Q3 Evaluate $\int \sec^6 x \, dx$ using $\int \sec^n x \, dx$ reduction formula.

Sol Reduction formula for $\int \sec^n x \, dx \Rightarrow$

$$\frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2} \, dx$$

$\int \sec^6 x \, dx$ Here $n = 6$.

$$\int \sec^6 x \, dx = \frac{\sec^{6-2} x \tan x}{6-1} + \frac{6-2}{6-1} \int \sec^{6-2} x \, dx$$

$$= \frac{\sec^4 x \tan x}{5} + \frac{4}{5} \int \sec^4 x \, dx$$

$$= \frac{\sec^4 x \tan x}{5} + \frac{4}{5}\left[\frac{\sec^2 x \tan x}{3} + \frac{2}{3}\int \sec^2 x \, dx\right]$$

$$= \frac{\sec^4 x \tan x}{5} + \frac{4}{5}\left[\frac{\sec^2 x \tan x + 2 \tan x}{3}\right]$$

$$= \frac{\sec^4 x \tan x}{5} + \frac{4}{15} \sec^2 x \tan x + \frac{8}{15} \tan x.$$

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Ques 4 Find the surface area swept by semi circle $x = \cos t$, $y = \sin t$, $0 \le t \le \pi$ about x axis

Sol
$$S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$S = \int_0^{\pi} 2\pi \sin t \sqrt{(-\sin t)^2 + (\cos t)^2}\, dt$$

$$S = \int_0^{\pi} 2\pi \sin t \sqrt{\sin^2 t + \cos^2 t}\, dt$$

$$S = \int_0^{\pi} 2\pi \sin t\, dt$$

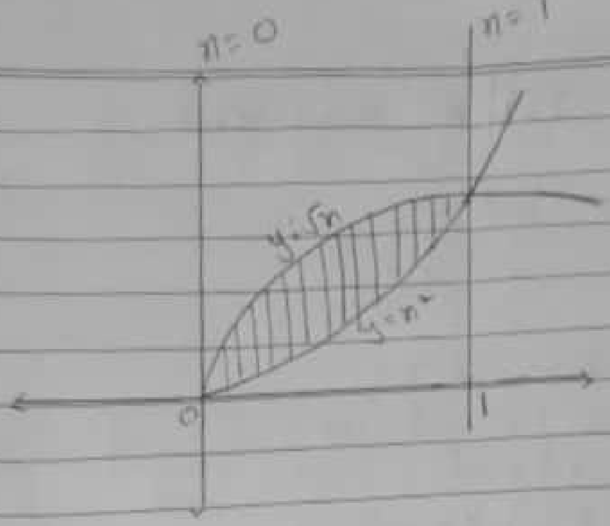
$$S = 2\pi \int_0^{\pi} \sin t\, dt$$

$$S = 2\pi \left[-\cos t\right]_0^{\pi}$$
$$S = -2\pi\left[\cos\pi - \cos 0\right]$$
$$S = -2\pi[-1-1]$$
$$S = 4\pi.$$

Ques 5 Sketch the graph of $r = 1 - 2\cos\theta$.

Sol The curve is symmetric about x axis. Thus we only need to graph over the interval $[0, \pi]$.

θ	0	$\pi/6$	$\pi/4$	$\frac{\pi}{3}$	$\pi/2$	$2\pi/3$	$3\pi/4$	[illegible]	π
$r = 1 - 2\cos\theta$	-1	$1-\sqrt{3}$ -0.73	$1-\sqrt{2}$ -0.414	0	1	2	2.414	[illegible]	3



$$V = \pi\int_0^1 (x^2)^2\,dx + \pi\int_0^1 (\sqrt{x})^2\,dx$$

$$V = \pi\int_0^1 x^4\,dx + \pi\int_0^1 x\,dx$$

$$V = \pi\left[\frac{x^5}{5}\right]_0^1 + \pi\left[\frac{x^2}{2}\right]_0^1$$

$$V = \frac{\pi}{5}[1-0] + \frac{\pi}{2}[1-0]$$

$$V = \frac{\pi}{5} + \frac{\pi}{2}$$

$$V = \frac{2\pi + 5\pi}{10}$$

$$V = \frac{7\pi}{10}$$

Ques Find the volume of solid that is form by the ~~the~~ bounded region by the graph, ~~f(x)~~

