18

$$\cancel{x^2} + 2xc + \cancel{y^2} = 4a^2 + \cancel{x^2} + \cancel{c^2} - 2xc + \cancel{y^2} - 4a\sqrt{y^2+(x-c)^2}$$

$$\cancel{4}xc = \cancel{4}a^2 - \cancel{4}a\sqrt{(x-c)^2+y^2}$$

$$xc - a^2 = -a\sqrt{(x-c)^2+y^2}$$

Again squaring both side we get

$$(xc-a^2)^2 = \left(-a\sqrt{(x-c)^2+y^2}\right)^2$$

$$(xc)^2 + a^4 - 2a^2(xc) = a^2[(x-c)^2+y^2]$$

$$x^2c^2 + a^4 - 2xa^2c = a^2[x^2+c^2-2xc+y^2]$$

$$x^2c^2 + a^4 - \cancel{2xa^2c} = a^2x^2 + a^2c^2 - \cancel{2xa^2c} + a^2y^2$$

$$x^2c^2 + a^4 = a^2x^2 + a^2c^2 + a^2y^2$$

$$a^4 = a^2x^2 + a^2c^2 + a^2y^2 - x^2c^2$$

$$= a^2x^2 - x^2c^2 + a^2c^2 + a^2y^2$$

$$a^4 - a^2c^2 = x^2(a^2-c^2) + a^2y^2$$

$$a^2(a^2-c^2) = x^2(a^2-c^2) + a^2y^2$$

divided By $a^2(a^2-c^2)$ both side:

$$\frac{a^2(a^2-c^2)}{a^2(a^2-c^2)} = \frac{x^2(a^2-c^2)}{a^2(a^2-c^2)} + \frac{a^2y^2}{a^2(a^2-c^2)}$$

$$\Rightarrow 1 = \frac{x^2}{a^2} + \frac{y^2}{a^2-c^2}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$$

$[\because c^2 = a^2 - b^2$

$b^2 = a^2 - c^2]$

$$\Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

this is the required eqn of ellipse when $a > b$

Qn Find the eqn of ellipse if $a < b$

i.e major axes lie on y-axes and minor axes lie on x-axis.

(Do. yourself)

{Sketching an ellipse from equation}

we have two standard eqn of ellipse

① $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $a > b$

② $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $a < b$

→ steps

(1) Identify major axes ~~or minor~~ axes lie on x-axes or y-axes

② Compare with standard equation to find the value of a and b

③ Draw a box extending a units on each side of the center along the major axis and b units on each side of the center along the minor axis

④ Using the box as a guide, sketch the ellipse so that its center is at origin and it touches the sides of the box where the sides intersect the co-ordinate axes.

Date

Expt. No.

Q. sketch the graphs of the ellipse

(a) $\frac{x^2}{9} + \frac{y^2}{16} = 1$ (b) $x^2 + 2y^2 = 4$

(a) $\frac{x^2}{9} + \frac{y^2}{16} = 1$

Compare with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

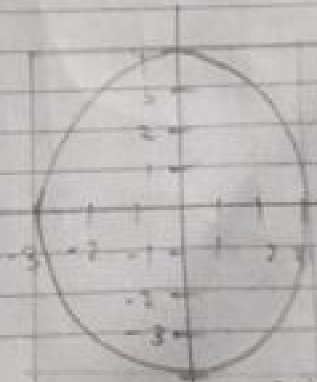
$\Rightarrow a^2 = 9 \quad b^2 = 16$

$\Rightarrow a = 3 \quad , b = 4$

$\Rightarrow a < b$

$\Rightarrow$ major axes lie on y-axes and minor axes lie on x-axes

→ Draw a box 4 unit from on y-axes and 3 unit on x-axes



this is the rough sketch of required ellipse

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(b) $x^2+2y^2=4 \Rightarrow \frac{x^2}{4}+\frac{y^2}{2}=1$

