Obtain Clausius-Mosotti relation

If there are *N atoms or molecules* per unit volume*, the* polarization of a material can be defined as ,

$$P = N\alpha E_{loc} = \chi_e \varepsilon_o E_{loc} \quad (1)$$

where the χ_e term is known as the electric susceptibility of the material given by the equation $\chi = \epsilon_r - 1$

The local field is given by

$$E_{loc} = E + \frac{P}{3\varepsilon_o} \qquad (2)$$

We can now write dielectric displacement vector relation is given by,

$$D = \varepsilon_o E + P$$

$$P = D - \varepsilon_o E$$

$$= \varepsilon_o E \varepsilon_r - \varepsilon_o E$$

$$P = \varepsilon_o (\varepsilon_r - 1) E \qquad (3)$$

Where ε_r is the dielctric constant or relative permittivity of the dielectric material

Putting the equation (3) in equation (2), we get

$$E_{loc} = \frac{E}{3}(\varepsilon_r + 2) \qquad (4)$$

Combining the equation (1) and equation (4), we get

$$P = N\alpha\frac{E}{3}(\varepsilon_r + 2) \qquad (5)$$

Combining the equation (3) and equation (5), we get

$$\varepsilon_o(\varepsilon_r - 1)E = N\alpha\frac{E}{3}(\varepsilon_r + 2)$$

$$\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha}{3\varepsilon_o} \qquad (6)$$

This is Clausius-Mosotti relation connecting macroscopic dielectric constant with the microscopic polarizabilities.

In general form , it is expression as;

$$\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{1}{3\varepsilon_o} \sum N_j \alpha_j \quad (7)$$

$Where, n = \sqrt{\varepsilon_r}$ $i.e$ for $optical$ $material$ and $dielectric, \mu_r = 1$

$$\frac{(n^2 - 1)}{(n^2 + 2)} = \frac{N\alpha}{3\varepsilon_o} \qquad (8)$$

This is knows as *Lorentz-Lorentz relation.*

Classical Theory of Electronic Polarizability

❖Classical picture of an atom is that there is a heavy, immobile, undeformable nucleus surrounded by an electronic shell of Ze charge connected to the nucleus by harmonic spring types F=-Kx.

If x is the displacement of the electron under the effect of local electric field, Then we get

$$-eE_{loc} = kx = m\omega_o x^2 \qquad (1)$$

Where K is the force constant b/w the positive heavy nucleus and the electron cloud and ω_o is the natural frequency of oscillation of atom. Pictorially shown in Fig .(1)

We want to develop a simple model that describes the frequency dependence of a electronic polarizability. Let ω frequency of local field , the field at any time is given by $E_{loc} \sin \omega t$

The equation of motion is given by

$$m\frac{d^2x}{dt^2} + m\omega_o x^2 = -eE_{loc} \sin \omega t \qquad (2)$$

In addition, under the influence of the varying electric field types is given by

$$x = x_o \sin \omega t \qquad (3)$$

Putting equation (3) in above equation (2), we obtain

$$m\,(-\omega^2 + \omega_o^2)x_o = -eE_{loc} \qquad (4)$$

The amplitude of corresponding dipole moment is given by

$$p_o = -ex_o \qquad (5)$$

Therefore,

$$p_o = \frac{e^2 E_{loc}}{m\,(\omega_{o^2} - \omega^2)} \qquad (6)$$

Now the electronic polarizability is given by

$$\alpha_{ele} = \frac{p_o}{E_{loc}} \qquad (7)$$

We get,

$$\alpha_{ele} = \frac{e^2/m}{(\omega_{o^2} - \omega^2)} \qquad (8)$$

This types the frequency dependence of a electronic polarizability

In the above equation (8), their arise cases

$$\text{if } \omega \ll \omega_o, \text{ then } \alpha_{ele} = \frac{e^2}{m\omega_o^2} \qquad (9)$$

The above equation (9) is static or independent of the frequency of the electric field.

See the Figure -1 at R. K. Puri & V. K. Babbar & R.J.Singh book

Reference book:

- Solid State Physics by R. K. Puri & V. K. Babbar
- Rudiments of Materials by S.O. Pillai and MRS. SIVAKAMI PILLAI (Author)
- Solid State Physics by Rita John
- Introduction To Solid State Physics by Charles Kittel
- Solid State Physics by R.J.Singh