(2) $y = x^2+1$ $\quad y = -x+3$ about y axis

Solution

$y = x^2+1$

$y = -x+3$ $\qquad y = -0+3$

$0 = -x+3$ $\qquad y = 3$

$x = 3$ $\qquad (0,3)$

$(3,0)$

Ques 2 Reduction formula for $\int \sec^n x \, dx$.

$$\int \sec^n x \, dx = \int \underset{I}{\sec^{n-2} x} \, \underset{II}{\sec^2 x} \, dx$$

$$\int \sec^n x \, dx = \sec^{n-2} x \int \sec^2 x \, dx - \int \frac{d}{dx} \sec^{n-2} x \int \sec^2 x \, dx \, dx$$

$$\int \sec^n x \, dx = \sec^{n-2} x \tan x - \int (n-2) \sec^{n-3} x \sec x \tan x \tan x \, dx$$

$$I_n = \sec^{n-2} x \tan x - (n-2) \int \sec^{n-2} x \tan^2 x \, dx$$

$$I_n = \sec^{n-2} x \tan x - (n-2) \int \sec^{n-2} x (\sec^2 x - 1) \, dx$$

$$I_n = \sec^{n-2} x \tan x - (n-2) \left[\int \sec^n x \, dx - \int \sec^{n-2} x \, dx \right]$$

$$I_n = \sec^{n-2} x \tan x - (n-2) [I_n - I_{n-2}]$$

$$I_n = \sec^{n-2} x \tan x - (n-2) [I_n - I_{n-2}]$$

$$I_n = \sec^{n-2} x \tan x - (n-2) I_n + (n-2) I_{n-2}$$

$$I_n + (n-2) I_n = \sec^{n-2} x \tan x + (n-2) I_{n-2}$$

$$(n-2+1) I_n = \sec^{n-2} x \tan x + (n-2) I_{n-2}$$

$$I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{(n-2)}{n-1} I_{n-2} \, dx,$$

Ques 7 Show that volume of sphere is $\frac{4}{3}\pi r^3$ whose radius is r

Solution Eq. of circle $x^2 + y^2 = r^2$ (radius = r
centre = $(0,0)$)

$$y^2 = r^2 - x^2$$
$$y = \sqrt{r^2 - x^2}$$

$$V = \int_{-a}^{a} \pi \left(\sqrt{r^2 - x^2}\right)^2 dx$$

$$V = \pi \int_{-a}^{a} r^2 - x^2 \, dx$$

$$V = \pi \left[\int_{-a}^{a} r^2 dx - \int_{-a}^{a} x^2 dx\right]$$

$$V = \pi \left[r^2 [x]_{-a}^{a} - \left[\frac{x^3}{3}\right]_{-a}^{a}\right]$$

$$V = \pi \left[r^2 [a + a] - \frac{1}{3}[a^3 + a^3]\right]$$

$$V = \pi \left[2r^2 a - \frac{2}{3} a^3\right]$$

$$V = 2\pi a \left[r^2 - \frac{1}{3} a^2\right]$$

$$V = \frac{2\pi}{3} a \left[3r^2 - a^2\right]$$

Ques 9 Find the arc length of the parametric curve $x = e^t \times \sin t$ $y = e^t \times \cos t$.
and t lies b/w ~~0~~ to $\pi/2$ $[0, \pi/2]$

$x = e^t \sin t$ $\quad x' = e^t \cos t + \sin t\, e^t$

$y = e^t \cos t$ $\quad y' = e^t(-\sin t) + \cos t\, e^t$.

$0 \le t \le \pi/2$

$$L = \int_0^{\pi/2} \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

$$L = \int_0^{\pi/2} \sqrt{(e^t \cos t + \sin t\, e^t)^2 + (e^t(-\sin t) + \cos t\, e^t)^2}$$

$$L = \int_0^{\pi/2} \sqrt{(e^t \cos t)^2 + (\sin t\, e^t)^2 + \cancel{2(e^t)^2 \cos t \sin t} + (e^t \sin t)^2 + (e^t \cos t)^2 - \cancel{2(e^t)^2 \cos t \sin t}}$$

$$L = \int_0^{\pi/2} \sqrt{e^{2t}\{\cos^2 t + \sin^2 t\} + e^{2t}\{\sin^2 t + \cos^2 t\}}$$

