Date :
Page No.:

Q. A Steady current I flows cylindrical wire of radius 'a'. Find the magnetic field both inside and outside the wire, if

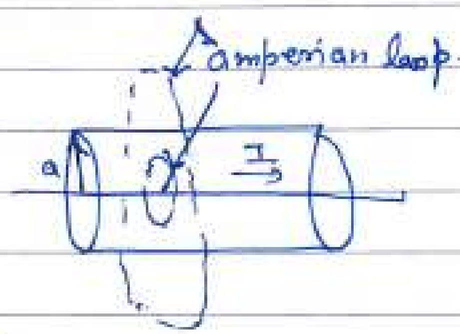
(a) The current is uniformly distributed over the outside surface of the wire.

(b) The current is distributed in such a say that J is proportional to s, the distance from the axis.

Sol (a)

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}.$$

$$|\vec{B}|2\pi s = \mu_0 I_{encl}.$$



we ~~have~~ will take two amperian loop one whose radius is (i) $s>a$
(ii) $s<a$.

$$\Rightarrow |\vec{B}|2\pi s = \mu_0 I_{encl}.$$

$$\vec{B} = 0 \quad \text{for } s<a. \quad (I_{encl}=0)$$

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi} \quad \text{for } s>a$$

(b)

$$J = ks;$$

$$I = \int_0^a J\,da = \int_0^a ks(2\pi s)\,ds$$

This can also be written in terms of cylindrical coordinates. Result will be same.

$$I = \frac{2\pi k a^3}{3} \Rightarrow k = \frac{3I}{2\pi a^3}$$

Spiral

in this problem we tried to find the 'k' ~~the sim~~ (unknown const) in terms of known parameter that is I.

$$I_{encl} = \int_0^s J\, da = \int_0^s k\bar{s}\,(2\pi\bar{s})\, d\bar{s}$$

$$= \frac{2\pi k s^3}{3} = \frac{I s^3}{a^3} \quad \text{for } s<a.$$

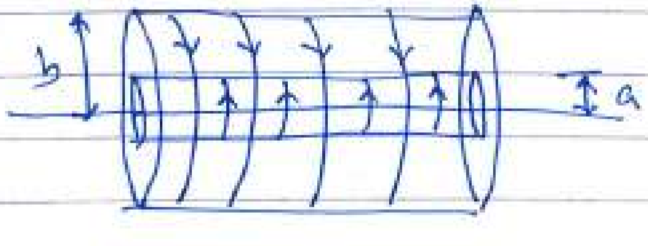
for ~~s<a~~ , $s>a$

$$I_{encl} = I$$

So

$$\vec{B} = \begin{cases} \dfrac{\mu_0 I s^2}{2\pi a^3}\,\hat{\phi} & \text{for } s<a. \\ \dfrac{\mu_0 I}{2\pi s}\,\hat{\phi} & \text{for } s>a \end{cases}$$

③ Problem:→ Two long coaxial solenoids each carry current I, but in opposite directions. The inner solenoid (radius a) has n_1 turns per unit length, and outer one (radius b) has n_2. Find B in each of the three regions (i) inside the inner solenoid (ii) between them & (iii) outside both.



Spiral

Date:
Page No.:

Solution:- Field inside any solenoid is $\cdot\ \mu_0 n I$ and outside is zero ('0'). The outer solenoid's field points to the right ($\hat{x}$). Whereas the inner one points to the left ($-\hat{x}$)

(i) $\vec{B} = \mu_0 (n_2 - n_1) I\ \hat{x}$.

(ii) $\vec{B} = \mu_0 I n_2\ \hat{x}$.

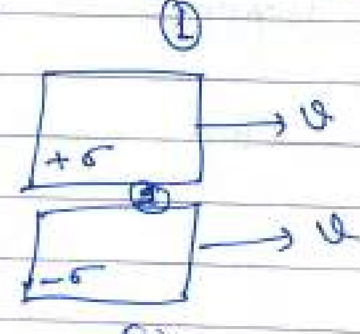
(iii) $\vec{B} = 0$.

Q. A large parallel-plate capacitor with uniform surface charge σ on the upper plate and $-\sigma$ on the lower is moving with a constant speed v.

(a). Find the magnetic field between the plates and also above & below them.

(b). Find the magnetic force per unit area on the upper plate, including its direction.

(c) At what speed v would the magnetic force balance the electrical force.



Solution

$$\vec{K} = \sigma \vec{v} \quad \text{(surface current density)}$$

For top plate with $+\sigma$. a field $\frac{\mu_0 K}{2}$ (aiming out of page, for point above it and into the page for point below.

