Course: (CBCS) B.Sc. (H)-Physics [Section-A]

(32221402) Elements of Modern Physics

Part & Semester – II & IV

Lecture-2

Dear Students

Hope all of you are well and taking all the necessary precautions in this difficult time.

In our last class, we have done Basic Nuclear Properties, Binding Energy and Liquid Drop Model: Semi-empirical Mass Formula. In continuation of my last lecture, I am going to discuss some of numerical problems on these topics and suggest you to do some related unsolved problems from your suggested text books. In our next lecture, we will try to discuss about Nature of Nuclear Force, NZ Graph and related problems.

Apart from this, all of you can contact me through Email, Whatsapp or Mobile for any quarry related to our course of Elements of Modern Physics.

Thanking you.

With Best Wishes

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Example-1: Calculate the binding energy in MeV of ^4He from the following data: Mass of $^4He = 4.003875$ a.m.u; Mass of $^1H = 1.008145$ a.m.u and mass of a neutron = 1.008986 a.m.u.

Soln. ^4He nucleus contains 2 protons and 2 neutrons. So, mass of the constituents

$= 2(1.008145 + 1.008986)$a.m.u. = 4.034262 a.m.u. But mass of ^4He nucleus = 4.003875 a.m.u.

Mass difference (loss) $= (4.034262 - 4.003875) = 0.030387$ a.m.u.

Binding energy, $E_B = 0.030387 \times 931$ MeV = 28.29 MeV

Example-2: The mass of the hydrogen atom and of neutron are 1.008142 and 1.008982 a.m.u. respectively. Calculate the packing fraction and the binding energy per nucleon of ^{16}O nucleus.

Soln. ^{16}O nucleus consists of 8 protons and 8 neutrons.

Mass of constituents = $8(1.008142 + 1.008882)$a.m.u. = 16.136992 a.m.u.

Mass of ^{16}O nucleus = 15.994915 a.m.u.

Mass difference (loss) = 0.142077 a.m.u.

Binding energy, $E_B = (0.142077 \times 931)$ a.m.u. = 132.27 MeV

Mass defect, $\Delta M = M(A, Z) - A = (15.994915 - 16)$a.m.u. = 0.005085 a.m.u.

Packing fraction = $\Delta M / A = 0.005085 / 16 = 3.178 \times 10^{-4}$

Example-3: The radius of a $^{64}_{29}Cu$ nucleus is measured to be 4.8×10^{-13}cm. Find the radius of a $^{27}_{12}\mu g$ nucleus.

Soln. We know that, $R = R_0 A^{1/3}$

Now, $R_1 = R_0 A_1^{1/3}$, $4.8 \times 10^{-13} = R_0(64)^{1/3}$

So, $R_0 = \dfrac{4.8 \times 10^{-13}}{4} = 1.2 \times 10^{-13}$

Again, $R_2 = R_0 A_2^{1/3} = 1.2 \times 10^{-13}$ Y $(27)^{1/3}$

$= 3.6 \times 10^{-13}$ cm

Example-4: Since $^{27}_{14}Si$ and $^{27}_{13}Al$ are mirror nuclei, their ground states are identical except for charge. If there mass difference is 6 MeV, estimate their radius (neglect the proton-neutron mass difference)

Soln. The mass-difference between mirror nuclei can be attributed to the difference in electrostatic energy. Now, the electrostatic energy of a charge Q distributed uniformly throughout a sphere of radius R is $W = 3Q^2 / 5R$

$$\text{So,} \quad \Delta W = \frac{3e^2}{5R}\left(Z_1^2 - Z_2^2\right) \Rightarrow R = \frac{3e^2}{5\Delta W}\left(14^2 - 13^2\right) = \frac{3hc}{5\Delta W}\left(\frac{e^2}{hc}\right) \times 27$$

where e^2 / hc = the fine structure constant = 1/137

$$\Rightarrow \quad R = \frac{3 \times 1.97 \times 10^{-11}}{5 \times 6} \times \frac{1}{137} \times 27 = 3.88 \times 10^{-11}\text{cm} = 388 \text{ fm}$$

Example-5: An α – particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. Calculate the distance of closest approach.