$$L = \int_0^{\pi/2} \sqrt{2e^{2t}}$$

$$L = \sqrt{2} \int_0^{\pi/2} \sqrt{e^{2t}}$$

$$L = \sqrt{2} \int_0^{\pi/2} e^{\frac{1}{2} \times 2t}$$

$$L = \sqrt{2} \int_0^{\pi/2} e^t$$

$$\sqrt{2} \int_0^{\pi/2}$$

$$L = \sqrt{2}\left[e^{\pi/2} - e^0\right]$$

$$L = \sqrt{2}\left[e^{\pi/2} - 1\right)$$

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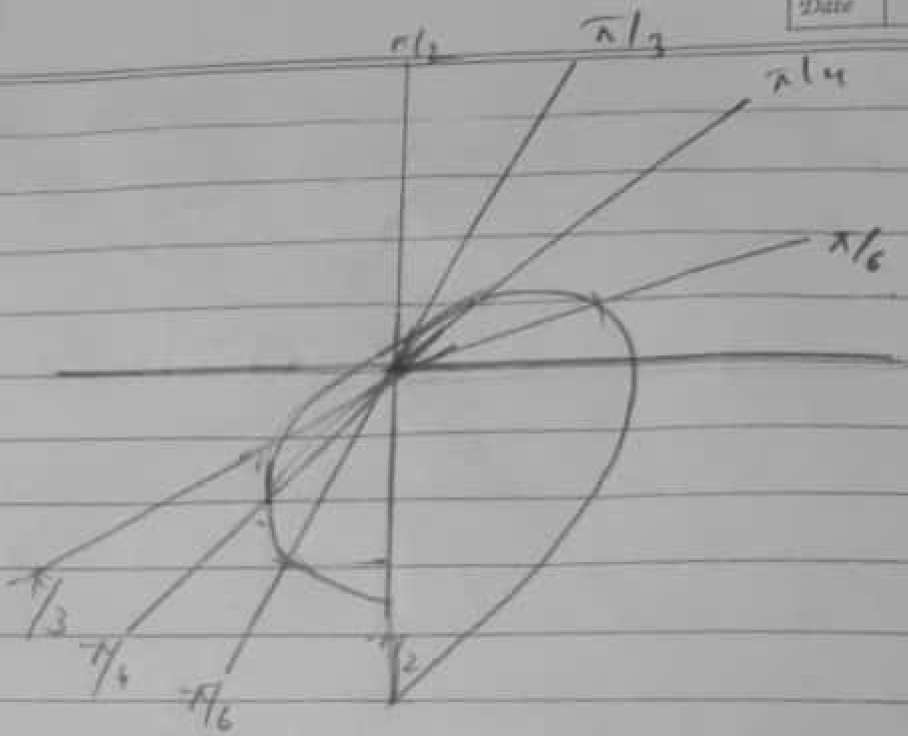
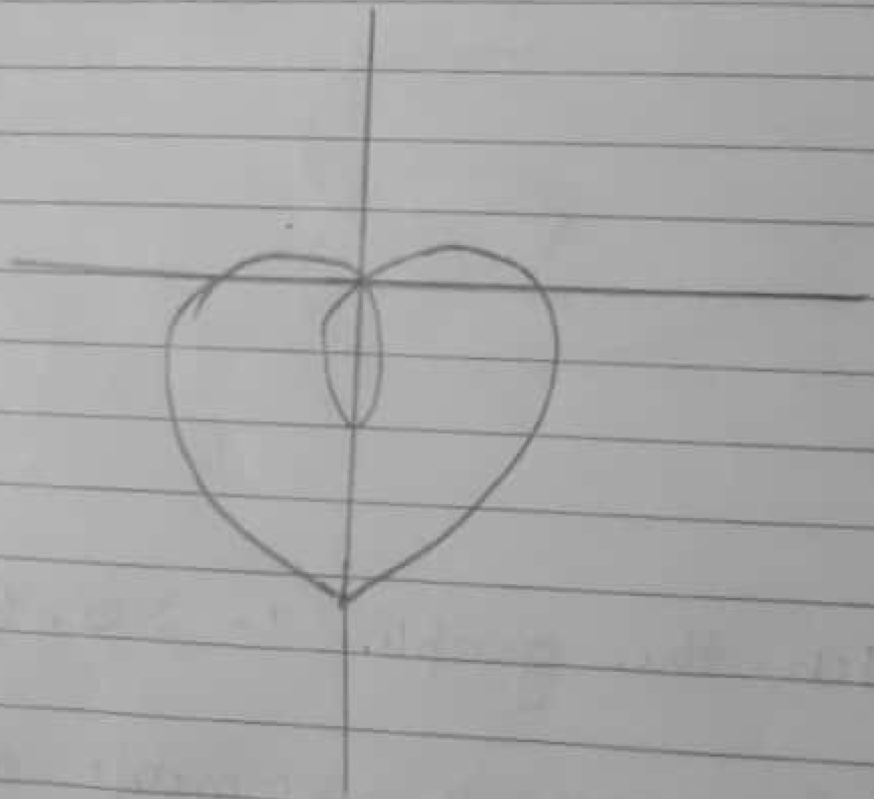

