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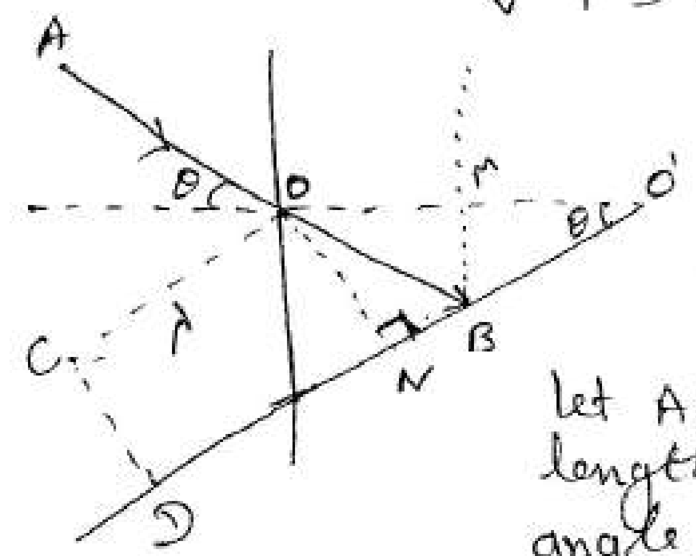
Course - B.Sc Physics (H), $\overline{VI}^{th}$ Sem
Sec. A

Spectral Distribution of Radiation Energy:-

Wien's Law:- According to Stefan-Boltzmann law the relationship b/w the total energy and the temperature of black body radiation. This law does not give the actual distribution of energy in the diff. parts of the spectrum.

Consider a spherical enclosure whose walls reflect perfectly but diffusely. Let it be full of radiation energy density of "u" at a uniform temp. T. Suppose the walls of enclosure move outwards slowly w/d a uniform velocity v, so that radiation ~~undr~~ undergo a reversible adiabatic expansion. So if the the V of the enclosure then we can write

$$V^{1/3}T = \text{Constant}$$



Wavelength change by a ray on reflection at slowly moving walls of the enclosure.

Let AO be the ray of wavelength λ incident at an angle θ on the wall as shown

In the fig.

Let the particular wave crest strike the wall at O. The crest will be reflected along OC. If OC = λ, then as the reflected wave crest reaches C, the next will reach at O in time T (Time period of wave motion). During this time the wall has traversed a distance OM = vT.

So the path followed by the second wave-crest will be ~~ABCD~~ OBD

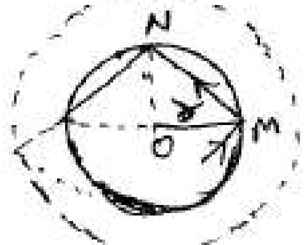
then Increase in wave length due to one reflection

$$= OB + BN = O'B + BN = OO'\cos$$

$$= 2(OM)\cos\theta = 2vT\cos\theta$$

$$= \frac{2v\lambda}{c}\cos\theta \quad \left[\because T = \frac{\lambda}{c}\ \text{c-velocity of ~~lig~~ rad.}\right]$$

Now for calculation of no. of reflections per second, consider a ray striking the spherical wall at angle θ and reflected along MN w/d the angle θ with normal



The distance traversed by beam before next suffers

$$= MN = 2r\cos\theta$$ [r → radi of Inner)

∴ Time b/w two successive reflections

$$= \frac{2r\cos\theta}{c}$$

∴ no. of reflections per second

$$= \frac{c}{2r\cos\theta}$$

If δr is increase in radius in time δt, then no. of reflection in time δt

$$= \frac{c}{2r\cos\theta}\cdot\delta t = \frac{c}{2r\cos\theta}\cdot\frac{\delta r}{v}$$

Change in wave-length in time δt

$\delta\lambda$ = change in wave-length in one reflection × No. of reflections in time δt

$$\delta\lambda = \frac{2v\lambda}{c}\cos\theta \times \frac{c}{2r\cos\theta}\cdot\frac{\delta r}{v}$$

$$= \lambda\frac{\delta r}{r}$$

$$\frac{\delta\lambda}{\lambda} = \frac{\delta r}{r}$$ $\left[\because V = \frac{4}{3}\pi r^3\right]$

(4)

Change In volume

$$\delta V = \frac{4}{3}\pi \cdot 3r^2 \cdot \delta r$$

$$\frac{\delta V}{V} = \frac{3\delta r}{r}$$

$$\frac{\delta r}{r} = \frac{1}{3}\frac{\delta V}{V}$$

$$\frac{\frac{4}{3}\pi\, 3r^2\, \delta r}{\frac{4}{3}\pi r^3} = \frac{3\delta r}{r}$$

Substituting this value of $\frac{\delta r}{r}$ then we get

$$\frac{\delta \lambda}{\lambda} = \frac{1}{3}\frac{\delta V}{V}$$

Integrating $\log \lambda = \frac{1}{3}\log V + \log K$

$$\lambda = K V^{1/3}$$

$$\Rightarrow$$

$$V^{1/3} = \frac{\lambda}{K}$$

from the first equation

$$V^{1/3} \cdot T = \text{Const}$$

$$\frac{\lambda}{K} = \frac{1}{T}$$

$$\lambda T = KC \text{ Contant}$$

Wien's displacement law $\boxed{\lambda T = \text{Constant}}$

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25/03/2020

Rayleigh - Jean's law of Distribution of Energy (of Normal spectrum)