Soln. The distance of closest approach is:

$$d = \frac{Ze^2}{\pi\varepsilon_0 Mv^2} = \frac{92\times\left(1.6\times10^{-19}\right)^2}{3.14\times8.85\times10^{-12}\times2\times5\times1.6\times10^{-13}} m = 5.3\times10^{-14}\ m$$

Example-6: Calculate the binding energies of the following Isobars and their B.E/nucleon: Given $_{28}Ni^{64}$ = 63.9280 a.m.u, $_{29}Cu^{64}$ = 63.9298 a.m.u., M_N = 1.008665 a.m.u. and M_H = 1.007825 a.m.u.

Soln. Hence for $_{28}Ni^{64}$: $NM_N + ZM_H$ = [36 × 1.008665 + 28 × 1.007825] a.m.u.= 64.531 a.m.u

ΔM = 64.531 – 63.98 = 0.603 a.m.u.

Hence, B.E = 561.4 MeV and B.E / nucleon = 8.77 MeV.

For $_{29}$ Cu^{64}: $NM_N + ZM_H = [35\times1.008665 + 29\times1.007825]$amu.

ΔM = 64.5272 – 63.9298 = 0.5974 a.m.u $\Rightarrow$ B.E. = 556.18 MeV and B.E/nucleon = 8.7 MeV

Example-7: Find the energy release if two, $_1H^2$ nuclei can fuse together to form $_2He^4$ nucleus. The B.E/nucleon of $_1H^2$ and $_2He^4$ is 1.1 MeV and 7.0 MeV respectively.

Soln. Number of nucleon in $_2He^4$ is 4, hence B.E for $_2He^4$ = 28.0 MeV
Similarly B.E for $_1H^2$ = 2.2 MeV

Energy equivalent of mass of $_2He^4$ nucleus = $\left[\left\{2M_p + 2M_n\right\}c^2 - 28.0\right]MeV = E\left({}_2He^4\right)$

Energy equivalent of mass of $_1H^2$ nucleus = $\left[\left\{M_p + M_n\right\}c^2 - 2.2\right]MeV = E\left({}_1H^2\right)$

$\Delta E = 2E\left({}_1H^2\right) - E\left({}_2He^4\right) = 23.6\ MeV$

Example-8: Find the binding energy of a nucleus consisting of equal no's of protons and neutrons and having the radius one and a half times smaller than that of Al^{27} nucleus.

Soln. Mass number of given nucleus $\frac{27}{(3/2)^3} = 8$; Nucleus is Be^8

$B.E. = \left\{(4\times1.00867) + (4\times1.00783) - 8\right\}\times931.5\ MeV = 61.48\ MeV\ (approx.)$

Example-9: The atomic mass of $_6C^{12}$ is 12.00 amu and that of $_6C^{13}$ 13.00354 amu, find the energy required to remove a neutron from $_6C^{13}$ in MeV. The mass of neutron is 1.008665 a.m.u.

Soln. The nuclear equation is $_6C^{13} \rightarrow {}_6C^{12} + {}_0n^1$. The mass of $_6C^{12} + {}_0n^1$ is 12.00 + 1.008665 = 13.008665 amu

Mass defect = 13.008665 – 13.003354 = 0.005311 amu

Its energy equivalent = ΔE = 0.005311 × 931.5 = 4.95 MeV

Example-10: Calculate the binding energy of the last neutron in $^{15}N^7$ and of the last proton in $^{15}O^8$, and contrast with the last neutron in $^{16}N^7$ and in $^{16}O^8$.

Soln. From the CRC handbook, we know that

$M(^1H^1) = 1.0078amu,\ m_n = 1.0087amu,$

$M(^{14}N^7) = 14.0031amu,\ M(^{15}N^7) = 15.0001amu,$

$M(^{16}N^7) = 16.0061amu,\ M(^{15}O^8) = 15.0030amu,$

$M(^{16}O^8) = 15.9949$amu.

