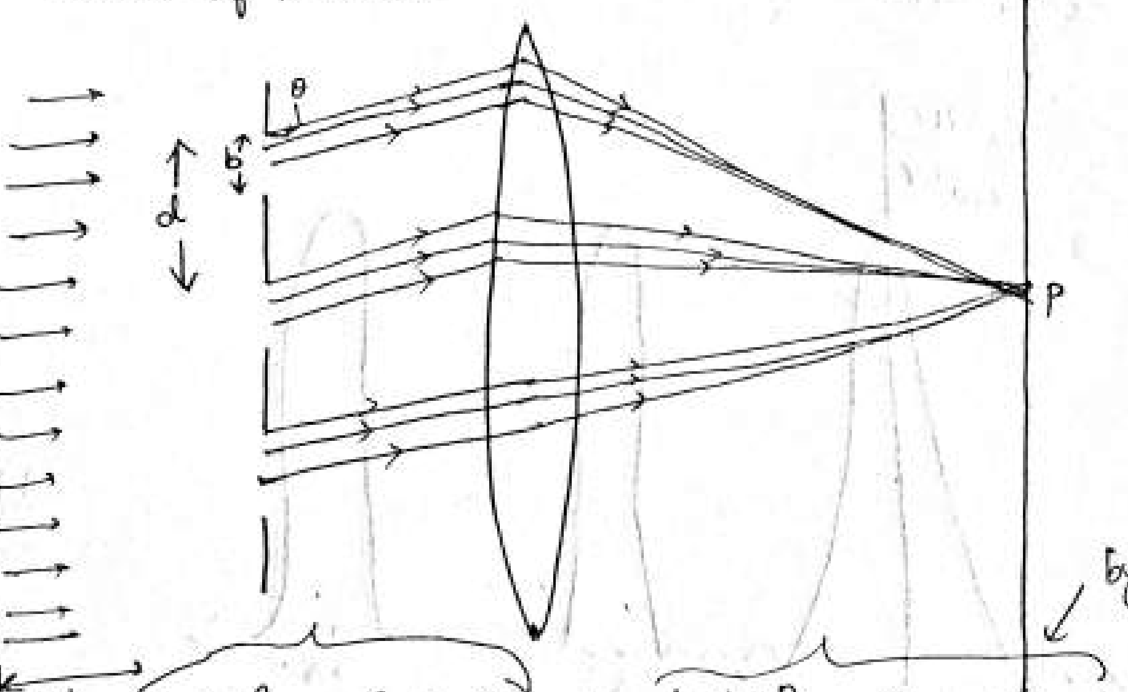
N-Slit Fraunhofer Diffraction Pattern ①

Let us consider the diffraction pattern produced by N parallel slits, each of width 'b', the distance between two consecutive slits is assumed to be 'd'.

As discussed before, each slit can be assumed to be consisting of n equally spaced point sources with spacing Δ.

∴ the field at an arbitrary point P will be a sum of N terms:



by second slit

amplitude due to first slit

$$E = \frac{A\sin\beta}{\beta}\cos(\omega t-\beta) + \frac{A\sin\beta}{\beta}\cos(\omega t-\beta-\phi_1) + \dots$$

$$+ \dots + \frac{A\sin\beta}{\beta}\cos\left[\omega t-\beta-(N-1)\phi_I\right]$$

all the terms have same meaning as discussed before.

$$E = \frac{A\sin\beta}{\beta}\left[\cos(\omega t-\beta) + \cos(\omega t-\beta-\phi_I) + \dots + \cos(\omega t-\beta-(N-1)\phi_I)\right]$$

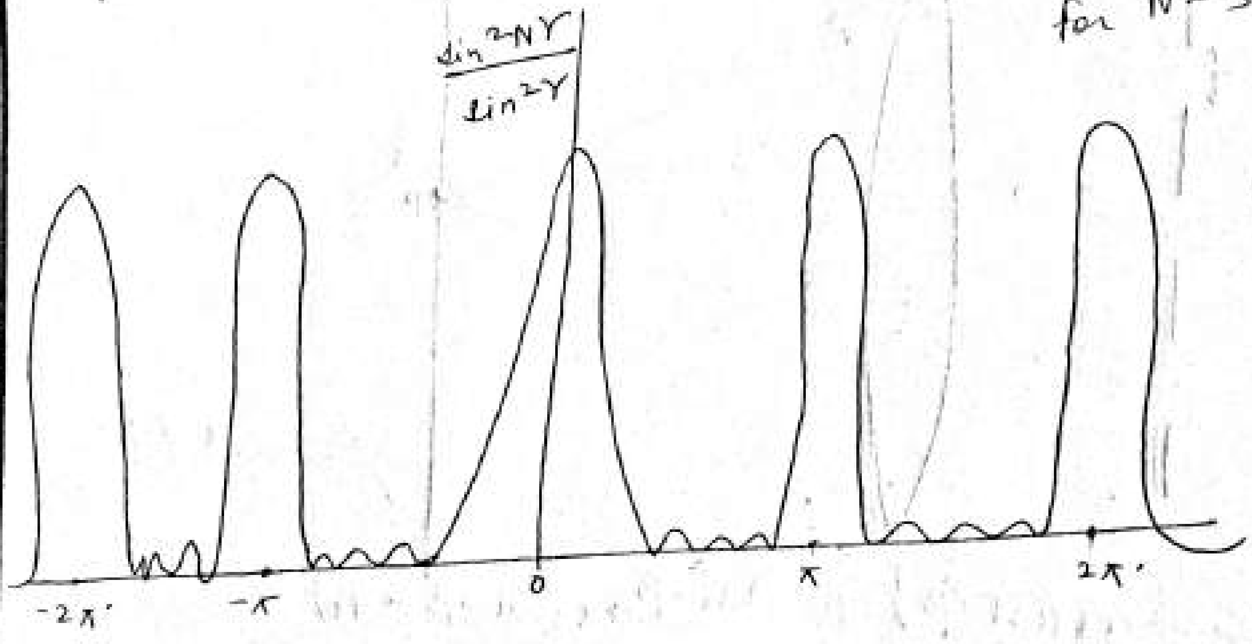
$$E = \frac{A\sin\beta}{\beta}\,\frac{\sin N\gamma}{\sin\gamma}\cos\left[\omega t - \beta - \frac{(N-1)}{2}\phi_I\right]$$

