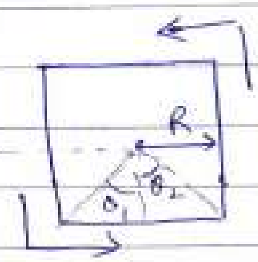
Problems 8:→

(a) find the magnetic field at the center of a square loop, which carries a steady current I. Let R be the distance from center to side



Solution:→ $$\vec{B} = \frac{\mu_0 I}{4\pi R}(\sin\theta_2 - \sin\theta_1)$$

$\theta_2 = -\theta_1 = 45°$ and four side: B=

$$\vec{B} = \frac{\sqrt{2}\,\mu_0 I}{\pi R} \odot \quad \boxed{\text{Ans}}$$

$\odot$ → out of the page.

(b) find the field at the center of a regular n-side polygon, carying a steady-current I. Again, let R be the distance from the center to any side.

Sol:→ $\theta_2 = -\theta_1 = \frac{\pi}{n}$ and n Sides: $\vec{B} = \frac{\mu_0 n I}{2\pi R}\sin \pi/n$

Check that your formula reduces to the field ~~that~~ at the center of a circular loop, in the limit $n \to \infty$

Sol:→ For small θ.

$$\sin\theta \approx \theta$$

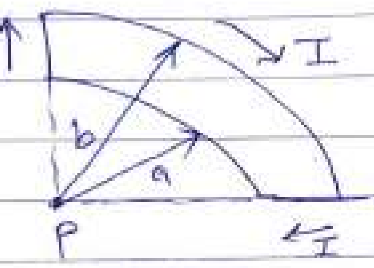
So as $n \to \infty$

$$\vec{B} \to \frac{n\mu_0 I}{2\pi R}\left(\frac{\pi}{n}\right) = \frac{\mu_0 I}{2R}$$

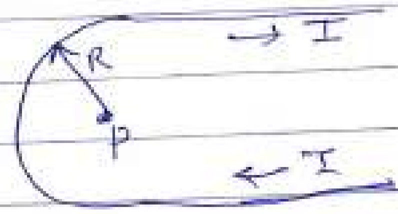
Problem 7.

Q. Find the magnetic field at point P for each of the steady current configuration shown.

①



②



Sol ①. The straight segments produce no field at P. The two quarter-circles give $\vec{B}$

$$\vec{B} = \left(\frac{1}{4}\frac{\mu_0 I}{2a}\right) + \left(-\frac{1}{4}\frac{\mu_0 I}{2b}\right) \odot$$

$$\vec{B} = \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right) \odot$$ → out of the page

Sol ②.

The two half-lines are the same as one infinite line;

$$\vec{B} = \frac{\mu_0 I}{2\pi R}$$ ⊗ → inside the page.

The ~~two~~ half-circle contributes $\frac{\mu_0 I}{4R}$

Total $\vec{B}$

$$\vec{B} = \frac{\mu_0 I}{4R}\left(1 + \frac{2}{\pi}\right) \otimes$$

Ampere's law :-

The curl of $\vec{B}$.

$$\vec{\nabla} \times \vec{B} = \mu_0 J$$

is called Ampere's law (in differential form) by applying fundamental Stokes' theorem.

$$\int (\vec{\nabla} \times \vec{B}) \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a}$$

$\int \vec{J} \cdot d\vec{a}$ is the total current passing through the surface, which is called I_{enc} (the current enclosed by the "Amperian loop")

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

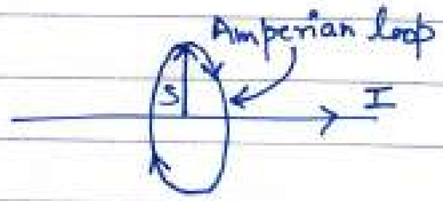
In Electrostatics : Coloumb → Gauss

In Magnetostatics : Biot-Savart → Ampere

Prob. Based on Amperes law:-

Q. Find the field a distance s from a long straight wire carrying a steady current I

Sol: We know the direction of $\vec{B}$ is "circumferential" circling around the wire as indicated by the right-hand thumb rule. By symmetry, the magnetic field 'B' magnitude is constant around an amperian loop of radius 's', centered on the wire, So Ampere's law gives