Using these values and the conversion between "amu" and "MeV" units, we can calculate the binding energy of the last neutron in $^{15}N^7$.

$$\text{B.E.} = -(M(^{14}N^7) + m_n - M(^{15}N^7))c^2$$

$$= -(14.0031 + 1.0087 - 15.0001)\text{amu} \times c^2$$

$$\approx -0.0117 \times 931.5\text{MeV}/c^2 \times c^2 = -10.8985\text{MeV}$$

Similarly, the binding energy of the last proton in $^{15}O^8$ is

$$\text{B.E.} = -(M(^{14}N^7) + M(^1H^1) - M(^{15}O^8))c^2$$

$$= -(14.0031 + 1.0078 - 15.0030)\text{amu} \times c^2$$

$$\approx -0.0079 \times 931.5\text{MeV}/c^2 \times c^2 = -7.3588\text{MeV}$$

Furthermore, the binding energy for the last neutron in $^{16}N^7$ is given by

$$\text{B.E.} = -(M(^{15}N^7) + m_n - M(^{16}N^7))c^2$$

$$= -(15.0001 + 1.0087 - 16.0061)\text{amu} \times c^2$$

$$\approx -0.0027 \times 931.5\text{MeV}/c^2 \times c^2 \approx -2.5150\text{MeV}$$

Finally, the binding energy of the last neutron in $^{16}O^8$ is

$$\text{B.E.} = -(M(^{15}O^8) + m_n - M(^{16}O^8))c^2$$

$$= -(15.0030 + 1.0087 - 15.9949)\text{amu} \times c^2$$

$$\approx -0.0168 \times 931.5\text{MeV}/c^2 \times c^2 = -15.6492\text{MeV}$$

Example-11: Chlorine-33 decays by positron emission with a maximum energy of 4.3 MeV. Calculate the radius of the nucleus from this.

Soln. The decay scheme is ${}_{17}Cl^{33} \rightarrow {}_{16}S^{33} + {}_{+1}e^0 + \nu + E_\beta$

When the positron emits with a maximum energy, the neutrino energy will be zero and the daughter nucleus S^{33} will be formed in the ground state.

$$\therefore \quad E_\beta = \frac{3}{5}\frac{e^2 A^{2/3}}{4\pi \in_0 R_0} - 1.80MeV$$

or $\dfrac{3}{5}\dfrac{e^2 A^{2/3}}{4\pi \in_0 R_0} = 6.1 \times 1.6 \times 10^{-13}$ Joule or $R_0 = \dfrac{3}{5}\dfrac{e^2 A^{2/3}}{4\pi \in_0 \times 6.1 \times 1.6 \times 10^{-13}}$

$$= \frac{3}{5}\frac{(1.6 \times 10^{-19})^2 (33)^{2/3} \times 9 \times 10^9}{6.1 \times 1.6 \times 10^{-13}} = 1.41 \times 10^{-15}\text{meter}$$

$$\therefore R = R_0 A^{1/3} = 1.41 \times 10^{-15} \times 33^{1/3} = 4.54 \times 10^{-15}\text{m}$$

Example-12: Compute the binding energy of the last proton in a nucleus of ^{12}C if the mass of ^{12}C-nucleus is 12.00052 was a.m.u. and the mass of the ^{11}B-nucleus is 11.01006 a.m.u. The mass of proton is 1.00759 a.m.u

Soln. On the addition of a proton, the ^{11}B-nucleus is converted into ^{12}C nucleus. The excess of mass of ^{12}C over ^{11}B is:

$$12.00052 - 11.01006 = 0.99046 \text{ a.m.u.}$$

The mass of proton is 1.00759 a.m.u. Thus the proton, when added to the nucleus, suffers mass loss.

$$\text{Mass loss, } \Delta m = 1.00759 - 0.99046 = 0.01713 \text{ a.m.u.}$$

$\therefore$ Equivalent energy, $\Delta E = 0.01713 \times 931 = 15.95$ MeV

$\therefore$ Binding energy of the last proton = 15.95 MeV

Example-13: Establish the relation $A = 2Z$ for light nuclei using the semi-empirical mass formula, given $a_c = 0.71$ MeV, $a_n = 22.7$ MeV, $M\left({}_1^1H\right) = 1.0078$, $M(n) = 1.0086$ unit.

