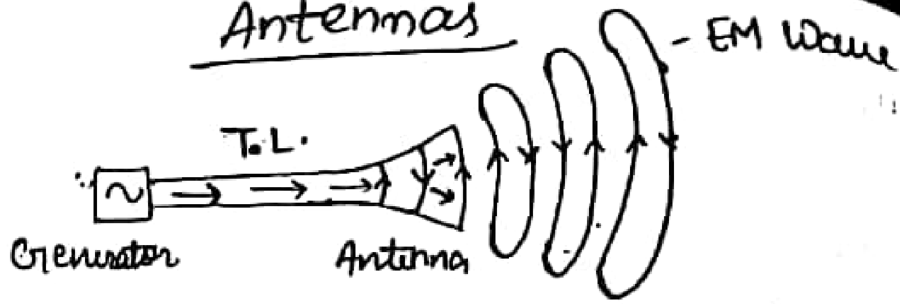
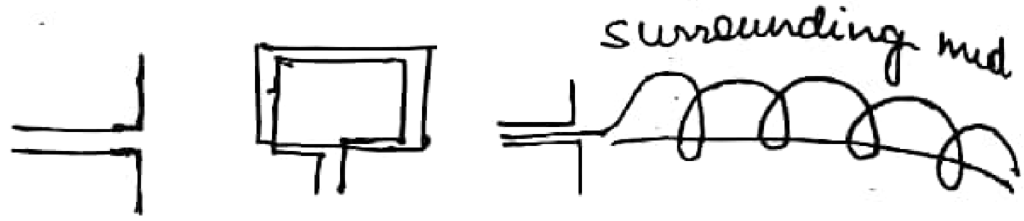
Antennas





a) Dipole (b) Loop (c) Helix

(d) Pyramidal (e) Parabolic dish reflector
(used in astronomy, radar)

* Radiation fields

S1- Select an appropriate coordinate syste
and determine the magnetic vector p
$\vec{A}$

S2- find H from $B = \mu H = \nabla \times A$

S3- Determine E from $\nabla \times H = \epsilon \frac{\partial E}{\partial t}$

or $E = \eta H \times a_k \quad (\sigma = 0)$

S4- find the far field and determine
the average power radiated by

$$\boxed{P_{rad} \int \mathcal{P}_{ave} \cdot ds}$$

$$\mathcal{P}_{ave} = \frac{1}{2} Re(E_s \times H_s^*)$$

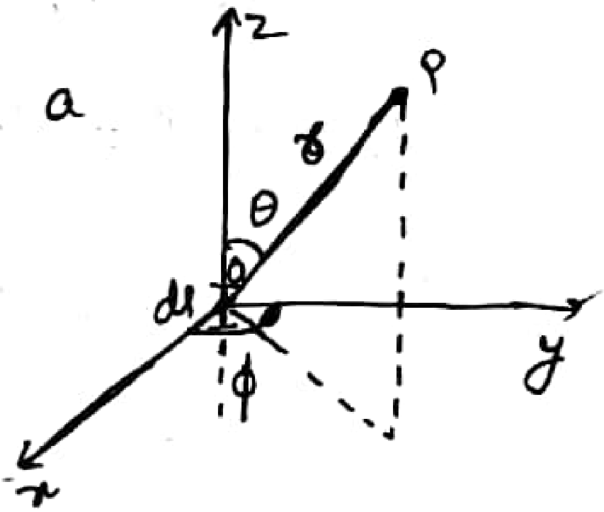
$$P_{rad} = P_{ave} = \int_s \mathcal{P}_{ave} \cdot ds$$

1. HERTZIAN DIPOLE :-

By "Hertzian dipole" means an infinitesimal current element Idl, where $dl \leq \frac{\lambda}{10}$.

consider it carries a uniform current,

$$\boxed{I = I_0 \cos \omega t}$$



At point P the retarded magnetic vector potential,

$$\vec{A} = \frac{\mu [I] dl}{4\pi r} \hat{a}_z \quad -①$$

when $[I]$ is retarded current

$$[I] = I_0 \cos \omega \left(t - \frac{r}{u}\right) = I_0 \cos(\omega t - \beta r)$$

$$\doteq Re[I_0 e^{j(\omega t - \beta r)}] \quad \left[\because \frac{\omega}{u} = \beta\right] \quad -②$$

$$\beta = \frac{\omega}{u} = \frac{2\pi}{\lambda} \;\&\; u = \frac{1}{\sqrt{\mu \varepsilon}}.$$

The current is said to be retarded at point P because there is a propagation time delay $\frac{r}{u}$ or phase delay βr from O to P.

