NBFM (Narrow band FM)

A Narrowband FM is FM with small bandwidth.

The Modulation Index m_f of NBFM is small as compared to one Radian.

The max. permissible freq. Deviation is restricted to about 5 KHZ. This system is used in FM Mobile comm. such as police wireless, ambulance, taxicab, etc.

$$x_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right) \quad \because \cos(A+B) = \cos A\cos B - \sin A \sin B$$

$$= A\cos\omega_c t \cos\left(k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right) - A\sin\omega_c t \cdot \sin\left(k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right)$$

Mathematical identity :- i) $\cos\theta \approx 1$ for small value of θ

(2) $\sin\theta \approx \theta$ for small value of θ.

for NBFM $k_f << 1$:

$$x_{NBFM}(t) = A\left[\cos\omega_c t - k_f \int_{-\infty}^{t} m(\tau)d\tau \sin\omega_c t\right]$$

$\because \sin A \sin B = \cos(A-B) - \cos(A+B)$

$$\boxed{\text{as } \lim_{\theta \to 0} \cos\theta = 1 \quad \lim_{\theta \to 0} \sin\theta \cong \theta}$$

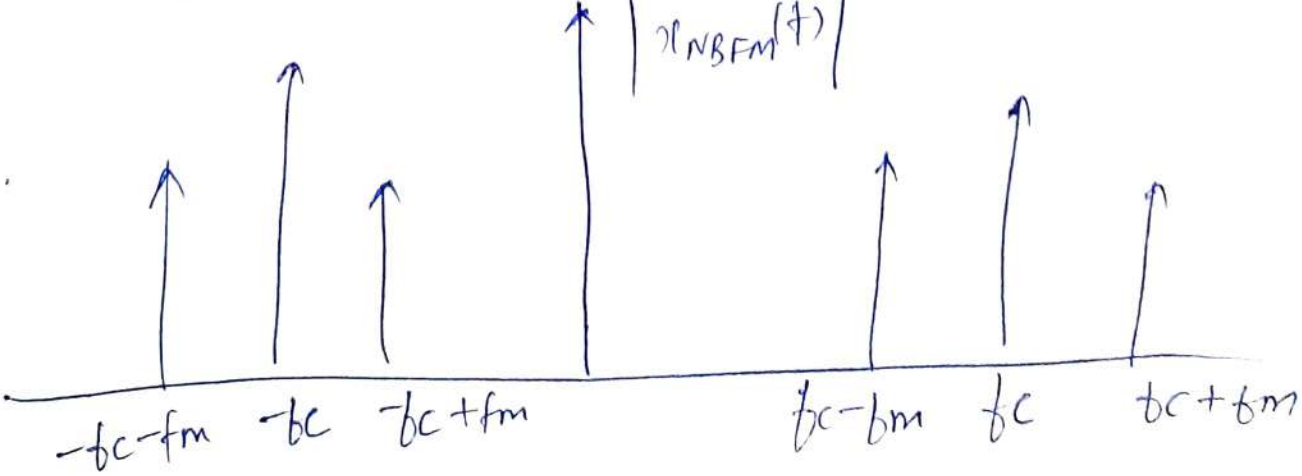
$$x_{NBFM}(t) = A\cos(2\pi f_c t) - \frac{A}{2}k_f \cos 2\pi(f_c - f_m)t + \frac{A}{2}k_f \cos 2\pi(f_c + f_m)t$$

Let $m(t) = A_m \cos\omega_m t$.

$$\text{so } x_{NBFM}(t) = A\left[\cos\omega_c t - \frac{k_f A_m}{\omega_m}\sin\omega_m t \cdot \sin\omega_c t\right]$$

$$x_{NBFM}(t) = \frac{A}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] - \frac{A k_f A_m}{4\omega_m}\left[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))\right] + \frac{A k_f A_m}{4\omega_m}\left[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))\right]$$

$$\boxed{BW = 2f_m}$$



Generation of NBFM :-

The expression of FM Signal in terms of modulation index is given by,

$$X_{NBFM}(t) = A_c\left(\cos\omega_c t - K_f\int_{-\infty}^{t} m(t)\,dt \cdot \sin\omega_c t\right)$$

