ from center $(0,0)$ to y-axis

Step find asymptotes. Draw the diagonal of box.

For finding asymptote

Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \Rightarrow y = \pm\frac{b}{a}x$

$\Rightarrow y = \pm\frac{3}{2}x$ i.e $y = \frac{3}{2}x$ and $y = -\frac{3}{2}x$

Q $y^2 - x^2 = 1$ Draw the hyperbola

[Do yourself]

$\rightarrow$ Asymptotes $y = -\frac{b}{a}x \Rightarrow bx + ay = 0$ — (i)

and $y = \frac{bx}{a} \Rightarrow bx - ay = 0$ — (ii)

are the Asymptotes of the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Now (i) $\times$ (ii)

$\Rightarrow (bx+ay)(bx-ay) = 0$

$\Rightarrow b^2x^2 - a^2y^2 = 0$

divided by eq,n a^2b^2

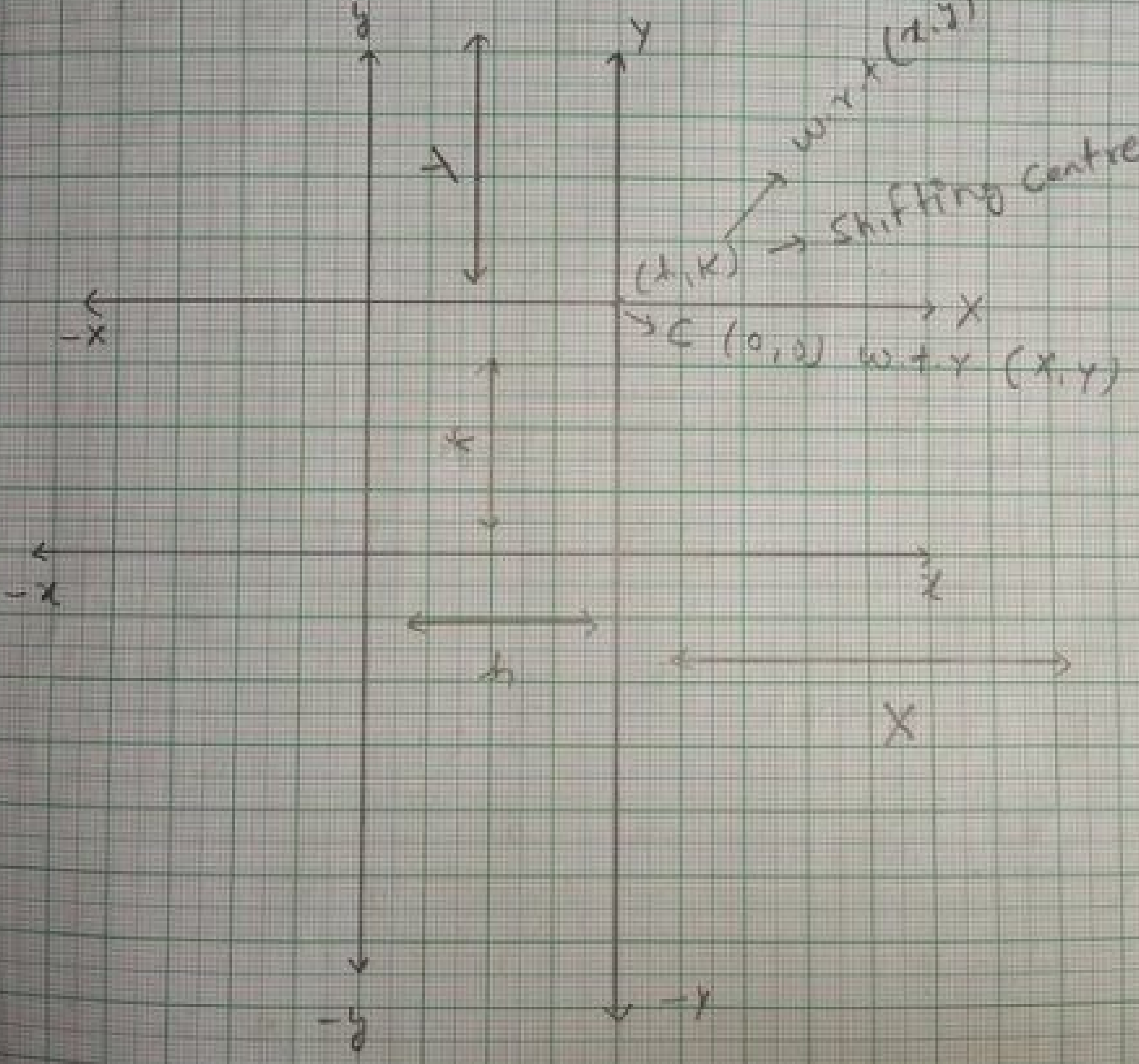
$\frac{b^2x^2}{a^2b^2} - \frac{a^2y^2}{a^2b^2} = 0$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ is eqn of asymptotes

