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Exercise: Experimental verification of Planck's law and comparison w/d <sup>other</sup> formulae:-

Show the no. of photon in "Black body" radiation at a Temp.  $T$  is equal to

$$N = \frac{V}{\pi^2} \left( \frac{kT}{hc} \right)^3 \int_0^\infty \frac{x^2 dx}{e^x - 1}$$

We already have the no. of resonator per unit volume in the freq range  $\nu$  and  $\nu + d\nu$

$$= \frac{8\pi\nu^2}{c^3} d\nu$$

And energy of each resonator  
 $= \frac{h\nu}{e^{h\nu/kT} - 1}$

Total Energy of resonator per unit volume

$$= \left( \frac{8\pi\nu^2 d\nu}{c^3} \right) \left( \frac{h\nu}{e^{h\nu/kT} - 1} \right)$$

photon energy =  $h\nu$

$\therefore$  no. of photons per unit volume in the enclosure  $\nu$  and  $\nu + d\nu$  (freq)

$$= \frac{8\pi\nu^2 d\nu}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

If  $V$  is volume of enclosure, then no. of photons in smaller freq range  $\nu$  and  $\nu + d\nu$

$$n = \frac{8\pi V \nu^2 d\nu}{c^3} \cdot \frac{1}{e^{h\nu/kT} - 1}$$

Total no. of photons

$$= \frac{8\pi V}{c^3} \int_0^{\infty} \frac{\nu^2 d\nu}{e^{h\nu/kT} - 1}$$

Let  $\frac{h\nu}{kT} = x$

$$\frac{xkT}{h} = \nu$$

$$d\nu = \frac{kT}{h} dx$$

$$\therefore n = \frac{V}{\pi^2} \left( \frac{2\pi kT}{hc} \right)^3 \int_0^{\infty} \frac{x^2 dx}{e^x - 1}$$

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$$k = h/2\pi$$

Ex: Find out the expression for the wavelengths corresponding to maximum energy of emission according to formula of Planck.

## Determination of Stefan's Constant

Apparatus for calculation of Stefan's Constant. Consist of a

hollow metallic hemisphere

H blackened inside and a wooden box W lined with tin which serves as a

steam chamber. Steam can be passed in the chamber when

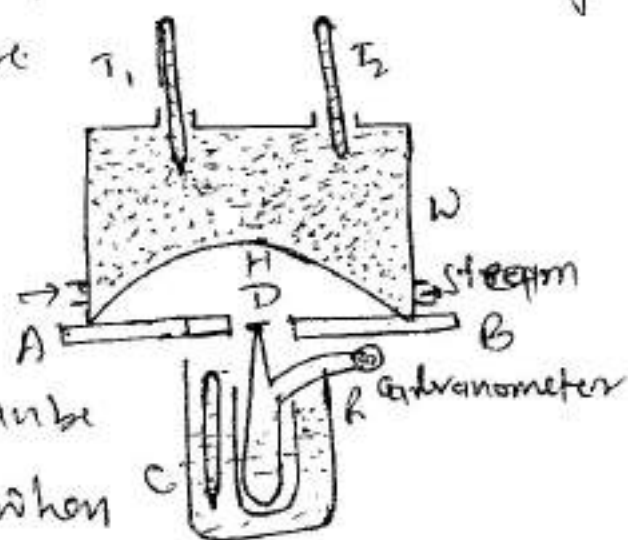
desired and then in steady state the temp. of hemisphere remains const; equal

to that of steam and may be measured by thermometers  $T_1$  and  $T_2$ . The hemisphere

H rest symmetrically on a platform AB

which has a small hole <sup>at centre</sup> D. A small silver disc  $ABD$  blackened at its top surface can be fitted or taken out from the hole.

one junction of the silver-constantan couple is soldered to the lower surface of D; while other is placed in the water bath or sand container C. A sensitive galvanometer (G) is introduced in thermocouple circuit with resistance R.



When the inner surface of H is heated by passing steam, it acts as a black body radiator. The disc D absorbs the radiation emitted by H and its temperature rises continuously, thus causing a difference of temperature in the two junctions of the thermocouple.

If  $T_1$  is the steady state temp. of H and  $T_0$  is that of disc D, when it is just exposed to radiation from H.

Then by Stefan-Boltzmann law the net energy gained by disc per second =  $EA$

$$= \sigma (T_1^4 - T_0^4) A \text{ joule, } A \text{ being area of disc.}$$

$$= \frac{\sigma (T_1^4 - T_0^4)}{J} A \text{ kilocal.}$$

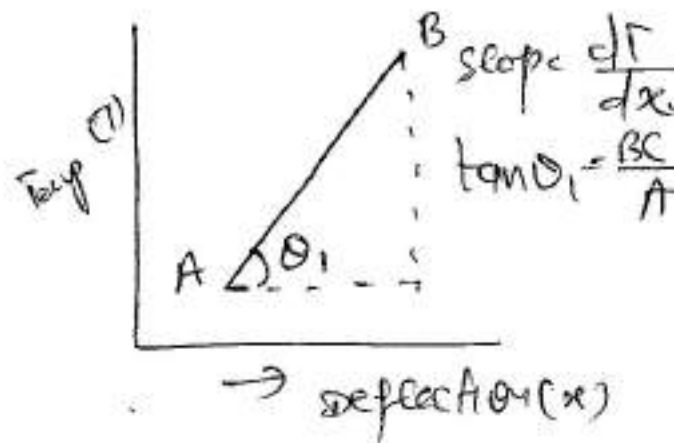
If  $m$  is the mass of disc,  $c$  is specific heat and  $dT/dt$  is rate of rise of temp. at  $T_0$  then the energy gained by disc per second

