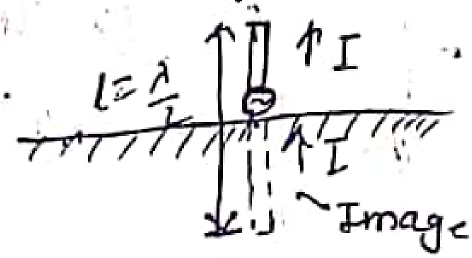
→ Quarter-wave Monopole Antenna

half of a half-wave dipole antenna.

$$\boxed{l = \frac{\lambda}{2}}$$



$$P_{rad} = 18.28\, I_o^2$$

$$R_{rad} = \frac{2P_{rad}}{I_o^2}$$

$$\boxed{R_{rad} = 36.5\,\Omega}$$

$$Z_{in} = 36.5 + j21.25\,\Omega$$

Ex. 13.2, 13.1

* ANTENNA CHARACTERISTICS :-

(a.) Antenna pattern, (b.) Radiation intensity, (c) directive gain

(d.) Power gain.

A. Antenna Patterns :-

When the amplitude of a specified component of the $\vec{E}$ field is plotted, it is called the field pattern or voltage pattern.

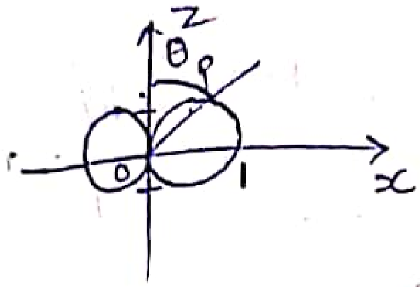
When square of $\vec{E}$ field is plotted, it is called the power pattern.

$|E_s|$ Vs θ for a ϕ = const :- Vertical pattern

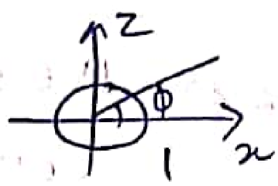
$|E_s|$ Vs ϕ for $\theta = 90°$ $\Rightarrow$ H- pattern.

for Hertzian dipole

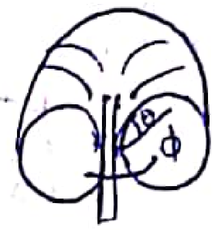
$$\boxed{f(\theta) = |\sin\theta|}$$



vertical pattern



Horizontal pattern



3-D pattern

$f^2(\theta) = \sin^2\theta$ for power pattern.

- An antenna pattern (or radn pattern) is a three-dimensional plot of its radiation at far field.

B. Radiation Intensity :-

$$\boxed{U(\theta,\phi) = r^2 P_{ave}} \quad \text{— (1)}$$

$$P_{rad} = \oint_S P_{ave}\, ds = \oint_S P_{ave} r^2 \sin\theta\, d\theta\, d\phi$$

$$= \int_S U(\theta,\phi) \underbrace{\sin\theta\, d\theta\, d\phi}$$ ↳ diff! solid angle (Sr)

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi)\, d\Omega$$

Scanned with CamScanner

$$\boxed{U_{ave} = \frac{P_{rad}}{4\pi}}$$

c. **Directive Gain** :- The directive gain $G_d(\theta,\phi)$ of an antenna is a measure of the concentration of the radiated power in a particular direction (θ,ϕ)

$$\boxed{G_d(\theta,\phi) = \frac{U(\theta,\phi)}{U_{ave}} = \frac{4\pi U(\theta,\phi)}{P_{rad}}} \quad -(1)$$

$$\boxed{P_{ave} = \frac{G_d}{4\pi r^2} P_{rad}} \quad -(2)$$

It is ability of antenna to direct radiated power in a given direction.

For an isotropic antenna, $G_d = 1$.