\# the joint equation of asymptotes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

is given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

Remark: we observe that the eq,n of hyperbola differ from the joint equation of asymptotes by a constant only "1"


y
Y
(h,k)
w.r.t (x,y)
→ Shifting centre
X
-x
C (0,0) w.r.t (X,Y)
k
-x
x
h
X
-y
-y

Ex Equation of asymptotes of hyperbola $y=x$ & $y=-x$ find equation of hyperbola passing through a pt $(\sqrt{2},1)$.

Soln Joint eqn of asymptotes is

$(y+x)(y-x)=0 \Rightarrow y^2-x^2=0$ — ①

or $x^2-y^2=0$

Let required equation of hyperbola be

$x^2-y^2+c=0$

it passes through $(\sqrt{2},1)$

$\Rightarrow 2-1+c=0$

$\Rightarrow c=-1$

$\Rightarrow$ eqn of hyperbola is $x^2-y^2+c=0$

$\Rightarrow x^2-y^2-1=0$

$\Rightarrow x^2-y^2=1$

case I

Imp Shifting of hyperbola with centre (h,k)

Shifting origin to centre (h,k) by using transformation eqn

$x=X+h, \quad y=Y+k$ — ①

then the centre "C" become origin w.r.t (X,Y)

at C $x=h$, & $y=k$

$\Rightarrow \cancel{h}=X+\cancel{h}, \quad \cancel{k}=Y+\cancel{k}$

$\Rightarrow X=0$ & $Y=0$

$\Rightarrow$ the resulting equation by case I in new co-ordinate system (X,Y) be

$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

or

$\frac{X^2}{a^2}-\frac{Y^2}{b^2}=1$

In this case case focal axis parallel to X-axis and conjugate axis is parallel to Y-axis

Q Trace the curve $x^2-y^2-4x+8y-21=0$

Soln $x^2-y^2-4x+8y-21=0$

use completing square method

$x^2-4x-y^2+8y-21=0$

$x^2 - 4x - (y^2 - 8y) - 21 = 0$

$(x^2 - 4x + 4 - 4) - (y^2 - 8y + 16 - 16) - 21 = 0$

$\Rightarrow (x-2)^2 - 4 - (y-4)^2 + 16 - 21 = 0$

$\Rightarrow (x-2)^2 - (y-4)^2 = 9$

divided By 9 both side

$\frac{(x-2)^2}{9} - \frac{(y-4)^2}{9} = 1$

$\Rightarrow \frac{(x-2)^2}{3^2} - \frac{(y-4)^2}{3^2} = 1 \quad (1)$

Comparing it with $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad (2)$

$\Rightarrow a = 3,\ b = 3$ and $h = 2,\ k = 4$

$\Rightarrow$ centre $= (h,k) = (2,4)$

Shifting origin to centre using

$x = X + h$ and $y = Y + k$

Substituting in (1)

$\frac{X^2}{9} - \frac{Y^2}{9} = 1$

which represent eqn of hyperbola with centre $(0,0)$ w.r.t (X,Y)

$\rightarrow$ Co-ordinate of vertices

$A(3,0)$, $A'(-3,0)$ w.r.t (X,Y)

& $A(2+3, 0+4)$, $A'(2-3, 0+4)$ w.r.t (x,y)

i.e $A(5,4)$, $A'(-1,4)$ w.r.t (x,y)

$\rightarrow$ Coordinate of foci

$c^2 = a^2 + b^2 = 18 \Rightarrow c = \pm 3\sqrt{2}$

$F(3\sqrt{2}, 0)$ and $F'(-3\sqrt{2}, 0)$ w.r.t (X,Y)

and $F(2+3\sqrt{2}, 0+4)$, $F'(2-3\sqrt{2}, 0+4)$ w.r.t (x,y)

i.e $F(2+3\sqrt{2}, 4)$ & $F'(2-3\sqrt{2}, 4)$ w.r.t (x,y)

$\rightarrow$ Length of transverse axis $= 6$

$\rightarrow$ End pts of conjugate axis

$B(0,3)$ & $B'(0,-3)$ w.r.t (X,Y)