would be  $mc(dT/dt)$ ; hence we must have

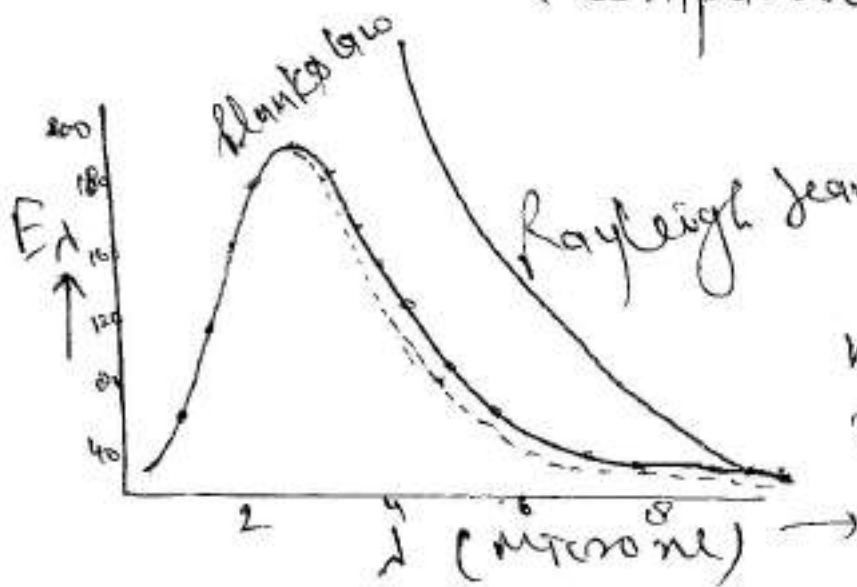
$$mc \frac{dT}{dt} = \frac{\sigma (T_1^4 - T_0^4) A}{J}$$

$$C = \frac{Jmc}{A(T_1^4 - T_0^4)} \frac{dT}{dt} \quad \text{watt m}^{-2} \text{K}^{-4}$$

evaluate  $\frac{dT}{dt}$



# Experimental verification of Planck's radiation law and comparison w/d other formula



spectrum of BB radiation was produced by ~~refra~~ refraction through a fluor spar prism

A concave mirror was used to obtain the image of the slit. This was focussed on Lummer-Kurlbaum linear spectrum calorimeter <sup>which</sup> was ~~employed~~ used to measure the radiant energy or distribution of energy.

Ex-1 Impacts and utility of Planck's laws.  
sol?

Ex-2.1 White dwarf star  
2.2 Chandrasekhar ~~star~~ limit  
Selkner Mass limit

Derive the Stefan's constant and Wien's constant in terms of Planck's and Boltzmann's constant. (Using Planck's constant.)

Using Planck formula - the total radiant energy in unit volume of an isothermal enclosure is given by

$$U = \int_0^{\infty} E_{\lambda} d\lambda = \frac{8\pi h c}{15} \int_0^{\infty} \frac{d\lambda}{\lambda^5} \left( \frac{h c}{\lambda k T} \right)^{-3} \quad \text{--- (1)}$$

But the total radiant energy in unit volume of an isothermal enclosure is also given by

$$U = AT^4 \quad \text{--- (2)}$$

Where  $A$  is constant and  $T$  is the absolute temp. of enclosure

Comparing (1) and (2)



$$AT^4 = 8\pi hc \int_0^\infty \frac{d\nu}{\nu^5 (e^{hc/\nu kT} - 1)}$$

Let  $x = \frac{hc}{\nu kT} \Rightarrow \nu = \frac{hc}{x kT}$

$d\nu = -\frac{hc}{x^2 kT} dx$ , we have

$$AT^4 = 8\pi hc \int_0^\infty \frac{-\frac{hc}{x^2 kT} dx}{\left(\frac{hc}{x kT}\right)^5 (e^x - 1)}$$

$\Rightarrow A = \frac{8\pi^5 k^4}{15 h^3 c^3}$

Since Stefan's constant ( $\sigma$ ) is given by

$$\sigma = \frac{AC}{4}$$

$$\sigma = \frac{8\pi^5 k^4}{15 h^3 c^3} \times \frac{c}{4} = \frac{2}{15} \frac{\pi^5 k^4}{h^3 c^2}$$

which is required expression for Stefan's const.

Wein's const.  $\rightarrow$  Acc. to Wein's law

$$b = \lambda_m T$$

wavelength  $\leftarrow \lambda_m = \frac{b}{T} = \frac{hc}{4.965 kT}$  (we have already)

$$\lambda_m T = \frac{hc}{4.965 k} = b$$