$[\because \cos(A+B) = \cos A \cdot \cos B - \sin A \sin B]$

Where $m(t) = \cos 2\pi f_m t$

$$X_{NBFM}(t) = \cos(2\pi f_c t)\cos(K_f \sin 2\pi f_m t) - \sin(2\pi f_c t)\sin(K_f \sin 2\pi f_m t)$$

Mathematical identification :

1. $\cos\theta \simeq 1$ for small value of θ
2. $\sin\theta \simeq \theta$ for small value of θ

for NBFM : $K_f << 1$

$$X_{NBFM}(t) = \cos\omega_c t - K_f \sin\omega_c t \sin\omega_m t$$

$= \cancel{\cos\omega_c t} -$

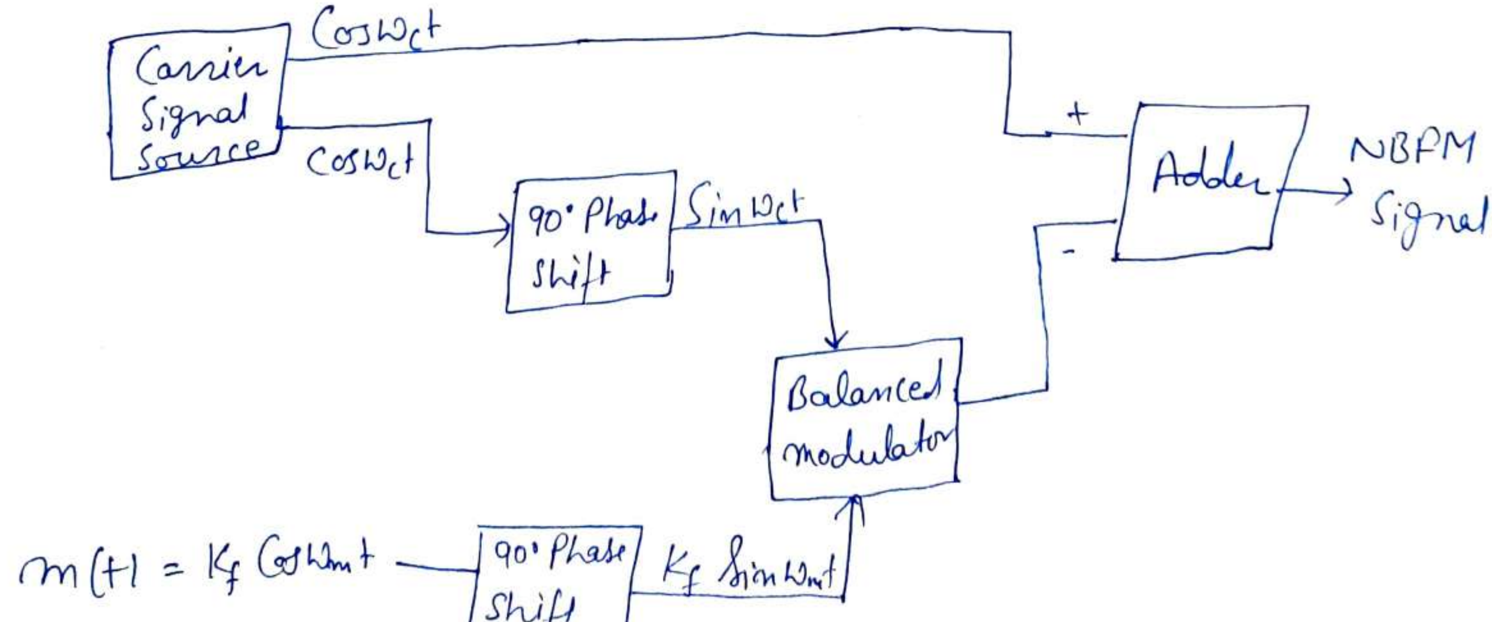
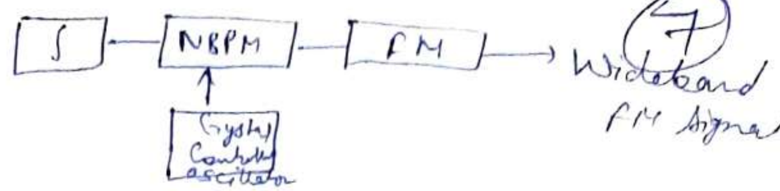