Soln. The mass M of a nucleus of mass number A and charge number Z according to the semi-empirical formula, is given by

$$M = Z\,M_H + (A-Z)M_n - \frac{1}{c^2}\left(a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - \frac{a_n (A-2Z)^2}{A} \pm E_\delta\right)$$

For odd A nuclei, we have $E_\delta = 0$

The mass of the most stable nucleus in a family of isobars is given by the condition $(\partial M / \partial Z)_A = 0$.

$$\Rightarrow \quad (\partial M/\partial Z)_A = (M_H - M_n)c^2 + 2a_c \frac{Z}{A^{1/3}} - 4a_n \frac{(A-2Z)}{A} = 0$$

$$\Rightarrow \quad 2a_c \frac{Z}{A^{1/3}} - 4a_n \frac{(A-2Z)}{A} = (M_n - M_H)c^2$$

$$\Rightarrow \quad 2Z\left(\frac{a_c}{A^{1/3}} + \frac{4a_n}{A}\right) = (M_n - M_H)c^2 + 4a_n$$

$$\Rightarrow \quad \frac{2Z}{A}\left(a_c A^{2/3} + 4a_n\right) = (M_n - M_H)c^2 + 4a_n$$

$$\therefore \quad \frac{2Z}{A}\left\{\frac{a_c A^{2/3}}{4a_n} + 1\right\} = \left\{\frac{(M_n - M_H)c^2}{4a_n} + 1\right\}$$

$$\text{Or,} \quad Z = \frac{A}{2}\left\{\frac{1 + (M_n - M_H)c^2 / 4a_n}{1 + \frac{a_c}{4a_n} A^{2/3}}\right\}$$

Now, $a_c = 0.71$ MeV, $a_n = 22.7$ MeV.

$\therefore$ $a_c / 4a_n = 0.0078$ and $(M_n - M_H)c^2 / 4a_n = 0.0082$

$$\therefore \quad Z = \frac{A}{2} / \left\{ \frac{1 + 0.0082}{1 + 0.0078 A^{2/3}} \right\} = \frac{A}{2}, \text{ for light nuclei.}$$

$\therefore$ $A = 2Z$, for light nuclei.

Example-14: : Using the semi-empirical binding energy formula, calculate the binding energy of ${}^{40}_{20}Ca$.

Soln. The semi-empirical binding energy formula is:

$$B.E. = a_v A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} + \delta A^{-3/4}$$

where $a_v = 15.75$ MeV, $a_s = 17.80$ MeV, $a_c = 0.71$ MeV;

$a_n = 22.7$ MeV and $\delta = 34$ as $A = \text{even} = 40$, $Z = \text{even} = 20$

$$\therefore \quad a_v A = 15.75 \times 40 = 630 \text{ MeV}; \; a_s A^{2/3} = 17.80 \times 40^{2/3} = 17.80 \times 11.696 = 208.2 \text{ MeV}$$

$$a_c \frac{Z(Z-1)}{A^{1/3}} = \frac{0.71 \times 20 \times 19}{40^{1/3}} = \frac{0.71 \times 20 \times 19}{3.2} = 84.3 \text{ MeV}$$

$$a_n \frac{(A-2Z)^2}{A} = a_n \times 0 = 0$$

$$\delta A^{-3/4} = 34 \times 40^{-3/4} = 34 \times 0.063 = 2.14 \text{ MeV}$$

$$\therefore \quad B.E. = 630 - [208.2 + 84.3 - 2.14] = 339.64 \text{ MeV}$$

Example-15: Using the semi-empirical binding energy formula, find the atomic number of the most stable nucle for a given mass number A. Hence explain which is the most stable among ${}^{5}_{2}He$, ${}^{6}_{2}Be$ and ${}^{6}_{3}Li$

Soln. Writing E_b for binding energy, $E_b = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$

where, $Z(Z-1) = Z^2$ has been taken.