$\pi/2$
$\pi/3$
$\pi/4$
$\pi/6$
$-\pi/3$
$-\pi/4$
$-\pi/6$
$-\pi/2$



Ques 1 Find the arc length of the circle whose radius is 'a' and $x = a\cos t$, $y = a\sin t$ and t lies b/w 0 to 2π.

Sol Arc length of the parametric curve

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{d}{dt}(a\cos t)\right)^2 + \left(\frac{d}{dt} a\sin t\right)^2}\, dt$$

$$L = \int_0^{2\pi} \sqrt{a^2(-\sin t)^2 + a^2(\cos t)^2}\, dt$$

$$L = \int_0^{2\pi} \sqrt{a^2(\sin^2 t + \cos^2 t)}\, dt$$

$$L = \int_0^{2\pi} a\, dt$$

$$L = a\int_0^{2\pi} dt$$

$$L = a[t]_0^{2\pi}$$

$$L = 2\pi a.$$

Qu. Prove that $\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{m+n} + \frac{m}{m+n}\int \cos^{m-1} x \sin(n-1)x \, dx$

$$\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{n} - \int m\cos^{m-1}x(-\sin x)\left(-\frac{\cos nx}{n}\right)$$

$$\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{n} - \frac{m}{n}\int \cos^{m-1}x \sin x \cos nx \, dx$$

$$\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{n} - \frac{m}{n}\int \cos^{m-1}x\left[\sin nx \cos x - \sin(n-1)x\right] dx$$

$$\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{n} - \frac{m}{n}\int \cos^{m-1}x \sin nx - \cos^{m-1}x \sin(n-1)x \, dx$$

$$\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{n} - \frac{m}{n}\int \cos^m x \sin nx + \frac{m}{n}\int \cos^{m-1}x \sin(n-1)x \, dx$$

$$\cancel{\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{n} - \frac{m}{n}}$$

$$\int \cos^m x \sin nx \, dx + \frac{m}{n}\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{n} + \frac{m}{n}\int \cos^{m-1}x \sin(n-1)x \, dx$$

$$\left(1+\frac{m}{n}\right)\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{n} + \frac{m}{n}\int \cos^{m-1}x \sin(n-1)x \, dx$$

$$\int \cos^m x \sin nx \, dx = -\frac{\cos^m x \cos nx}{m+n} + \frac{m}{m+n}\int \cos^{m-1}x \sin(n-1)x \, dx \quad \text{--- (1)}$$

Let $I_{m,n} = \int \cos^m x \sin nx \, dx$

(A) Becomes

$$I_{m,n} = -\frac{\cos^m x \cos nx}{m+n} + \frac{m}{m+n} I_{m-1,\, n-1}$$

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Ques 8 Find the volume of surface generated by the region enclosed by the curve

$$y = \sqrt{x} \; , \; y = 6 - x \text{ and } y = 0$$

about x axis.

$y = \sqrt{x}$.

$y = 6 - x$.

$0 = 6 - x$

$x = 6$.

$y = 6 - 0$

$y = 6$

$(6, 0)$ and $(0, 6)$

Pt. of intersection.