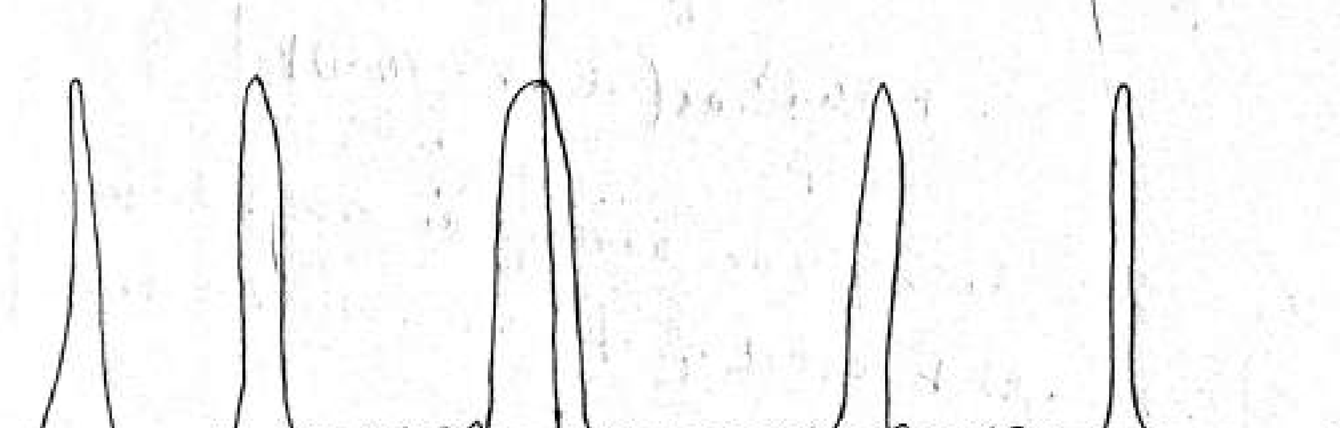
where $\gamma = \dfrac{\phi_I}{2} = \dfrac{\pi}{\lambda} d\sin\theta$

corresponding intensity distribution will be.

$$I = I_0 \underbrace{\frac{\sin^2\beta}{\beta^2}}_{\text{Intensity distribution produced by single slit}}\;\underbrace{\frac{\sin^2 N\gamma}{\sin^2\gamma}}_{\text{Intensity distribution due to interference pattern produced by N equally spaced point sources}} \quad (1)$$

for $N = 5$.





(See book for proper plots).

as N becomes very large the intensity takes form of very sharp peaks at $\gamma = 0, \pi, 2\pi \ldots$.

Between two peaks, the function vanishes when

$$\gamma = \frac{p\pi}{N} \quad p = \pm 1, \pm 2 \ldots \text{ but } p \neq 0, \pm N, \pm 2N$$

which are referred as secondary minima

<u>Positions of maxima & minima</u>

When N is very large, one obtains intense maxima at $\gamma = m\pi$

i.e when $d\sin\theta = m\lambda$.

$$\lim_{\gamma \to m\pi} \frac{\sin N\gamma}{\sin \gamma} = \lim_{\gamma \to m\pi} \frac{\frac{d}{d\gamma}(\sin N\gamma)}{\frac{d}{d\gamma}(\sin\gamma)} = \lim_{\gamma \to m\pi} \frac{N\cos N\gamma}{\cos\gamma}$$

$$= \pm N$$

$\therefore$ the resultant amplitude and the corresponding intensity distribution are given by

$$E = \frac{NA\sin\beta}{\beta}.$$

$$I = \frac{N^2 I_0 \sin^2\beta}{\beta^2}$$

where $\beta = \frac{\pi b \sin\theta}{\lambda} = \frac{\pi b}{\lambda}\frac{m\lambda}{d} = \frac{\pi b m}{d}$

such maxima are known as principal maxima.