Based on the principle of equipartition of Energy for all the possible modes of free vibration which might be assigned to radiation. Thus they considered average energy of an oscillator

$$\bar{E} = KT$$

we already have e the no. of modes of vibration per unit volume in the freq range ν and $\nu + d\nu$ is given by

$$N_\nu d\nu = \frac{8\pi\nu^2}{c^3} d\nu$$

Energy density ~~of~~ within freq^n range ν and $\nu + d\nu$, given by

$$E_\nu d\nu = \frac{8\pi\nu^2}{c^3} d\nu \times KT$$

Rayleigh - Jean's law In terms of freq

In terms of wavelength $E_\lambda d\lambda = \frac{8\pi}{c^3}\left(\frac{c}{\lambda}\right)^2 \left[\frac{c}{\lambda^2}d\lambda\right] \cdot KT$

$$\boxed{E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} \cdot d\lambda}$$

for long wavelength

While Wien's law for short wavelength.

Plank's law

This is based upon the plank's hypothesis

(1) A radiation chamber is filled with black radiation and the molecules which can vibrate with all possible freq^n and these mol^n are called resonators and can exchange energy with radiation.

[because chamber formed by the molecules]

2. Resonator or vibrator can not absorb or radiate energy continuously; but an oscillator of freq^n ν can radiate or absorb energy. so in other words exchange of energy b/w radiation and matter can not take place continuously but are limited to discrete set of value 0, $h\nu$, $2h\nu$, $3h\nu$ ... $nh\nu$ in multiples of some small unit called the quantum

If total no. of resonator N, and E is

is total Energy then $\bar{E} = \frac{E}{N}$ (average Energy)

So acc. to Maxwell's law, if ϵ is the energy, the no. of mole having ~~energy~~ energies. $0, \epsilon, 2\epsilon \ldots r\epsilon \ldots$ are in the ratio

$1 : e^{-\epsilon/KT} : e^{-2\epsilon/KT} : \ldots \ldots e^{-r\epsilon/KT} \ldots$ etc

If N_0 is the no. of resonator having energy zero and N_1 is the having energy energy ϵ. will be

$$N_1 = N_0 e^{-\epsilon/KT}$$

$$N_2 = N_0 e^{-2\epsilon/KT}$$

So $N = N_0 + N_1 + N_2 + \ldots N_r + \ldots$

$$= N_0(1 + e^{-\epsilon/KT} + e^{-2\epsilon/KT} + \ldots + e^{-r\epsilon/KT} + \ldots]$$

Let $y = e^{-\epsilon/KT}$ => say.

$$N = N_0(1 + y + y^2 + \ldots y^r + \ldots)$$

$$\text{or } N = \frac{N_0}{1-y},$$

Total energy of Plank resonator

$$E = 0 \times N_0 + \epsilon \times N_1 + 2\epsilon \times N_2 + \ldots r\epsilon \times N_r$$

$$= \epsilon N_0 e^{-\epsilon/KT} + 2\epsilon N_0 e^{-2\epsilon/KT} + \ldots$$

$$r\epsilon N_0 e^{-r\epsilon/KT} + \ldots$$

$E = N_0 \epsilon S$

$S = y + 2y^2 + 3y^3 + \cdots r y^r + \cdots$

$Sy = y^2 + 2y^3 + \cdots (r-1) y^r + \cdots$

$S(1-y) = y + y^2 + y^3 + \cdots \cancel{(ry^r} - \cancel{y} + 1) y^r + \cdots + y^r + \cdots$

$$S = \frac{y}{1-y}$$

$$E = N_0 \epsilon \cdot \frac{y}{(1-y)^2}$$

$\therefore$ average Energy of Resonator

$$\bar{E} = \frac{E}{N} \quad \frac{N_0 \cdot \epsilon \, y/(1-y)^2}{\frac{N_0}{1-y}}$$

$$\bar{\epsilon} = \frac{\epsilon}{e^{\epsilon/kT} - 1}$$

Acc. to plank's hypothesis $\epsilon = h\nu$

$$\therefore \bar{\epsilon} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

we already have the no. of resonator b/w ν + and $\nu + d\nu$

$$N(\nu)\, d\nu = \frac{8\pi\nu^2}{c^3}$$

So energy density belonging to $d\nu$

= ave. energy of ~~oscill~~ resonator × No. of resonators/unit volume

$$E_\nu d\nu = \frac{8\pi h\nu^3}{c^3} \times \frac{1}{e^{h\nu/kT}-1} d\nu$$

or $$E_\lambda d\lambda = \frac{8\pi h}{c^3}\left(\frac{c^3}{\lambda^3}\right)\frac{1}{e^{hc/\lambda kT}-1}\left(-\frac{c}{\lambda^2}d\lambda\right)$$

$$\boxed{E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}}$$

Radiation law of Planck

Ex. Q) Show that planck's law reduces to Wein's law for shorter wavelength and Rayleigh Jean's law for larger wavelength.

Ex. Show that in the frequency range ν and $\nu+d\nu$ the no. of resonator per unit volume

is $$= \frac{8\pi\nu^2}{c^3} d\nu .$$