Put ② in ①

$\frac{1}{r^3}$ and $\frac{1}{r^2}$ can be neglected in

$$r = \frac{2d^2}{\lambda} \quad — (7)$$

d is the large and
of the antenn

$$\boxed{H_{\phi s} = \frac{j I_0 \beta dl}{4\pi r} \sin\theta \, e^{-j\beta r}} \quad E_{\theta s} = \eta H_{\phi s} \quad — (8a)$$

$$\boxed{H_{rs} = H_{\theta s} = E_{rs} = E_{\phi s} = 0} \quad — (8b)$$

radiation terms of $H_{\phi s}$ and $E_{\theta s}$ are in time phase and orthogonal just as the field of a uniform plane wave.

The time-average power density is obtained as

$$\mathcal{P}_{ave} = \frac{1}{2} Re(E_s \times H_s^*) = \frac{1}{2} Re(E_{\theta s} H_{\phi s}^* \hat{a}_r)$$

$$\boxed{\mathcal{P}_{ave} = \frac{1}{2} \eta |H_{\phi s}|^2 \hat{a}_r} \quad — (9)$$

$$P_{rad} = \int \mathcal{P}_{ave} \cdot ds$$

$$= \int_{\phi=0}^{2\pi} \left[\int_{\theta=0}^{\pi} \frac{I_0^2 \eta \beta^2 dl^2}{32\pi^2 r^2} \sin^2\theta \, r^2 \sin\theta \, d\theta \right] d\phi$$

$$P_{rad} = \frac{I_0^2 \eta \beta^2 dl^2}{32\pi^2} \, 2\pi \int_0^{\pi} \sin^3\theta \, d\theta \quad — (10)$$

where $ds = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$.

But $\int_0^{\pi} \sin^3\theta \, d\theta = \int_0^{\pi} (1-\cos^2\theta) \, d(-\cos\theta)$

$$= \frac{\cos^3\theta}{3} - \cos\theta \Big|_0^{\pi} = \frac{4}{3}$$

and $\beta^2 = \frac{4\pi^2}{\lambda^2}$. hence eqn (10) becomes

$$P_{rad} = \frac{I_0^2 \pi \eta}{3}\left[\frac{dl}{\lambda}\right]^2 \quad \text{— (11)}$$

in free space, $\eta = 120\pi$ and

$$P_{rad} = 40\pi^2\left[\frac{dl}{\lambda}\right]^2 I_0^2 \quad \text{— (11a)}$$

This power is equivalent to the power dissipated in a fictitious Resistance R_{rad} by current $I = I_0 \cos \omega t$,

i.e. $P_{rad} = I^2_{rms} R_{rad}$

or $$\boxed{P_{rad} = \frac{1}{2} I_0^2 R_{rad}} \quad \text{— (12)}$$

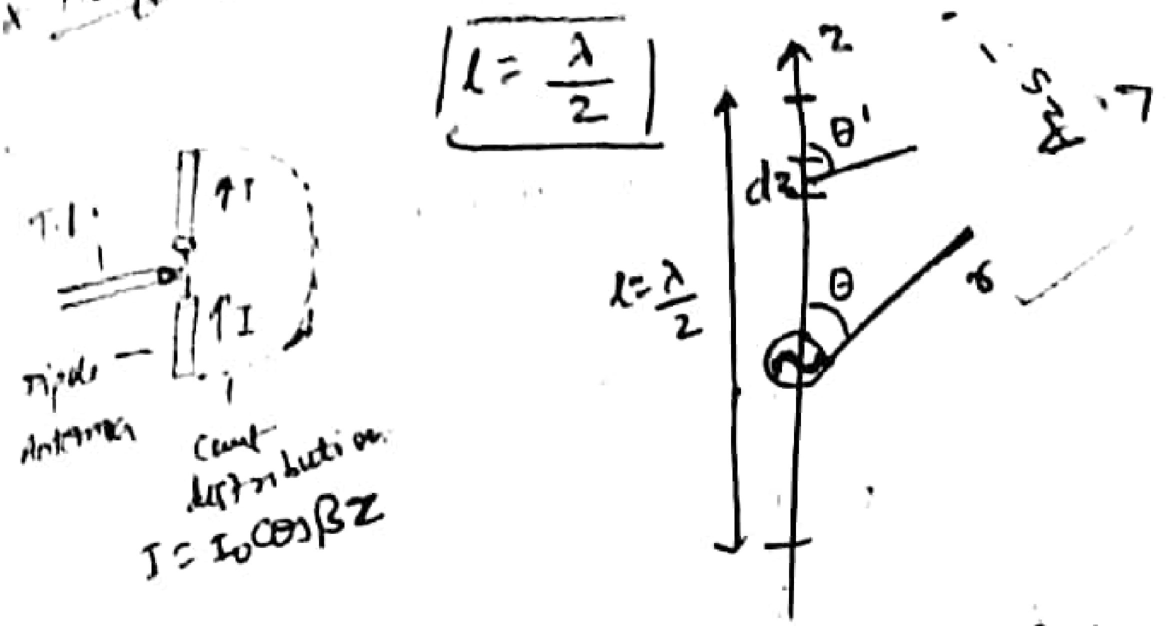
where I_{rms} is the root-mean-square value of I. from (11) and (12)

$$\boxed{R_{ad} = \frac{2 P_{rad}}{I_0^2}}$$

or $$\boxed{R_{ad} = 80\pi^2\left[\frac{dl}{\lambda}\right]^2}$$

Half Wave Dipole Antenna.