Intensity will be zero when (from eq^n ①)

$$b\sin\theta = n\lambda \quad , \; n = 1, 2, 3 \ldots \quad \text{(minima corr. to single slit-diffraction)}$$

$$\text{or } N\gamma = p\pi \quad p \neq N, 2N \ldots$$

Diffraction Grating

An arrangement of a large number of equidistant slits is known as diffraction grating.

Since the position of principal maxima depend on wavelength λ, the grating gives us an method to determine wavelengths.

Usually a grating is constructed by ruling grooves with a diamond point on an optically transparent sheet of material, the grooves act as opaque spaces. Commercially gratings are produced by taking the cast of an actual grating on a transparent film like that of cellulose acetate. These impressions of gratings are then preserved by mounting the film between two glass sheets.

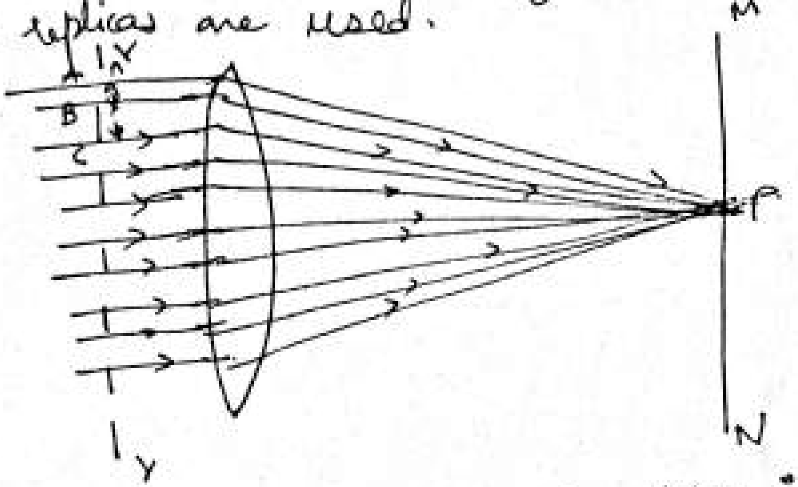
Diffraction spectrum due to a plane diffraction Grating

As discussed in N-slit diffraction, the principal maxima are given by $d\sin\theta = m\lambda$, $m = 0, 1, 2 \ldots$

Various spectral components (having different λ) are obtained at different position. Hence by measuring the angle of diffraction for various colours one can determine the values of wavelength. The intensity is maximum for the zeroth order spectrum and it falls off as the value of m increases.

Plane diffraction grating.

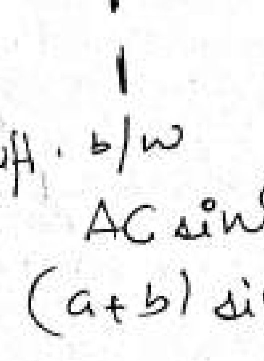
Usually replicas are used.



All the secondary waves travelling in the same direction as that of the incident light will come to focus at the point P on the screen.

If inclined at angle θ w.r.t incident light then they secondary waves comes to focus at point P_1.

Intensity at P_1 will depend on the path diff b/w the secondary waves originating from the corr. points A & C of 2 neighbouring slits.



If path diff b/w A & C increase by $\frac{\lambda}{N}$ then. $\left(\frac{\lambda}{N}\right) N = \lambda$.

then it will give minima

then $\frac{2\lambda}{N}, \frac{3\lambda}{N} \dots$

path diff. b/w secondary waves starting from A & C is $AC \sin\theta = (a+b)\sin\theta$

$(a+b)\sin\theta_n = n\lambda \rightarrow$ maxima.

direction of n^{th} principal maxima

if the incident light consists of more than one λ beam gets dispersed & the ∠s of diffraction for diff. λ will be diff. Let λ & $\lambda + d\lambda$ be nearby λ's present in the incident light & θ & $(\theta + d\theta)$ be corr. ∠ of diff.

then. $(a+b)\sin\theta = \lambda$

$(a+b)\sin(\theta + d\theta) = \lambda + d\lambda$

Putting $n = 1, 2$ we will get directions of images diff. ord

$(a+b)$ is called as <u>grating constant</u>.

$$(a+b) = \frac{2.54}{15000}\text{ cm}.$$

→ Determination of λ using grating

→ <u>Dispersive power of a Grating</u>

is defined as the ratio of the difference in the angle of diffraction of any 2 neighbouring spectral lines to the difference in wavelength between the 2 spectral lines.

i.e $\dfrac{d\theta}{d\lambda}$ = Dispersive power.

$$(a+b)\sin\theta = n\lambda$$

$$(a+b)\cos\theta \, d\theta = n\,d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

$$= \frac{nN'}{\cos\theta}$$

directly $\propto$ 'n' order.

$\propto$ no. of lines /cm.

$\frac{1}{\propto} \rightarrow \cos\theta$.