$\Rightarrow \frac{x^2}{(2)^2}+\frac{y^2}{(\sqrt{2})^2}=1$

compare with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

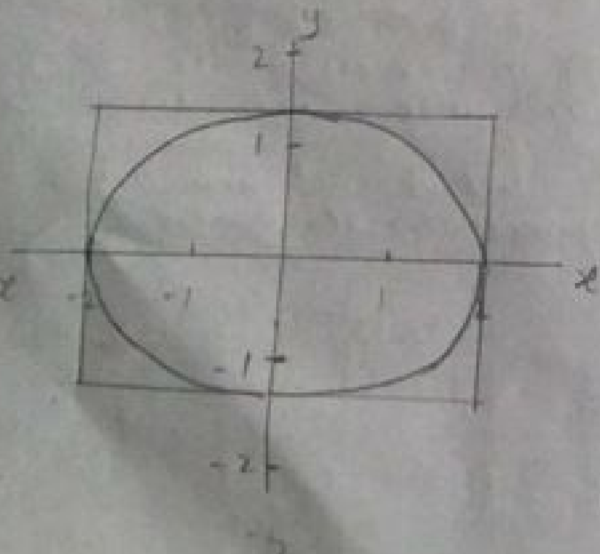
$\Rightarrow a^2=2^2,\ b^2=(\sqrt{2})^2$

$\Rightarrow a=2$ and $b=\sqrt{2} \Rightarrow a>b$

$\because a>b \Rightarrow$ major axis lie on x-axis and minor axis lie on y-axis

$c=\sqrt{a^2-b^2}=\sqrt{4-2}=\sqrt{2}$

Foci $(\pm\sqrt{2},0)$



$\rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ [centre of ellipse is origin]

If $a>b$ then [case (1)]

(1) vertices $A(a,0)$ and $A(-a,0)$

(2) major axis [length] $=2a$

(3) minor axis [length] $=2b$

(4) Foci $(\pm c,0)$ where $c=\sqrt{a^2-b^2}$

$\rightarrow$ If $a<b$ [case (2)]

(1) vertices $(0,-b)$ & $(0,+b)$

(2) length of major axis $=2b$ [$\because a<b$]

(3) length of minor axis $=2a$

(4) Foci: $F(0,\pm c)$ i.e $F_1(0,c)$, $F_2(0,-c)$

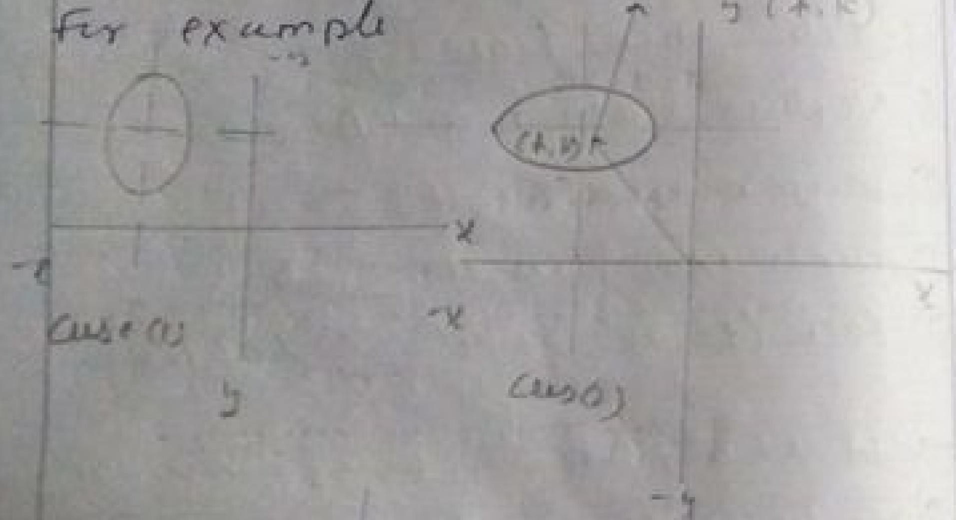
where $c=\sqrt{b^2-a^2}$ $\quad b>a$

LECTURE 4

Shifting origin

Equation of ellipse having centre at (h, k) and major axes is $\parallel$ to x-axes or y axes