fig :- Block diagram of generation of an NBFM signal

Wideband FM



For large value of m_f, FM is ideally contain the carrier and an infinite No. of sideband located symmetrically around the carrier. Such FM has infinite Bandwidth and called WBFM.

Max permisible freq is 75 KHZ

$$x_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right)$$

$$= A\cos\left[\omega_c t + \frac{k_f A_m}{\omega_m} \sin\omega_m t\right]$$

$$\Downarrow$$

$$= A\cos\left[\omega_c t + \beta \sin\omega_m t\right.$$

$$= A\cos\omega_c t \cdot \underbrace{\cos(\beta \sin\omega_m t)}_{\text{Periodic}} - \sin\omega_c t \underbrace{\sin(\beta \sin\omega_m t)}_{\text{Periodic.}} \quad 1$$

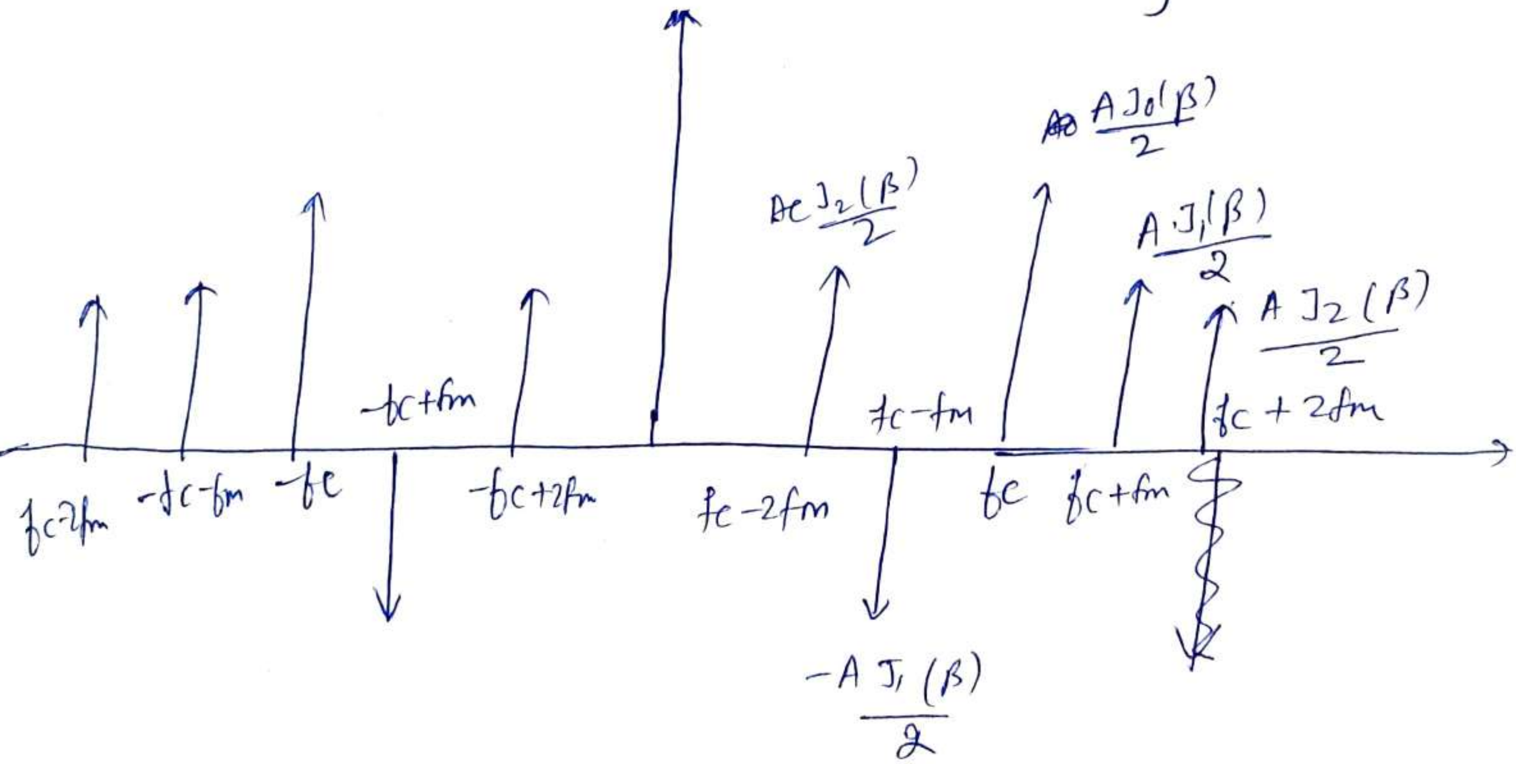
$$\cos(\beta \sin\omega_m t) = J_0(\beta) + 2J_2(\beta)\cos 2\omega_m t + 4J_4(\beta)\cos 4\omega_m t + \cdots \quad 2$$

