Roll No. ……..

Unique Paper Code : **32221201**

Name of Paper : **Electricity and Magnetism**

Name of Course : **B.Sc. (Hons) Physics**

Semester : **II**

Duration: **3 Hours** Maximum Marks: **75**

(Write your Roll No. on the top immediately on receipt of this question paper)

Question **No.1** is compulsory

Answer any **four** of the remaining six, attempting any **two** parts from each question.

1. Attempt **all** parts of this question.

(i) Suppose the electric field in some region is found to be, $\vec{E} = kr^3\hat{r}$ in spherical coordinates where, k is a constant with appropriate units.

(a) Find the charge density.

(b) Find the total charge contained in a sphere of radius R centered at the origin. 3

(ii) What is the physical significance of $\vec{\nabla} \cdot \vec{B} = 0$? 1

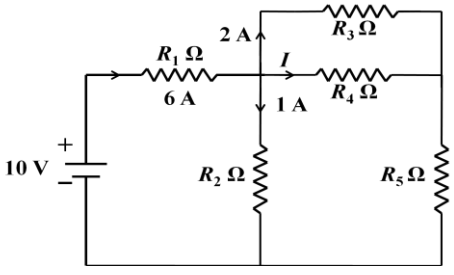
(iii) The plates of a parallel-plate capacitor are arranged so that the distance between them can be varied. When the distance between them is d, the capacitor is charged until the difference of potential is V. The plates are then separated until the distance between them is $2d$. Assuming that the charge Q on the plates is unchanged, determine the difference of potential between the plates of this capacitor. 3

(iv) A sphere of radius R has magnetization $\vec{M} = a\hat{k}\left(\hat{k} \cdot \vec{r}\right)^2$, where $\hat{k}$ is a unit vector along the *z*-direction, and a is a constant with appropriate units. Find,

(a) The bound magnetic charge density in the sphere.

(b) The bound magnetic surface charge density on the sphere. 3

(v) For the circuit given below, find *I* and calculate R_2 if $R_1 = 1\Omega$. 3



(vi) Identify which one of the following is an impossible electrostatic field. Give reason.

(a) $\vec{E} = k\left[xy\,\hat{\imath} + 2yz\,\hat{\jmath} + 3xz\,\hat{k}\right]$

(b) $\vec{E} = k\left[y^2\,\hat{\imath} + (2xy + z^2)\,\hat{\jmath} + 2yz\,\hat{k}\right]$

Here, k is a constant with appropriate units. 3

(vii) Find the resonance frequency, bandwidth and quality factor of a series *LCR* circuit having, $C = 0.1\mu F, L = 0.16\ H$ and $R = 10\Omega$. 3

2. **(a)** Find the electric field at a distance s from an infinitely long straight wire which carries a uniform line charge λ. 7

(b) Use Gauss's law to determine the electric field inside and outside a spherical shell of radius R, which carries uniform surface charge density σ. 4, 3

(c) An infinite plane slab of thickness $2d$ lies in the $x - z$ plane. It carries a uniform volume charge density ρ. Find the electric field as a function of y, where $y = 0$ at the centre. Plot $\vec{E}$ as function of y, taking $\vec{E}$ as positive along $+y$ direction. 5, 2

3. **(a)** Using the method of images, discuss the problem of a point charge q inside a hollow, grounded, conducting sphere of inner radius a. Find

(i) the potential inside the sphere
(ii) the induced surface-charge density
(iii) the magnitude and direction of the force acting on q. 3, 2, 2

(b) State and prove the second Uniqueness theorem. 2, 5

(c) Suppose the electric potential is given by the following expression.

$$V(r,\theta,\phi) = \frac{V_o}{2}\left(3 - \frac{r^2}{R^2}\right);\ r < R$$

$$= \frac{V_o R}{r} \qquad ;\ r > R$$

for all r where V_o and R are constants. Find the corresponding electric field distribution and to what charge distribution would this correspond? 4, 3

