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Assignment No.: 02, GE-02 : Linear Algebra

Q.1 Verify that the set

$$B = \left\{ v_1 = \left[\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right], v_2 = \left[-\frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}, \frac{-1}{3\sqrt{2}}\right], v_3 = \left[\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right] \right\}$$

is an orthonormal basis for $\mathbb{R}^3$. Then find $[v]_B$ for the vector $v = [-1, 5, 3]$

Q.2. Write the Gram-Schmidt orthogonalization process. Use Gram-Schmidt process to find an orthogonal basis for the subspace of $\mathbb{R}^3$ spanned by the set $\{[3,1,-2], [5,-3,-1]\}$

Q.3 For the subspace $W = \{[x,y,z] \in \mathbb{R}^3 : 2x - 5y + z = 0\}$ of $\mathbb{R}^3$, find a basis for the orthogonal complement $W^\perp$ and verify that

$$\dim(W) + \dim(W^\perp) = \dim(\mathbb{R}^3)$$

Q.4. For each of the following subspace W of $\mathbb{R}^n$ & given vector $v \in \mathbb{R}^n$, find $\mathrm{Proj}_W v$ & decompose v into $w_1 + w_2$; $w_1 \in W$ & $w_2 \in W^\perp$

(a) In $\mathbb{R}^3$, W = the plane $3x - y + 4z = 0$, $v = [2,2,-3]$

(b) in $\mathbb{R}^3$, W = plane $2x - 2y + z = 0$, $v = [1,-4,3]$

Is the decomposition unique ?

Q.5 Define the following with example

(a) orthogonal Projection onto a subspace

(b) orthogonal complement

(c) orthogonal & orthonormal set of vectors

(d) orthogonal & orthonormal Basis.