For example

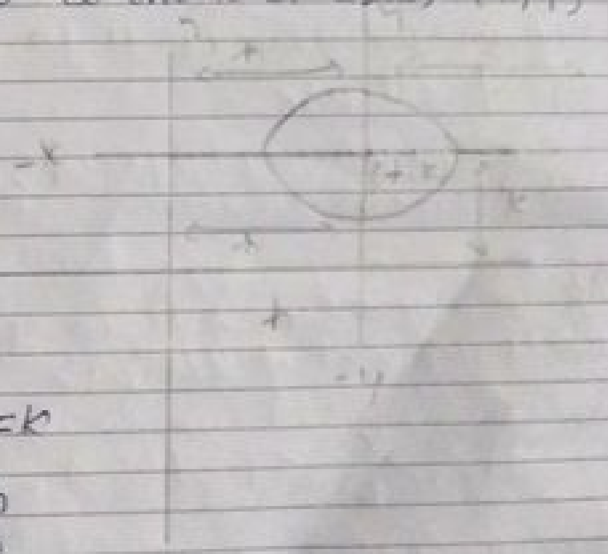


→ In each case centre of ellipse is not $(0,0)$ these type ellipse is called shifted / translated ellipse

Shifting origen to centre (h, k) using transformation eqn

$x = X + h \quad , \quad y = Y + k$

the centre $C(h, k)$ becomes origen w.r.t new co-ordinate axes (X, Y)



→ By figure

$x = h, \; y = k$

so $x = X + h$

$h = X + h$

$X = 0$

$y = Y + k$

$k = Y + k \Rightarrow Y = 0$

$\Rightarrow (X, Y) = (0, 0)$ centre of new co-ordinate axes w.r.t (x, y)

$\Rightarrow$ The resulting eqn of ellipse in new co-ordinate system (X, Y) is given by

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad \text{if } a > b$$

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad \text{if } a < b$$

$\rightarrow$ if major axis is $\parallel$ to x-axis then eqn of ellipse

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad [\because a > b]$$

$\rightarrow$ If major axis is $\parallel$ to y-axis then we take eqn of ellipse

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad [\because a < b]$$

Standard eqn of translated ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{with center } (h, k)$$

first we find co-ordinates of vertices, foci, etc. w.r.t the new co-ordinate axis (X, Y) and then we get new co-ordinate of vertices foci etc. w.r.t (x, y) old co-ordinate by using transformation equation

Q. Trace the eqn of ellipse

$$16x^2 + 9y^2 - 64x - 54y + 1 = 0$$

$\Rightarrow 16x^2 - 64x + 9y^2 - 54y + 1 = 0$

$\Rightarrow 16(x^2 - 4x) + 9(y^2 - 6y) + 1 = 0$

$\Rightarrow 16(\underbrace{x^2 - 4x + (2)^2} - (2)^2] + 9[\underbrace{y^2 - 6y + (3)^2} - (3)^2] + 1 = 0$