$$\boxed{l = \frac{\lambda}{2}}$$



The magnetic vector potential at pt P due to $dl\,(dz)$ carrying a phasor current $I_s = I_0 \cos\beta z$ is

$$dA_{zs} = \frac{\mu I_0 \cos\beta z\, dz}{4\pi r'} e^{-j\beta r'} \quad (1)$$

If $r >> l$

$$r - r' = z\cos\theta$$

$$r' = r - z\cos\theta$$

or $r' \cong r$ in (1)

$$A_{zs} = \frac{\mu I_0}{4\pi r}\int_{-\lambda/4}^{\lambda/4} e^{-j\beta(r - z\cos\theta)} \cos\beta z\, dz$$

$$= \frac{\mu I_0}{4\pi r} e^{-j\beta r}\int_{-\lambda/4}^{\lambda/4} e^{j\beta z\cos\theta} \cos\beta z\, dz \quad (2)$$

$$\because \int e^{az}\cos bz\, dz = \frac{e^{az}(a\cos bz + b\sin bz)}{a^2 + b^2} + c$$

and $\frac{\mu I_o}{4\pi r} e^{-j\beta r} \, e^{j\beta z\cos\theta} \frac{(j\beta\cos\theta\cos\beta z + \beta\sin\beta z)}{-\beta^2\cos^2\theta + \beta^2}$ — (3)

$$\left.\begin{aligned} a &= j\beta\cos\theta \\ b &= \beta \end{aligned}\right\}$$

Since $\beta = \frac{2\pi}{\lambda}$, or $\frac{\beta\lambda}{4} = \frac{\pi}{2}$

$-\cos^2\theta + 1 = \sin^2\theta$

$$A_{zs} = \frac{\mu I_o e^{-j\beta r}}{4\pi r\beta^2\sin^2\theta}\left[e^{j\frac{\pi}{2}\cos\theta}(0+\beta) - e^{-j(\frac{\pi}{2})\cos\theta}(0-\beta)\right] \quad -(4)$$

$\because \; e^{jx} + e^{-jx} = 2\cos x.$

$$\boxed{A_{zs} = \frac{\mu I_o e^{-j\beta r}\cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r\beta\sin^2\theta.}} \quad -(5)$$

$\because \; B_s = \mu H_s = \nabla\times A_s$ & $\nabla\times H_s = j\omega\epsilon E_s$

$$\boxed{H_{\phi s} = \frac{jI_o e^{-j\beta r}\cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r\sin\theta}} \quad -(6)$$

$$\boxed{E_{\theta s} = \eta H_{\phi s}} \quad -(7)$$

$$P_{ave} = \frac{1}{2}\eta|H_{\phi s}|^2 a_r$$

$$= \frac{\eta I_o^2\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{8\pi^2 r^2\sin^2\theta} \quad -(8)$$

$$P_{rad} = \int_S \mathcal{P}_{ave} \cdot ds$$

$$= \int_{\phi=0}^{2\pi} \left[\int_{\theta=0}^{\pi} \frac{\eta I_0^2 \cos^2\left(\frac{\pi}{2}\cos\theta\right)}{8\pi^2 \cancel{r^2} \sin^{\cancel{2}}\theta} \cancel{r^2} \cancel{\sin\theta}\, d\theta \right] d\phi$$

$$= \frac{\eta I_0^2}{8\pi^2} 2\pi \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$= 30 I_0^2 \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$P_{rad} = 60 I_0^2 \int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right) d\theta}{\sin\theta} \quad — (9)$$

$$P_{rad} = 15 I_0^2 \left[\frac{(2\pi)^2}{2(2!)} - \frac{(2\pi)^4}{4(4!)} + \frac{(2\pi)^6}{6(6!)} - \frac{(2\pi)^8}{8(8!)} + \cdots \right]$$

$$\boxed{P_{rad} \simeq 36.56 I_0^2} \quad — (10)$$

$$\boxed{R_{rad} = \frac{2P_{rad}}{I_0^2} \simeq 73\Omega} \quad — (11)$$

$$\boxed{Z_{in} = R_{in} + jX_{in}} \quad — (12)$$

wher $R_{in} = R_{rad}$ for a lossless antenna.

These factors in addition to the resonance property are the reasons for the dipole antenna's popularity and its extensive use.