$y = 6 - y^2$

$y^2 + y - 6 = 0$

$y^2 + 3y - 2y - 6 = 0$

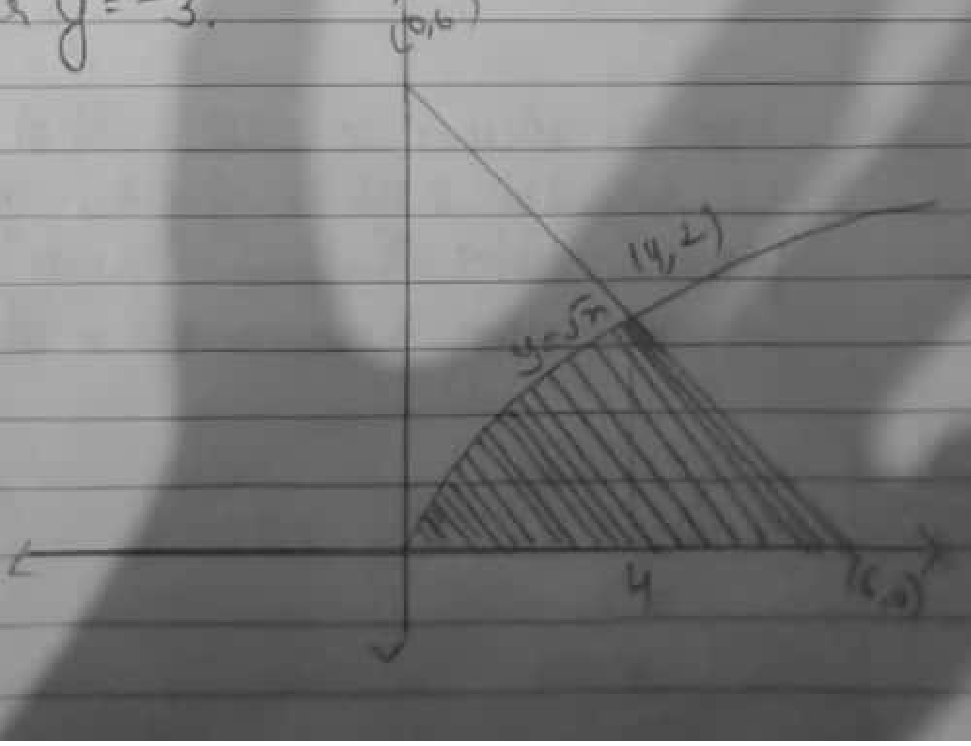
$y(y+3) - 2(y+3) = 0$

$(y-2)(y+3) = 0$

$y = 2$ or $y = -3$.

If $y = 2$,

$x = 4$.



$$V = \pi \int_0^4 [\sqrt{x}]^2 dx + \pi \int_4^6 (6-x)^2 dx.$$

$$V = \pi \int_0^4 x\,dx + \pi \int_4^6 (6-x)^2 dx$$

$$V = \pi \left[\frac{x^2}{2}\right]_0^4 + \pi \int_4^6 36 + x^2 - 12x\, dx$$

$$V = \frac{\pi}{2}[16] + \pi \left[\left[36x\right]_4^6 + \left[\frac{x^3}{3}\right]_4^6 - 12\left[\frac{x^2}{2}\right]_4^6 \right]$$

$$V = 8\pi + \pi\left[36\times 6 - 36\times 4 + \frac{1}{3}[6^3 - 4^3] - 6[6^2 - 4^2]\right]$$

$$V = 8\pi + \pi\left[72 + \frac{1}{3}[216 - 64] - 6[36 - 16]\right]$$

$$V = 8\pi + \pi\left[\cancel{\frac{152}{3}}\ 72 + \frac{152}{3} - 6\times 20\right]$$

$$V = 8\pi + \pi\left[\frac{152}{3} - 48\right]$$

$$V = 8\pi + \pi\left[\frac{152 - 144}{3}\right]$$

$$V = 8\pi + \frac{8\pi}{3}$$

$$V = \frac{32\pi}{3}$$

Ques9 Find the volume of solid that is formed by the bounded region by the graph $f(x) = \sqrt{x}$ $g(x) = x^2$ b/w the line $x = 0$ and $x = 1$ revolving about the x axis.

Sol

$f(x) = \sqrt{x}$

$g(x) = x^2$

$x = 0$

$x = 1$

Reduction formula

Que (1) $I_n = \int \tan^n x \, dx$.

$I_n = \int \tan^n x \, dx \; \frac{\tan^2 x}{\tan^2 x}$.