Spiral

For bottom plate with $(-\sigma)$ a field $\frac{\mu_0 K}{2}$ (aiming into the page for point above it and out of the page for point below.

(a) $B = \mu_0 \sigma v$ (in) between the plate. ②
$B = 0$ elsewhere outside ① & ③

(b) The Lorentz force law say $f = \int (\vec{K} \times \vec{B})\, da$

so the force per unit area $f = (\vec{K} \times \vec{B})$

$$f_m = \frac{\mu_0 \sigma^2 v^2}{2} \quad (up)$$

where

$K = \sigma v$

$\vec{B}$ (the field of the lower plate.) $= \frac{\mu_0 \sigma v}{2}$

(c) The electric field of the lower plate is $\frac{\sigma}{2\epsilon_0}$; the electric force per unit area on the upper plate is $f_e = \frac{\sigma^2}{2\epsilon_0}$ (down)

They balance if: $f_e = f_m$

$$\mu_0 v^2 = \frac{1}{\epsilon_0}$$

$$v^2 = \frac{1}{\epsilon_0 \mu_0}$$

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c \text{ (speed of light)}$$

Date:
Page No:

Magnetostatics.

$$\vec{\nabla}.\vec{B} = 0 \qquad \text{(No name)}$$
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \qquad \text{(Ampere's law)}$$

So. $\vec{\nabla}.\vec{B} = 0$

Divergance of curl is alway zero.

$$\vec{\nabla}\cdot\vec{\nabla}\times\vec{A} = 0.$$

$$\Rightarrow \quad \vec{B} = \vec{\nabla}\times\vec{A}.$$

So $\vec{B}$ can be written curl of a vector. this is called the vector potential.

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0 J$$

we can add to $\vec{A}$ any function whose curl vanish. (gradient of any scalar), with no effect on B. So we can ~~explicit~~ eliminate the divergence of A.

$$\vec{\nabla}.\vec{A} = 0 \qquad \text{—— (1)}$$

To prove that this is always possible, suppose that our orignal potential, A_0 is not divergenceless. If we add to it the gradient of λ ($A = A_0 \nabla\lambda$), the new divergence is

$$\vec{\nabla}.\vec{A} = \vec{\nabla}.A_0 + \nabla^2\lambda$$

using eq (1).

$$\nabla^2\lambda = -\vec{\nabla}.\vec{A}_0$$

this is mathematically identical to Poisson's eqn.

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

Spiral

with $\vec{\nabla}\cdot\vec{A}_0$ inplace of ρ/ϵ_0. And we know how to solve Poisson's equation: ρ goes to zero at infinity the solution is:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{r} d\tau'$$

by same token $\vec{\nabla}\cdot A_0$ goes to zero at infinity.

$$\lambda = \frac{1}{4\pi}\int \frac{\vec{\nabla}\cdot A_0}{r} d\tau'$$

It is always possible to make the vector potential divergenceless. To put it the other way around. $\vec{B} = \vec{\nabla}\times\vec{A}$ specifies the curl of $\vec{A}$, but it doesn't say anything about the divergence - we are at liberty to pick. With this condition of A. Ampere's law becom:

$$\nabla^2 A = -\mu_0 \vec{J}$$

Assuming $\vec{J}$ goes to zero at infinity.

$$\boxed{\vec{A}(r) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r')}{r} d\tau'}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}}{r} dl' \quad \text{also} \quad \vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{K}}{r} da'$$

Q. Find the vector potential of an infinite solenoid with n turns per unit length, radius R and current I.

Sol. We can't use the above eqn. since the current itself extends to infinity. But

$$\oint \vec{A}\cdot d\vec{l} = \int (\vec{\nabla}\times\vec{A})\cdot d\vec{a} = \int \vec{B}\cdot d\vec{a} = \Phi$$

Φ is the flux of $\vec{B}$ through the loop in question

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl.}$$

Using a circular amperian loop at radius s inside the solenoid.

$$\oint \vec{A}\cdot d\vec{l} = A(2\pi s) = \int \vec{B}\cdot d\vec{a} = \mu_0 n I \pi s^2$$

$$\vec{A} = \frac{\mu_0 n I}{2} s\,\hat{\phi} \text{ for } s \leq R.$$

For an amperian loop outside the solenoid.

$$\oint \vec{B}\cdot d\vec{a} = \mu_0 n I (\pi R^2)$$

$$\vec{A} = \frac{\mu_0 n I}{2}\frac{R^2}{s}\,\hat{\phi} \quad \text{for } s \geq R$$

Spiral