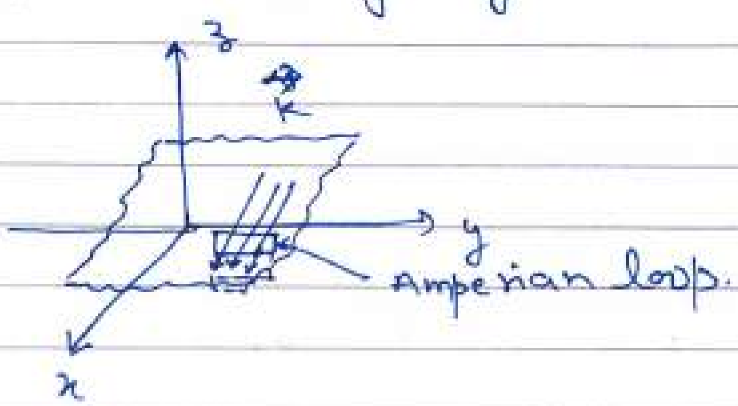
$$\oint \vec{B}\cdot d\vec{l} = B\oint dl = B \cdot 2\pi s$$

$$\mu_0 I_{enc} = \mu_0 I.$$

$$B = \frac{\mu_0 I}{2\pi s}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s}\,\hat{\phi}}$$

Q2 Find the magnetic field of an infinite uniform surface current $\vec{K} = K\hat{x}$ flowing over the xy plane.



Sol.

Any component of $\vec{B}$ perpendicular to $\vec{K}$ (vertical contribution) will be canceled by +y & -y filament. So $\vec{B}$ can have only "y" direction component. Along $-\hat{y}$ above the plane and "$+\hat{y}$" below the plane.

$$\oint \vec{B}\cdot d\vec{l} = 2Bl = \mu_0 I_{enc} = \mu_0 K l.$$

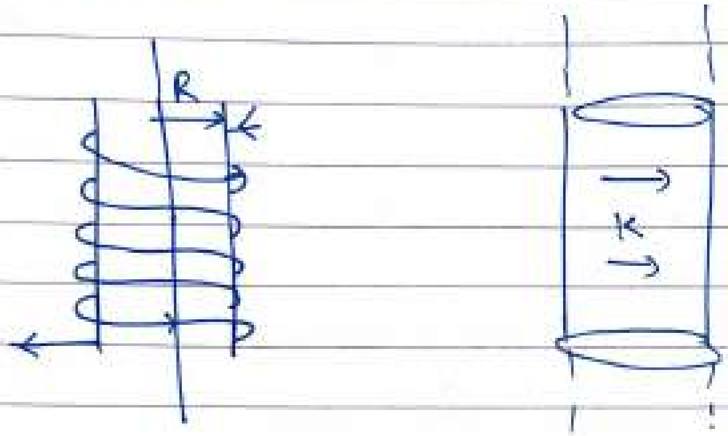
one Bl comes from the top segment and other from the bottom.

$$\vec{B} = +\frac{\mu_0}{2} K\,\hat{y} \quad \text{for } z<0$$

$$B = -\frac{\mu_0}{2} K\,\hat{y} \quad \text{for } z>0$$

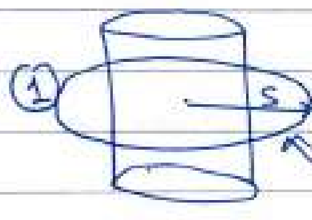
$\vec{B}$ is independent of the distance from the plane.

Q. Find the magnetic field of a very long solenoid, consisting of n closely wound turns per unit length on a cylinder of radius R, each carrying a steady current 'I'



Sol. First of all direction of $\vec{B}$

windings are so close that one can take it as a circular loop.



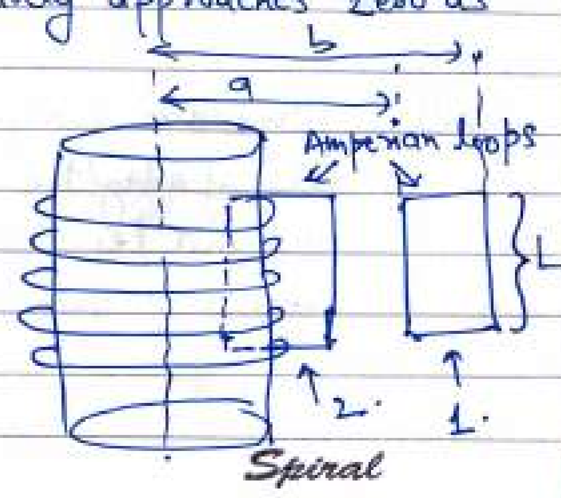
In that case if Amperian loop concentric ① with the solenoid and hence.

$$\oint \vec{B}\cdot d\vec{l} = B_\phi (2\pi s) = \mu_0 I_{enc} = 0$$

Since the loop enclose no current.