$$\sin(\beta \sin \omega_m t) = 2J_1(\beta)\sin\omega_m t + 2J_3(\beta)\sin 3\omega_m t + \cdots \quad (3)$$

Put (3) & (2) in (1)

$$= A\left[J_0(\beta)\cos\omega_c t - 2J_1(\beta)\sin\omega_m t \sin\omega_c t + 2J_2(\beta)\cos 2\omega_m t \cos\omega_c t - 2J_3(\beta)\sin 3\omega_m t \cdot \sin\omega_c t + \cdots\right]$$

$$= A\left[J_0(\beta)\cos\omega_c t - J_1(\beta)\cos(\omega_c - \omega_m)t + J_1(\beta)\cos(\omega_c + \omega_m)t + J_2(\beta)\cos(\omega_c + 2\omega_m)t + J_2(\beta)\cos(\omega_c - 2\omega_m)t + \cdots\right]$$



$\boxed{BW = \infty}$

But practically the BW Depend Depend on the No. of band we take for our use

If we take 1 band

$BW = 2 f_m$

2 band $BW = 4 f_m$

n band $\boxed{BW = 2n f_m}$

Imp. where $n = \beta + 1$ at this value 98% Power is contained.

so, $\boxed{BW = 2(\beta+1) f_m}$ Carlson Bandwidth.

$\beta = \frac{\Delta f}{f_m}$

$BW = 2\left(\frac{\Delta f}{f_m} + 1\right) f_m$

$= 2(\Delta f + f_m)$

POWER of FM

$$= A \Big[J_0(\beta) \cos\omega_c t - J_1(\beta)\cos(\omega_c - \omega_m)t + J_1(\beta)\cos(\omega_c+\omega_m)t$$

$$+ J_2(\beta)\cos(\omega_c + 2\omega_m)t + J_2(\beta)\cos(\omega_c - 2\omega_m)$$

$$+ \ldots$$

$$\boxed{P_t = A^2 \left[\frac{J_0^2(\beta)}{2} + \sum_{n=1}^{\infty} J_n^2(\beta) \right]}$$

equivalence b/w FM & PM

(1) eqn	(1) eqn
(2) Amplitude of FM wave is constt	(2) Amplitude of PM is const
(3) It is possible to Receive FM on PM Receiver	(3) It is possible to Receive PM on FM Receiver
(4) Noise immunity is better than AM & PM	(4) Noise immunity is better than AM but worst than FM
(5) FM is widely used	(5) PM is used in some mobile system
(6) frequency Deviation is proportional to modulating voltage	(6) frequency Deviation is proportional to modulating voltage

Comparison b/w AM / FM / PM

AM	FM	PM
(1) Amplitude is not constt	(1) Amplitude is constt	(1) Amplitude is constt
(2) Amplitude is varied	(2) Freq. is varied	(2) Phase is varied
(3) freq. of carrier is constt & Phase is also constt	(3) Amplitude is carrier is constt & phase is also constt	(3) Amplitude & freq. is constt
(4) AM types include DSB-SC, SSB, VSB etc	(4) FM include FSK, GFSK etc	(4) PM include BPSK, QAM etc.