$B(0+2, 3+4)$ & $B'(0+2, -3+4)$ w.r.t (x,y)

i.e $B(2,7)$ & $B'(2,1)$ w.r.t (x,y)

$\rightarrow$ eqn of ~~Asymptotes~~ Asymptotes

$Y = \pm \frac{b}{a} X \Rightarrow Y = \pm X$ (w.r.t (X,Y))

$y - 4 = \pm(x-2)$ w.r.t (x,y)

$\Rightarrow y-4 = 1(x-2) \Rightarrow x-y+2=0$ (w.r.t (x,y))

and $y-4 = -(x-2) \Rightarrow x-2+y-4=0$

$\Rightarrow x+y=6$ w.r.t (x,y)

point of intersection with co-ordinate axis

$x^2-y^2-4x+8y-21=0$

for intersecting $\underline{x}$-axis

put $y=0$

$\Rightarrow x^2-4x-21=0$

$x^2-7x+3x-21=0$

$(x-7)(x+3)=0$

$x=7,-3$

→ for intersecting y-axis

put $x=0$

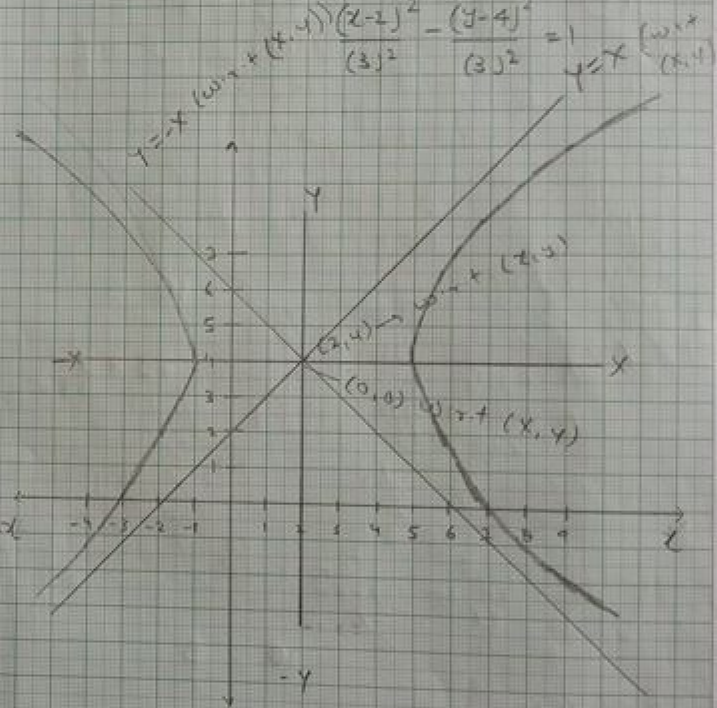
$\Rightarrow -y^2+8y-21=0$

$y^2-8y+21=0$

$\therefore$ this eqn has no real roots

$\Rightarrow$ hyperbola does not intersect y-axis

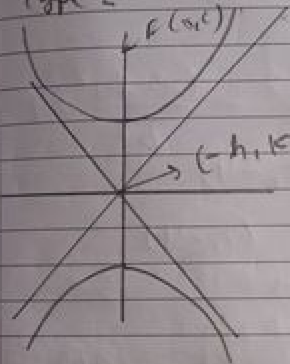
→ hyperbola intersect x-axis at $(7,0)$ & $(-3,0)$



Lecture - 8

Case II To find eqn of hyperbola having centre (h, k) and transverse axis ∥ to to y-axis.

Type 1



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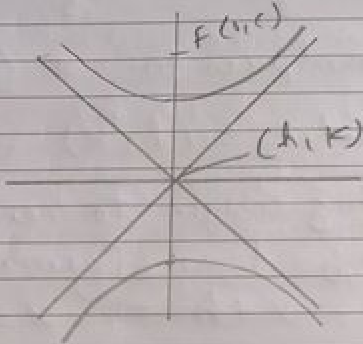