Directivity :- D of an antenna is the ratio of the maximum radiation intensity to the average radiation intensity.

$$\boxed{D = \frac{U_{max}}{U_{ave}} = G_{d\,max}} \quad -(3)$$

or $$D = \frac{U_{max}}{\frac{P_{rad}}{4\pi}} = \frac{4\pi U_{max}}{P_{rad}} \quad -(4)$$

for Hertzian dipole,

$$G_d(\theta, \phi) = 1.5 \sin^2\theta, \quad D = 1.5 \quad -(5)$$

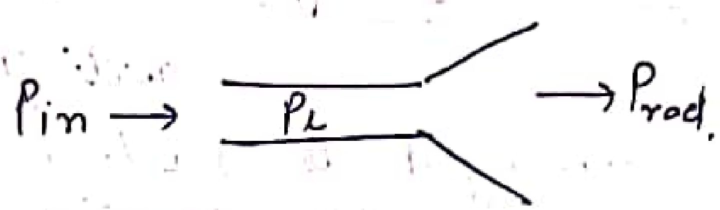
$\therefore$ the max. value of $\sin^2\theta = 1$. for the $\frac{\lambda}{2}$ dipole

$$G_d(\theta, \phi) = \frac{\eta}{\pi R_{rad}} f^2(\theta), \quad D = 1.64 \quad -(6)$$

$$\eta = 120\pi, \quad R_{rad} = 73\Omega$$

$$f(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \quad -(7)$$

D. <u>Power Gain</u> :-



$$P_{in} = P_L + P_{rad}$$

$$= \frac{1}{2}|I_{in}|^2 (R_L + R_{rad}) \quad -(8)$$

I_{in} = peak current

R_L = ohmic resistance.

$$\boxed{G_p(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{in}}} \quad -(9)$$

<u>Radiation efficiency (η_r)</u> :- Ratio of the power gain in any specified direction (θ, ϕ) to directive gain in that direction.

$$\eta_r = \frac{G_p}{G_d} = \frac{P_{rad}}{P_{in}}$$

$$\boxed{\eta_r = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad}+R_L}} \quad -(10)$$

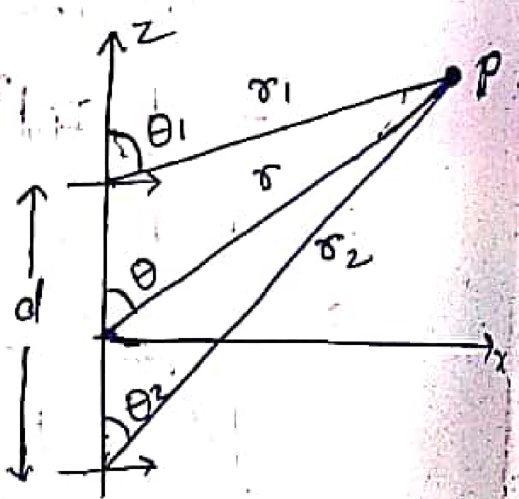
$$\left.\begin{aligned} D(dB) &= 10\log_{10} D \\ G(dB) &= 10\log_{10} G \end{aligned}\right\} -(11)$$

* $r_{min} = \dfrac{2d^2}{\lambda}$.

$d = l$ for electric dipole antenna

$d = 2\rho_0$ small-loop antenna

Antenna Arrays :- It is a group of radiating elements arranged to produce particular radiation characteristics.



Consider two Hertzian dipoles place in free space.

at $(0, 0, \frac{d}{2})$

$I_{1s} = I_0 \angle \alpha$

at $(0, 0, -\frac{d}{2})$

$I_{2s} = I_0 \angle 0$

α = phase diff. b/w two current

at P

$E_s = E_{1s} + E_{2s}$

$$= \frac{j\eta \beta I_o dl}{4\pi}\left[\cos\theta_1 \frac{e^{-j\beta r_1}}{r_1} e^{j\alpha} a_{\theta_1} + \cdots\right] \quad \text{— (1)}$$