$\Rightarrow 16[(x-2)^2 - 4] + 9[(y-3)^2 - 9] + 1 = 0$

$\Rightarrow 16(x-2)^2 + 9(y-3)^2 - 64 - 81 + 1 = 0$

$\Rightarrow 16(x-2)^2 + 9(y-3)^2 - 144 = 0$

$\Rightarrow 16(x-2)^2 + 9(y-3)^2 = 144$

divided By 144 both side we get

$$\frac{(x-2)^2}{9} + \frac{(y-3)^2}{16} = 1$$

⇒ we get

$$\frac{(x-2)^2}{(3)^2}+\frac{(y-3)^2}{(4)^2}=1$$

compare with

$$\frac{(x-h)^2}{a^2}+\frac{(y-3)^2}{b^2}=1$$

$\Rightarrow a=3,\ b=4,\ h=2,\ k=3$

$(h,k)=(2,3)$

$\because a<b \Rightarrow$ major axis lie on y-axis and $\parallel$ to y axis

⇒ shifting origin to center (h,k) using transformation eqn

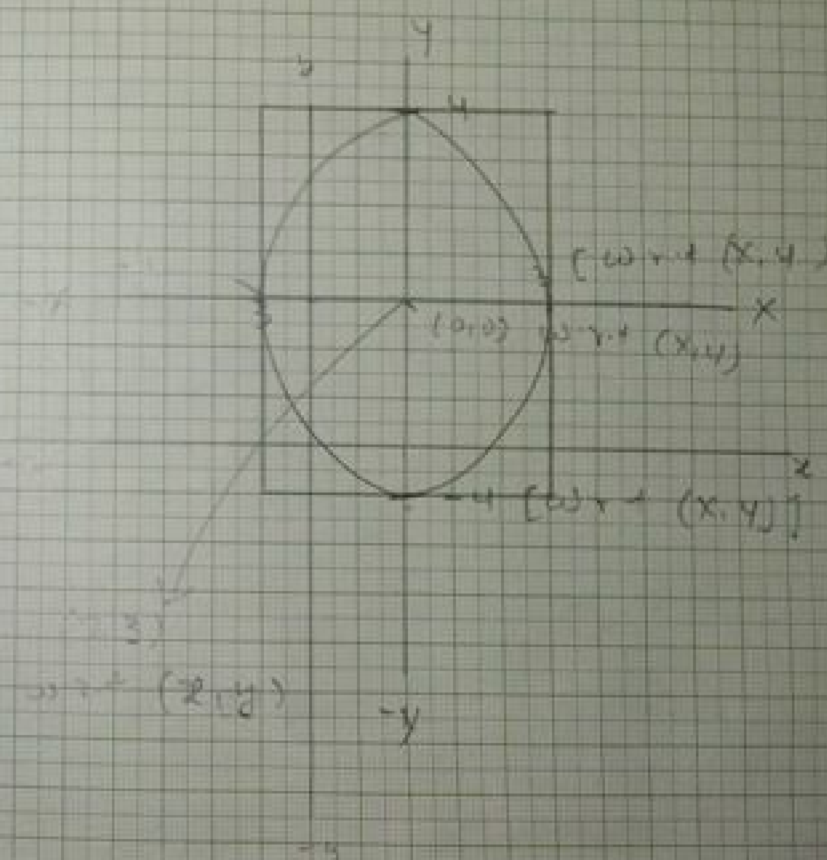
$x=X+h, \quad y=Y+K$

→ center w.r.t new co-ordinates (X,Y) $(0,0)$

→ center w.r.t old coordinate axes (x,y) $(0+2,0+3)=(2,3)$

→ Focus w.r.t new coordinate axes (X,Y) is $F(0,\pm c)$

$c=\sqrt{b^2-a^2}=\sqrt{16-9}=\sqrt{7}=\sqrt{7}$



$F_1(0, \sqrt{7})$ and $F_2(0, -\sqrt{7})$

→ Focus w.r.t old axes (x, y) :

$F_1(2+0, 3+\sqrt{7})$ & $F_2(2+0, 3-\sqrt{7})$

$F_1(2, 3+\sqrt{7})$ & $F_2(2, 3-\sqrt{7})$

→ Length of major axes $= 2b = 2\times4$
$= 8$ [w.r.t (x, y)]

→ length of ~~major~~ minor axes $= 2\times3$
$= 6$

→ vertices w.r.t (X, Y) $(0, 4)$
and $(0, -4)$

→ vertices w.r.t (x, y) $(0+h, 4+k)$
and $(0+h, -4+k)$

i.e $(0+2, 4+3)$ and $(0+2, -4+3)$

i.e $(2, 7)$ and $(2, -1)$

Qn sketch the ellipse and label the foci, vertices and ends of the minor, major axes