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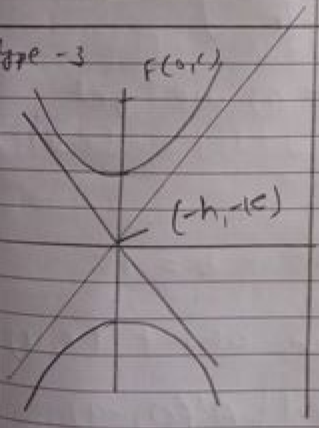
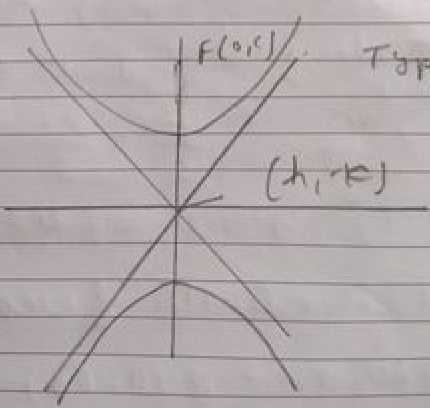


Type-3



$\Rightarrow \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Trace the curve

$\frac{(y+4)^2}{3} - \frac{(x-2)^2}{5} = 1$ — (1)

$\Rightarrow$ Compare to $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\Rightarrow$ centre $= (h, k) = (2, -4)$

Shifting origin to centre $(2, -4)$ using transformation

$x = X + h \Rightarrow x = X + 2$
$y = Y + k \Rightarrow y = Y - 4$] — (11)

putting x & y in eqn (1) we get

$\frac{Y^2}{3} - \frac{X^2}{5} = 1$

$\Rightarrow a^2 = 3 \Rightarrow a = \sqrt{3} \quad b = \sqrt{5}$

$\Rightarrow$ co-ordinates of vertices w.r.t (X, Y)

$A(0, a) = A(0, \sqrt{3})$ and $A'(0, -a)$
$= A'(0, -\sqrt{3})$

$\rightarrow$ co-ordinates of vertices w.r.t (x, y)

$A(0+h, a+k)$ & $A'(0+h, -a+k)$

$\Rightarrow A(0+2, \sqrt{3}-4)$ & $A'(0+2, -\sqrt{3}-4)$

$\Rightarrow A(2, \sqrt{3}-4)$ & $A'(2, -\sqrt{3}-4)$

$\rightarrow$ co-ordinates of Foci

$c^2 = a^2 + b^2 = 3 + 5 = 8 \Rightarrow c = 2\sqrt{2}$

$\rightarrow$ Foci w.r.t (X, Y)

$F(0, c)$ & $F'(0, -c)$

$\Rightarrow F(0, 2\sqrt{2})$ & $F'(0, -2\sqrt{2})$

$\rightarrow$ Foci w.r.t (x, y)

$F(0+h, c+k)$ & $F'(0+h, -c+k)$

$\Rightarrow F(0+2, 2\sqrt{2}-4)$ & $F'(0+2, -2\sqrt{2}-4)$

$\Rightarrow F(2, 2\sqrt{2}-4)$ & $F'(2, -2\sqrt{2}-4)$
w.r.t (x, y)

Asymptote w.r.t ~~(x,y)~~ (X, Y)

$y = \pm \frac{b}{a} X$

$\Rightarrow y = \frac{\sqrt{3}}{\sqrt{5}} x$ an $y = -\frac{\sqrt{3}}{\sqrt{5}} x$

→ Asymptote w.r.t (x, y)

$(y-k) = \pm \frac{b}{a}(x-h)$

$\Rightarrow (y+4) = \frac{b}{a}(x-2)$

$\Rightarrow y+4 = \frac{\sqrt{5}}{\sqrt{3}}(x-2) \Rightarrow \sqrt{3}y + 4\sqrt{3} = \sqrt{5}x - 2\sqrt{5}$

$\sqrt{5}x - \sqrt{3}y = 2\sqrt{5} + 4\sqrt{3}$

$\Rightarrow (y+4) = -\frac{b}{a}(x-2) \Rightarrow$ ~~$y+4 = -\frac{\sqrt{3}}{\sqrt{5}}$~~

$y+4 = -\frac{\sqrt{5}}{\sqrt{3}}(x-2) \Rightarrow \sqrt{3}y + \sqrt{3}4 = -(x-2)\sqrt{5}$

$\Rightarrow \sqrt{5}(x-2) + \sqrt{3}y + \sqrt{3}4 = 0$

$\Rightarrow \sqrt{5}x - 2\sqrt{5} + \sqrt{3}y + 4\sqrt{3} = 0$

$\Rightarrow \boxed{\sqrt{5}x + \sqrt{3}y = 2\sqrt{5} - 4\sqrt{3}}$

Date

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Expt. No.