$I_n = \int \tan^n x \tan^2 x \tan^{-2} x \, dx$.

$I_n = \int \tan^{n-2} x \tan^2 x \, dx$.

$I_n = \int \tan^{n-2} x (\sec^2 x - 1) dx$.

$I_n = \int \tan^{n-2} x \sec^2 x \, dx - \int \tan^{n-2} x \, dx$.

let $\tan x = t$

$\sec^2 dx = dt$

$I_n \; \int t^{n-2} x \, dt - \int \tan^{n-2} x \, dx$

$I_n = \int t^{n-2} x \, dt - I_{n-2}$.

$I_n = \frac{t^{n-1}}{n-1} - I_{n-2}$

$I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$

$I_n + I_{n-2} = \frac{\tan^{n-1} x}{n-1}$

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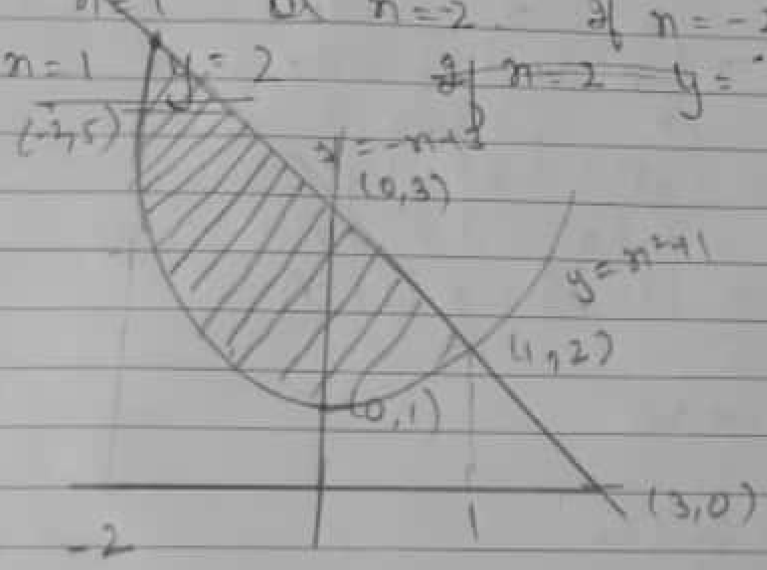
Pt of Intersection

$y = x^2+1$
$y = -x+3.$

$-x+3 = x^2+1$
$x^2+x-2=0$
$x^2+2x-x-2=0$
$x(x+2)-1(x+2)=0$
$(x-1)(x+2)=0$
$x=1$ or $x=-2$. If $x=-2$ $y=5$.
If $x=1$ $y=2$ ~~If $x=2$ $y=$~~



$$V=\pi\int_{-2}^{1}\left(-x+3-x^2-1\right)^2 dx.$$

$$V=\pi\int_{-2}^{1}\left(-x^2-x+2\right)^2 dx$$

$$V=\pi\int_{-2}^{1}\left(x^4+x^2+4+2x^3-4x-4x^2\right) dx$$

~~$V\int_{-2}^{1}$~~

$$V=\pi\left[\frac{x^5}{5}\right]_{-2}^{1}+\left[\frac{x^3}{3}\right]_{-2}^{1}+\left[4x\right]_{-2}^{1}+2\left[\frac{x^4}{4}\right]_{-2}^{1}-4\left[\frac{x^2}{2}\right]_{-2}^{1}-4\left[\frac{x^3}{3}\right.$$

$4\pi x$

$\pi\left[\frac{1}{5}(1+32)+\frac{1}{3}(1+8)+(1+2)+\frac{1}{2}(1-16)-2(\right.$

$\left.-\frac{4}{3}[1+8]\right]$

$\pi\left[\frac{33}{5}+\frac{9}{3}+3-\frac{15}{2}+6-\frac{4}{3}\times 9\right.$

$\pi\left\{\frac{33}{5}+3+3-\frac{15}{2}+6-12\right\}\pi$

$\pi\left[\frac{33}{5}+6-\frac{15}{2}-6.\right]$

$\pi\left[\frac{33}{5}-\frac{15}{2}\right]$

$\pi\left[\frac{66+60-75-60}{10}\right]$

$=\frac{117\pi}{5}.$