Now, for most stable nucleus, E_b must be maximum for a given mass number A, i.e.,

$$\left(\frac{\partial E_b}{\partial Z}\right)_{A=\text{const}} = -2a_c A^{-1/3} Z + 4a_n (A-2Z) A^{-1} = 0$$

$$\Rightarrow \quad 4a_n - 8a_n A^{-1} Z = 2a_c A^{-1/3} Z; \; Z\left(4a_n + a_c A^{2/3}\right) = 2a_n A$$

$$\therefore \quad Z = \frac{A}{2 + (a_c / 2a_n) A^{2/3}} = \frac{A}{2 + 0.015 A^{2/3}}$$

Substituting the values of a_c and a_n.

He, Be and Li are all light nuclei for which $0.015\ A^{2/3}$ is negligible and Z = A/2. This shows that of three nuclei, ${}^{6}_{3}Li$ is most stable.

Example-16: ow, by way of computation, which nuclei you would expect to be more stable:

$${}^{7}_{3}Li \text{ or } {}^{8}_{3}Li;\ {}^{9}_{4}Be \text{ or } {}^{10}_{4}Be$$

Soln. For a given mass number A, the atomic number Z of the most stable nucleus is:

$$Z = \frac{A}{2 + 0.015\ A^{2/3}}$$

Now, for $A = 7, Z = \frac{7}{2 + 0.015 \times 7^{2/3}} = \frac{7}{2 + 0.055} = \frac{7}{2.055} = 3.4$

for $A = 8, Z = \frac{8}{2 + 0.15 \times 8^{2/3}} = \frac{8}{2 + 0.060} = \frac{8}{2.060} = 3.88$

Since of the two Z-values, 3.4 is nearer to 3, the ${}^{7}_{3}Li$ nucleus is more stable.

Again, for $A = 9,\ Z = \frac{9}{2 + 0.015 \times 9^{2/3}} = \frac{9}{2 + 0.065} = \frac{9}{2.065} = 4.36$

for $A = 10,\ Z = \frac{10}{2 + 0.015 \times 10^{2/3}} = \frac{10}{2 + 0.067} = \frac{10}{2.067} = 4.80$

Since the two Z-values, 4.36 is nearer to 4, the ${}^{9}_{4}Be$ nucleus is more stable.

Example-17: The difference in the coulomb energy between the mirror nuclei ${}^{49}_{24}Cr$ and ${}^{49}_{25}Mn$ is 6MeV. Assuming that the nuclei have a spherically symmetric charge distribution and that e^2 is approximately 1.0 MeV-fm. Find the radius of the ${}^{49}_{25}Mn$ nucleus.

Soln. Kinetic coulomb energy of uniformly charged sphere of radius R is

$$E_C = \frac{3}{5}\frac{z^2 e^2}{R}$$

According to the question, $E_{Cr} - E_{Mn} = 6MeV$

$$\frac{3}{5}\frac{(Z_{Cr}^2 - Z_{Mn}^2)e^2}{R} = 6MeV \Rightarrow R = \frac{3(Z_{Cr}^2 - Z_{Mn}^2)e^2}{6 \times 5MeV}$$

$= 4.9$ fm $= 4.9 \times 10^{-15}$ fm

where $Z_{Cr} = 25$ and $Z_{Mn} = 24$

References:

1- Concepts of Modern Physics (Sixth Edition, TMH Pvt. Ltd.) by Arthur Beiser et.al.

2- Nuclear Physics (S. Chand Limited, 2008) by S. N. Ghoshal.

3- Nuclear Physics (Himalaya Publishing House, Mumbai) by D. C. Tayal.

4- Last year examination papers.

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