$\because$ P is far from the array

$$\theta_1 \simeq \theta \simeq \theta_2 \;\&\; a_{\theta_1} \simeq a_\theta \simeq a_{\theta_2}$$

$$r_1 \simeq r \simeq r_2 \quad \text{(in amplitude)}$$

$$\left.\begin{aligned} r_1 &\simeq r - \frac{d}{2}\cos\theta \\ r_2 &\doteq r + \frac{d}{2}\cos\theta \end{aligned}\right\} \text{in phase} \quad (3)$$

$$E_s = \frac{j\eta \beta I_o dl}{4\pi r}\cos\theta\, e^{-j\beta r} e^{j\alpha/2}\left[e^{j(\beta d\cos\theta)/2} e^{j\alpha/2} + e^{-j(\beta d\cos\theta)/2} e^{-j\alpha/2}\right] a_\theta$$

$$= \frac{j\eta \beta I_o dl}{4\pi r}\cos\theta\, e^{-j\beta r} e^{j\alpha/2}\, 2\cos\left[\frac{1}{2}(\beta d\cos\theta + \alpha\right]\hat{a}_\theta \quad \text{— (4)}$$

$$\boxed{AF = 2\cos\left[\frac{1}{2}(\beta d\cos\theta + \alpha)\right] e^{j\alpha/2}} \quad \text{— (5)}$$

the far field due to a two element array –

E(total) = (E due to single element at origin) * (array factor)

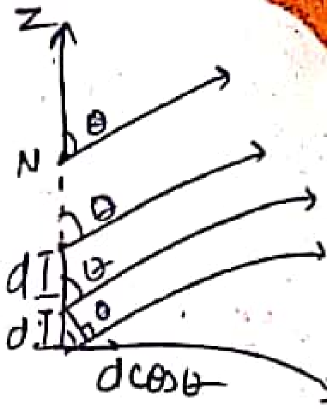
$\boxed{\text{resultant pattern} = \text{unit pattern} \times \text{group pattern}}$

This is known as pattern multiplications — (6)

Now for N-element array

$$I_{1s} = I_o\angle 0, \quad I_{2s} = I_o\angle\alpha, \quad I_{3s} = I_o\angle 2\alpha \;\ldots$$

For the uniform linear array



$$AF = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(N-1)\psi} \quad — (7)$$

where $\underline{\psi = \beta d \cos\theta + \alpha}$ — (8)

$$\beta = \frac{2\pi}{\lambda}.$$

$\because \quad 1 + x + x^2 + x^3 + \cdots + x^{N-1} = \frac{1 - x^N}{1 - x}$

$$AF = \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} \quad — (9)$$

$$AF = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1} = \frac{e^{jN\psi/2}\left(e^{jN\psi/2} - e^{-jN\psi/2}\right)}{e^{j\psi/2}\left(e^{j\psi/2} - e^{-j\psi/2}\right)} \quad — (10)$$

$$= e^{j(N-1)\psi/2}\,\frac{\sin(N\psi/2)}{\sin(\psi/2)} \quad — (11)$$

$$\boxed{|AF| = \left|\frac{\sin\frac{N\psi}{2}}{\sin\frac{\psi}{2}}\right|}, \quad \boxed{\psi = \beta d \cos\theta + \alpha} \quad — (12)$$

1. for Principle max $\psi = 0$

$$\beta d \cos\theta + \alpha \Rightarrow \cos\theta = \frac{-\alpha}{\beta d}$$

2. when $|AF| = 0$, $|AF|$ has nulls.

$$\frac{N\psi}{2} = \pm k\pi$$

3. $\psi = 0,\ \theta = 90^\circ,\ \alpha = 0$ (broadside array)

4. $\psi = 0;\ \theta = \begin{cases} 0 \\ \pi \end{cases},\ \alpha = \begin{cases} -\beta d \\ \beta d \end{cases}$ [end fire array]