So the magnetic field of an infinite, closely wound solenoid runs parallel to the axis. From the right-hand rule, we expect that ~~if~~ it points upward inside the solenoid and downward outside. Moreover, it certainly approaches zero as we go far (very far way.

We Draw two Amperian rectangular loops. Loop 1. lies entirely outside the solenoid with its sides at distance a & b from the axis



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For loop 1 :

$$\oint \vec{B}.d\vec{l} = [B(a) - B(b)]L = \mu_0 I_{enc} = 0.$$

$$B(a) = B(b)$$

Evidently the field outside does not depend on the distance from the axis. But we agreed that it goes to zero for large distance. ~~They~~ Therefore $\vec{B}$ must be zero outside everywhere.

For loop 2 :, which is half inside and half outside, Ampere's law gives.

$$\oint \vec{B}.d\vec{l} = BL = \mu_0 I_{enc} = \mu_0 n I L.$$

$n \to$ no. of turns per unit length.

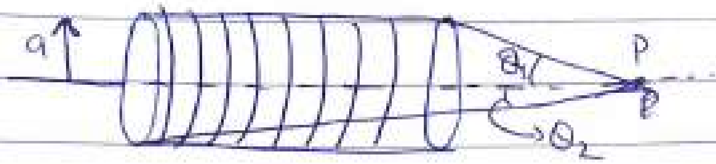
where B is the field inside the solenoid (The right side of the loop contributes nothing since B = 0 outside).

$$\boxed{\begin{array}{lll} \vec{B} = & \mu_0 n I \hat{z} & \text{inside the solenoid.} \\ \vec{B} = & 0 & \text{outside the solenoid.} \end{array}}$$

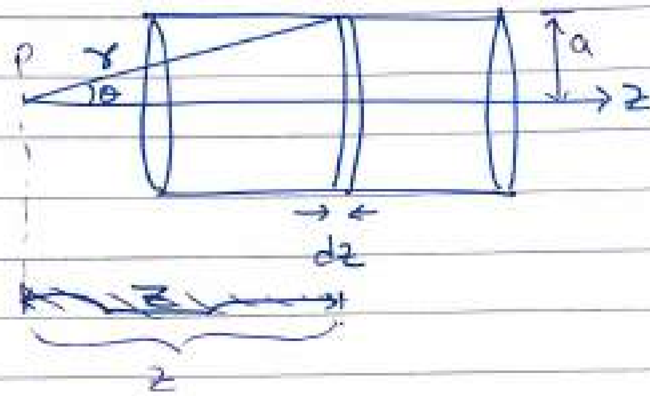
FOR Comparison

The same problem using Biot-Savart law.

Q.11 Find the magnetic field at Point P on the axis of a tightly wound Solenoid Consisting of 'n' turns per unit length wrapped around a Cylindrical tube of radius 'a' and carrying current I.



Solution :-



Let us consider a ring of width dz at a distance of z from the point 'p' or point of interest.

$$I \to nI\,dz$$

$$\vec{B} = \frac{\mu_0 n I}{2}\int \frac{a^2}{(a^2+z^2)^{3/2}}\,dz \qquad z = a\cot\theta$$

$$dz = -\frac{a}{\sin^2\theta}\,d\theta \quad \text{and} \quad \frac{1}{(a^2+z^2)^{3/2}} = \frac{\sin^3\theta}{a^3}$$

$$\vec{B} = \frac{\mu_0 n I}{2}\int \frac{a^2 \sin^3\theta}{a^3 \sin^2\theta}(-a)\,d\theta$$

$$\vec{B} = -\frac{\mu_0 n I}{2}\int \sin\theta\,d\theta = \frac{\mu_0 n I}{2}\cos\theta\Big|_{\theta_1}^{\theta_2}$$

$$\vec{B} = \frac{\mu_0 n I}{2}(\cos\theta_2 - \cos\theta_1)$$ For an infinite solenoid.

$\theta_2 = 0,\ \theta_1 = \pi$ So $(\cos\theta_2 - \cos\theta_1) = 2 \Rightarrow \boxed{\vec{B} = \mu_0 n I\,\hat{z}}$

Sperat

Reference:- Introduction to Electrodynamics:- David J. Griffiths