→ hyperbola intersect co-ordinate axes (x, y)

For intersecting x-axis put y = 0 in ① we get

$\frac{(0+4)^2}{3} - \frac{(x-2)^2}{5} = 1$

$\frac{16}{3} - \frac{(x-2)^2}{5} = 1$

$80 - (x-2)^2 = 15 \Rightarrow 80 = 15 + (x-2)^2$

$\Rightarrow 80 = 15 + x^2 + 4 - 4x \Rightarrow 80 = x^2 - 4x + 19$

$x^2 - 4x - 61 = 0$

$b^2 - 4ac = 16 + 244 = 260 > 0 \Rightarrow$ roots real

$x = \frac{+4 \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 + 244}}{2 \times 1}$

$x = \frac{4 \pm \sqrt{260}}{2} \Rightarrow x = \frac{4 + \sqrt{260}}{2}$

$x = \frac{\cancel{4} + \cancel{2}\sqrt{65}}{\cancel{2}} \Rightarrow x = 2 + \sqrt{65}$

$x = 2 + 8.06 = 10.06$

Teacher's Signature

$$x = \frac{4-\sqrt{260}}{2} = \frac{4-2\sqrt{65}}{2} = 2-\sqrt{65}$$

$$x = 2 - 8.06 \approx -6.06 \Rightarrow x \approx -6.06$$

for intersecting y-axis put $x=0$ in ①
we get

$$\frac{(y+4)^2}{3} - \frac{(0-2)^2}{5} = 1 \qquad \frac{(y+4)^2}{3} - \frac{4}{5} = 1$$

$$5(y+4)^2 - 12 = 15 \Rightarrow 5(y^2+16+8y) - 12 = 1$$

$$5y^2 + 80 + 40y = 27 \Rightarrow 5y^2 + 40y + 53 = 0$$

$$b^2 - 4ac = 1600 - 4 \times 5 \times 53 = 540 > 0$$

$\Rightarrow$ real roots

$$y = \frac{-40 \pm \sqrt{b^2-4ac}}{2a} = \frac{-40 \pm \sqrt{540}}{2\times 5}$$

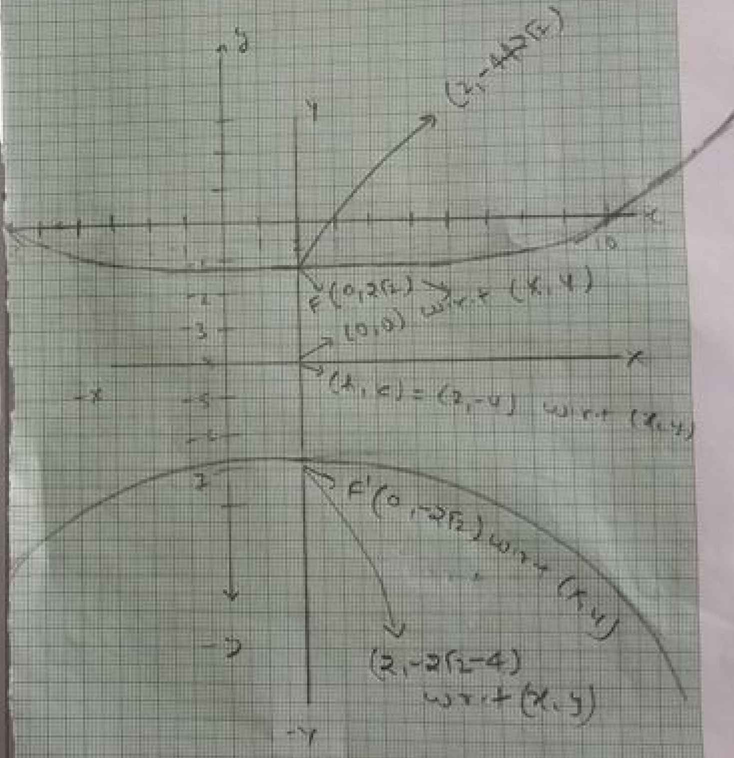
$$y = \frac{-40+\sqrt{540}}{10} = \frac{-40+6\sqrt{15}}{10} \approx -1.6762$$