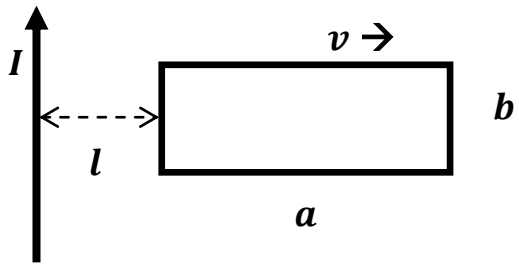
4. **(a)** Consider two infinite intersecting, conducting planes $x = 0$ and $y = 0$. A charge of $100\ nC$ is placed at (3,4,0). Find the electric potential and electric field at (3,5,0). 4,3

(b) Show that the potential energy of a system of charges can be calculated from electric field by assigning an amount of energy $\frac{1}{2}\varepsilon_0 E^2 d\tau$ to every volume element $d\tau$ and integrating over the entire space. 7

(c) Four point charges $q, 2q, 3q$ and $4q$ are placed at the corners of a square with side $\sqrt{2}\ m$. Calculate the electric potential and the electric field at centre of the square. 4, 3

5. **(a)** For a dielectric medium, derive a relation between electric displacement $\vec{D}$, electric field $\vec{E}$ and polarization density $\vec{P}$. 3, 2, 2

(b) A current I flows through a long straight filamentary wire. A conducting rectangular loop of side, a and b, is placed at a distance l from the wire. The loop is moving with a velocity v in the direction as shown in diagram. Calculate the magnitude of e.m.f induced in the loop after a time t. 7



(c) A parallel-plate capacitor is filled with a non-uniform dielectric, such that the relative permittivity is given by,

$$\varepsilon_r = \varepsilon_0 + \alpha x$$

Here, α is a constant having units of (1/length) and x is distance from one plate. Assume that area of the plates is A and separation between them is d. What is the capacitance of the capacitor? 7

6. **(a)** Obtain the differential and integral form of Ampere's Circuital Law for an arbitrary current density $\vec{J}$. 4, 3

(b) A deutron is travelling in a straight line along the x-direction with a speed $2 \times 10^7 \text{m/s}$. It enters a region of uniform magnetic field of strength 0.2 T along the z-direction. How much will be its displacement in the y-direction from the original position after it has travelled a distance of 1.0 cm in the x-direction? After how much time will the particle's velocity be instantaneously in the y-direction? 4, 3

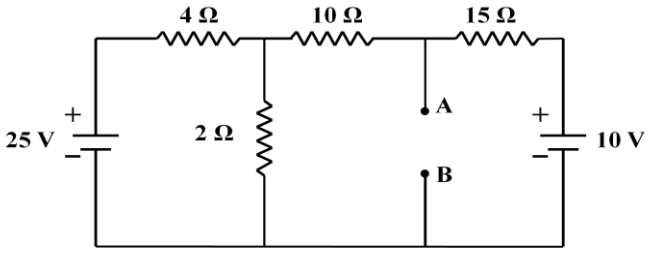
(c) A very long air core solenoid of radius b has n turns per unit length and carries a current $i = i_0 \sin \omega t$.

(i) Write an expression for the magnetic field inside the solenoid as a function of time.
(ii) Assuming the magnetic field to be zero outside, write an expression for the electric field inside and outside the solenoid as a function of time.

3, 2, 2

7. **(a)** Derive an expression for the resonance frequency of a series *LCR* circuit. How can we increase the quality factor of a series *LCR* circuit without changing its resonance frequency? Explain. 4, 2, 1

(b) State and prove the Thevenin theorem. Obtain the Thevenin equivalent for the circuit given below. 1, 3, 3



(c) A series *LCR* circuit has $R = 4\Omega, L = 0.5H$ and $V = 100 \cos \pi t$. Find the value of *C* for resonance. Also determine the voltage across *C*. Plot the variation of quality factor with resistance for a series *LCR* circuit. 3, 2, 2