Generation of FM

(1) Direct Method

In direct method the instantenous freq. of carrier is changed directly in proportion with message signal. For this a device called VCO is used.

Let us consider an LC tank Ckt. whose freq.

is varying by varying any one of L or C

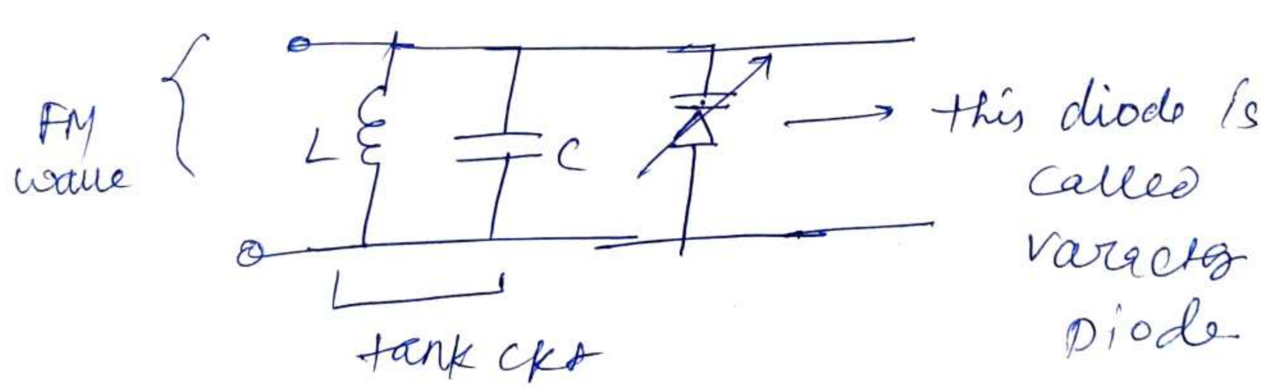
But practically it is very difficult to vary 'L'

So we will vary the capacitor.

as we know when an diode is operated in Reverse bias a capacitance is developed called Depletion / Jxn capacitor

$$\boxed{C_D = \frac{\varepsilon A}{W}}$$

which vary when we vary the R.B voltage across diode which full fill our Requirement.



as we know $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ in absence of $m(t)$

$$C(t) = C + C_{varactor}$$

effective capacitor $\Rightarrow$ $C(t) = C - K_c\, m(t)$

$$\cancel{f(t) = \frac{1}{2\pi\sqrt{L(C}}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{L\left(C - K_c m(t)\right)}}$$

$$f_i = \frac{1}{2\pi\sqrt{LC}\left[1 - \frac{Kc\, m(t)}{C}\right]^{1/2}}$$

$$f_i = \frac{1}{2\pi\sqrt{LC}\left[1 - \frac{Kc\, m(t)}{C}\right]^{1/2}}$$

$$= f_0 \left[1 - \frac{Kc\, m(t)}{C}\right]^{-1/2}$$

$$= f_0 \left(1 + \frac{Kc}{2C} m(t)\right)$$

$$\boxed{f_i = f_0 + \frac{f_0 Kc}{2C} m(t)}$$

$$\boxed{\frac{f_0 Kc}{2C} = K_f}$$

$$\boxed{f_i = f_0 + K_f m(t)}$$

K_f is freq. sensitivity of Modulator.

Drawback of Direct Method

- Non-linearity of varactor diode produce of a freq. variation due to harmonics of modulating or baseband signal hence FM s/g distorted.

We will have to take the proper cares to keep the type of distortion minimum.

- ~~It~~ In direct method it is not easy get high order stability in carrier freq.