$$\Rightarrow y \approx -1.6762$$

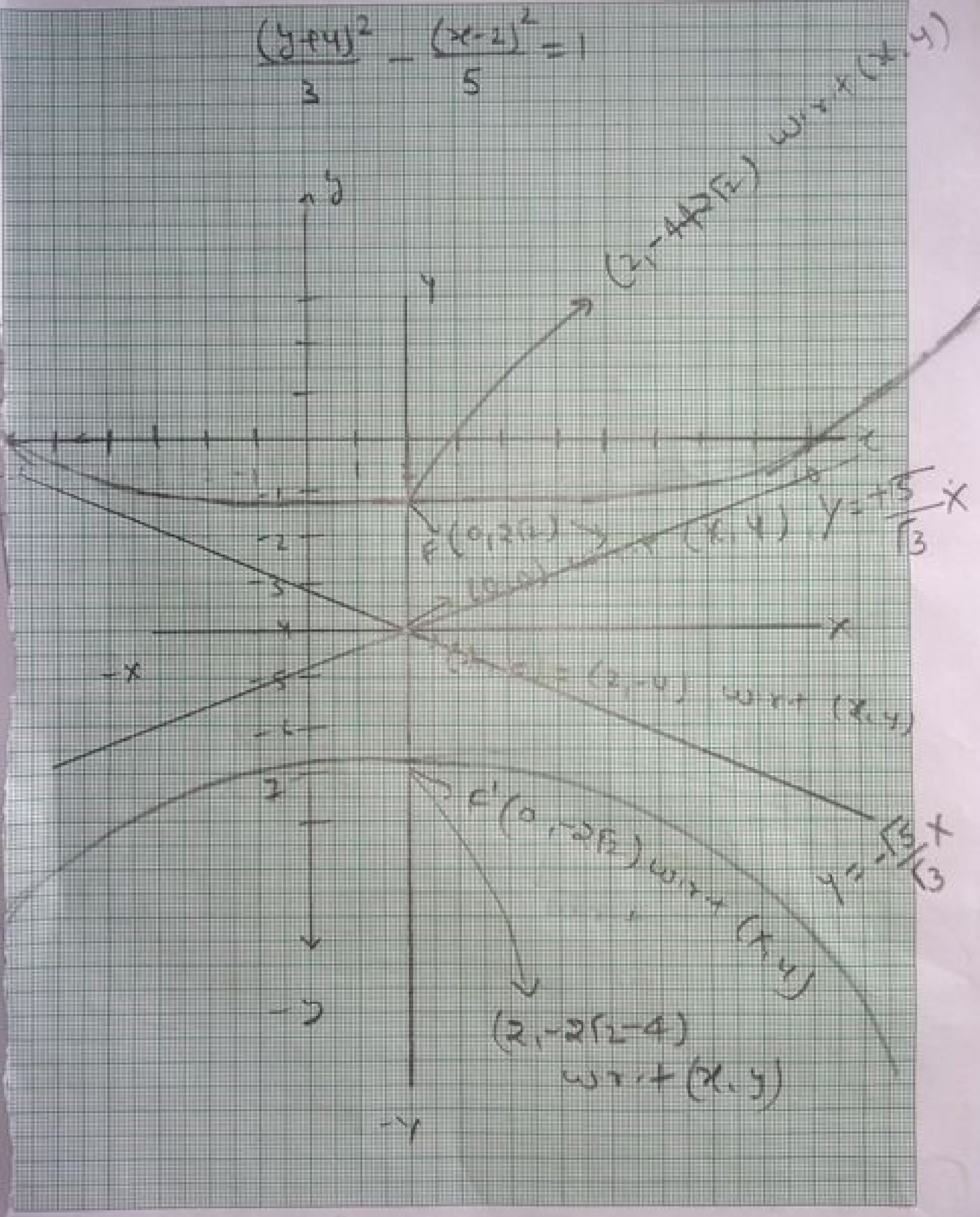
Now $y = \frac{-40-\sqrt{540}}{2\times 5} = \frac{-40-6\sqrt{15}}{10} \approx -6.3237$

$$\Rightarrow y \approx -6.3237$$

$$\frac{(y+4)^2}{3} - \frac{(x-2)^2}{5} = 1$$



$$\frac{(y+4)^2}{3} - \frac{(x-2)^2}{5} = 1$$



Q1 Sketch the hyperbola, and label the vertices, foci and asymptotes

(1) $16x^2 - 25y^2 = 400$

(2) $9y^2 - x^2 = 36$

[Do yourself]

Qn2 Sketch the hyperbola.

(a) $\dfrac{(y+4)^2}{3} - \dfrac{(x+2)^2}{5} = 1$

(b) $16(x+1)^2 - 8(y-3)^2 = 16$

(c) $x^2 - 4y^2 + 2x + 8y - 7 = 0$

(d) $16x^2 - y^2 - 32x - 6y = 57$

Do yourself]