● **Liquid drop model:**

The liquid drop model of the nucleus was first proposed by Niels Bohr and F.Kalcar in the year 1937. They observed that there exists many similarities between the drop of a liquid and a nucleus. For instance,

(i) both the liquid drop and the nucleus possess constant density,

(ii) The constant binding energy per nucleon of a nucleus is similar to the latent heat of vaporization of a liquid,

(iii) The evaporation of a drop corresponds to the radioactive properties of the nucleus, and

(iv) The condensation of drops bears resemblance with the formation of compound nucleus, etc.

- According to this model, the nucleus is supposed to be spherical in shape in the stable state with radius $R = r_0 A^{1/3}$, just as a liquid drop is spherical due to symmetrical surface tension forces. The surface tension effects are analogous to the potential barrier effects on the surface of the nucleus.
- The density of a liquid drop is independent of the volume, as is the case with the nucleus. But whereas the nuclear density is independent of the type of nucleus, the density of a liquid does depend on its nature.
- Like the nucleons inside the nucleus, the molecules in the liquid drop interact only with their immediate neighbours.
- The non-independence of the binding energy per nucleon on the number of nucleons in the nucleus is analogous to the non-independence of the heat of vaporization of a liquid drop on the size of the drop.
- Molecules in a liquid drop evaporate from the liquid surface in raising the temperature of the liquid due to their increased energy of thermal agitation. Analogously, if high energy nuclear projectiles bombard the nucleus, a compound nucleus is formed in which the nucleons quickly share the incident energy and the emission of nucleons occurs.
- The phenomenon of nuclear fission is easily explained as the splitting of the liquid drop into two more or less equals parts if set into vibration with sufficient energy.

● **Semi-empirical binding energy or mass formula:**

C.V. Weizsacker, a German physicist, proposed the following semi-empirical formula for the nuclear binding energy B.E. (in MeV) for the nucleus (Z, A)

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

with the constants having the value, $a_v = 15.75, a_s = 17.80, a_c = 0.71, a_n = 22.7$ and $\delta = 33.6$ are in MeV.

(i) Volume energy:

The first term, $E_V = a_v A$, is the volume effect representing the volume energy of all nucleous. The more the total number of nucleons A, more difficult it becomes to remove an individual nucleon from the nucleus. Since the nuclear density is nearly constant, the nuclear mass is proportional to the nuclear volume, which again is proportional to R^3. But $R \propto A^{1/3} \Rightarrow R^3 \propto A$. So, the volume energy $E_v \propto A$. $\Rightarrow \quad E_v = a_v A$

This energy corresponds to the amount of heat energy (the heat of vaporisation) required to transform a liquid to its vapour state being proportional to the mass of the liquid.

(ii) Surface energy:

The second term, $E_s = a_s A^{2/3}$, is the surface effect being similar to the surface tension in liquids like the molecules on the surface of a liquid, the nucleons at the surface of the nucleus are not completely surrounded by other nucleons. It results in reducing the total binding energy due to nucleons on the surface. This correction due to surface energy E_s, which is proportional to the surface area of the nucleus i.e. to $4\pi R^2$ i.e. $E_s \propto R^2$

$$\Rightarrow \quad E_s \propto A^{2/3} \qquad \Rightarrow \quad E_s = a_s A^{2/3}$$

(iii) Coulomb energy:

The third term, E_c, is the Coulomb electrostatic repulsion between the charged particles in the nucleus. Since each charged particle repulses all the other charged particles, this term would be directly proportional to the possible number of combinations for a given proton number Z, which is Z(Z–1)/2. The energy of interaction between protons is again inversely proportional to the distance of separation R, so the energy associated with Coulomb repulsion is:

$$E_c = k\frac{Z(Z-1)}{R} = k\frac{Z(Z-1)}{r_0 A^{1/3}} = a_c\frac{Z(Z-1)}{A^{1/3}} \quad (\text{–ve quantity})$$

(iv) Asymmetry energy:

The fourth term E_a originates from the asymmetry between the number of protons and the number of neutrons in the nucleus. Nuclear data for stable nuclei indicate that for lighter nuclei, the number of protons is almost equal to that of neutrons: N = Z. As A increases, the symmetry of proton and neutron number is lost and the number of neutrons exceeds that of protons to maintain the nuclear stability. This excess of neutrons over protons, i.e. N–Z, is the measure of the asymmetry and it decreases the stability or the B.E. of medium or heavy nuclei.

So, $E_a \propto (N-Z)$, and $E_a \propto (N-Z)A$

$$\Rightarrow \quad E_a = a_a\frac{(N-Z)^2}{A} = a_a\frac{(A-2Z)^2}{A}$$

(v) Pairing energy:

The last term, a pure corrective term, is called pairing energy E_p.

$$E_p = \pm\frac{\delta}{A^{3/4}}$$

Z	N	A	δ	E_p
even	even	even	34	$+\delta/A^{3/4}$
even	odd	odd	0	0
odd	even	odd	0	0
odd	odd	even	35	$-\delta/A^{3/4}$

- Binding fraction, $f_B = \frac{B.E.}{A} = a_v - \frac{a_s}{A^{1/3}} - \frac{a_c Z(Z-1)}{A^{4/3}} - a_a\frac{(A-2Z)^2}{A^2} \pm \frac{\delta}{A^{7/4}}$
- Mass of nucleus,

$${}^{A}_{Z}M = Z M_p + (A-Z)M_n - B.E./c^2$$

$$= ZM_p + (A-Z)M_n - \frac{1}{c^2}\left[a_0 A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}\right]$$

The above formula is known as the semi-empirical mass formula.

Assuming, $F_A = A(M_n - a_v + a_n) + a_s A^{2/3}$

$$p = -4a_n - (M_n - M_p);\ q = \frac{1}{A}\left(a_c A^{2/3} + 4a_n\right)$$

$\Rightarrow$ $M(A,Z) = F_A + pZ + qZ^2$

$\Rightarrow$ $\left(\frac{\partial M}{\partial Z}\right)_A = p + 2qZ = 0$ at $Z = Z_A$, whence, we get

$$\Rightarrow \quad Z_A = -\frac{p}{2q} = \frac{(M_n - M_p + 4a_n)A}{2\left(a_c A^{2/3} + 4a_n\right)} \quad \Rightarrow \quad Z_A = A/\left(1.98 + 0.015A^{2/3}\right)$$

In most cases, the value of Z nearest to Z_A gives the actual stablest nucleus for a given A.