$(x+3)^2+4(y-5)^2=16$

$\frac{(x+3)^2}{16}+\frac{\cancel{4}(y-5)^2}{\cancel{16}}=1$

$\frac{(x+3)^2}{(4)^2}+\frac{(y-5)^2}{(2)^2}=1$

compare with $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

$h=-3 \quad k=5 \quad a=4,\ b=2$

$(h,k)=(-3,5)$

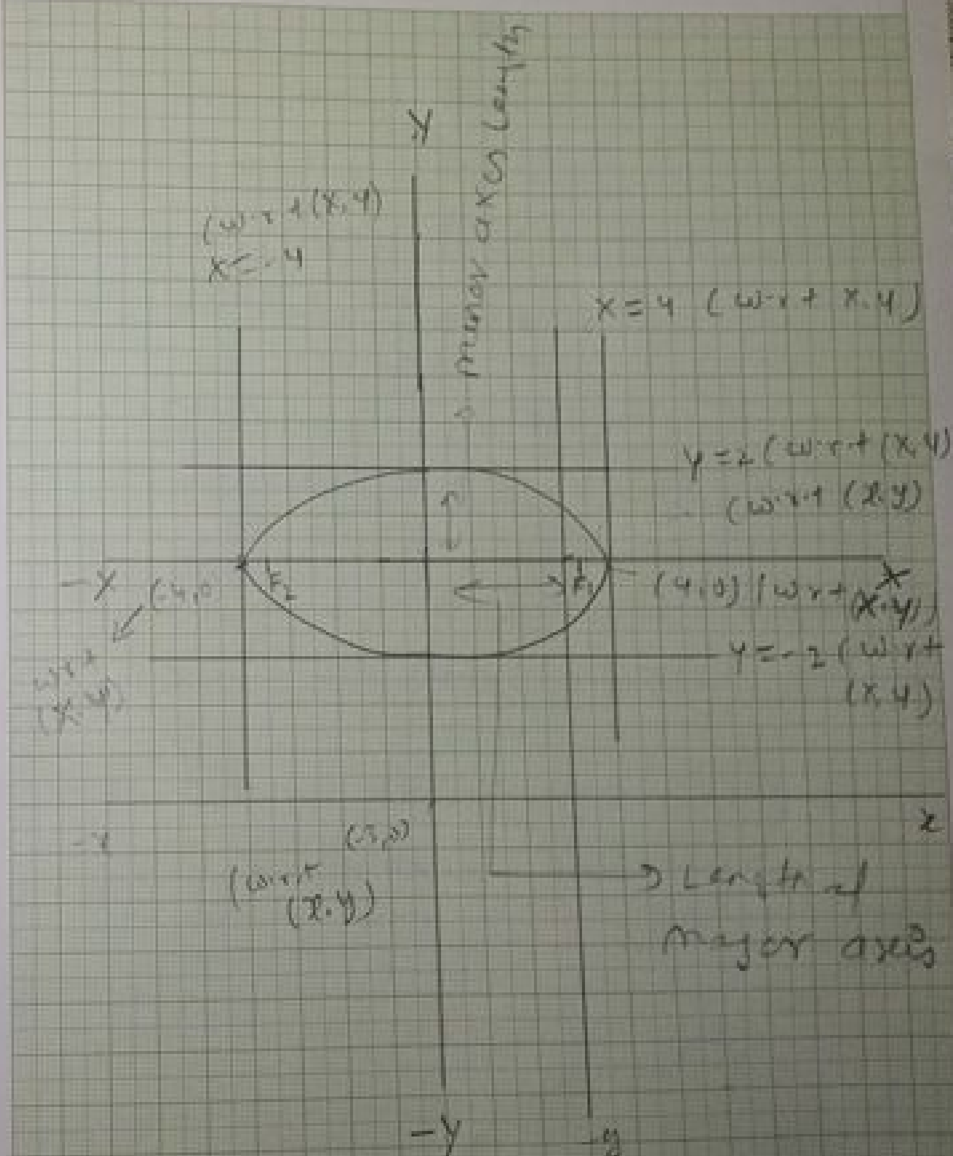
$\because a>b \Rightarrow$ major axes $\parallel$ to x-axes

$\Rightarrow$ ~~Shifting~~ shifting origin to center (h,k) using transformation eqn

$x=X+h,\ y=Y+k$

vertices w.r.t (X,Y) are $(4,0)$ & $(-4,0)$

Vertices w.r.t (x,y) are $(4+h, 0+k)$ & $(-4+h, 0+k)$



Date

Expt. No.

Page No.

i.e $(4+(-3), 0+5)$ and $(-4+(-3), 0+5)$

i.e $(1, 5)$ and $(-7, 5)$

→ Foci w.r.t (X, Y) $F_1(c, 0)$ and $F_2(-c, 0)$

$c = \sqrt{a^2 - b^2}$ ∵ $a > b$

$= \sqrt{16-4} = 2\sqrt{3}$

$F_1(2\sqrt{3}, 0)$ and $F_2(-2\sqrt{3}, 0)$

→ Foci w.r.t (x, y) $F_1(c+h, 0+k)$ and $F_2(-c+h, 0+k)$

i.e $F_1(2\sqrt{3}-3, 0+5)$ & $F_2(-2\sqrt{3}-3, 0+5)$

i.e $F_1(2\sqrt{3}-3, 5)$ and $F_2(-2\sqrt{3}-3, 5)$

→ length of major axes $= 2a = 2 \times 4 = 8$

→ length of minor axes $= 2b = 2 \times 2 = 4$

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