LECTURE -2

WEEK - 2 [23/03/2020 - 28/03/2020]

Course : B.Sc (Hons) Physics - Section (A)

Semester : Second (II)

Paper : Electricity and Magnetism

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Lecture : 2

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Types of Materials

- Conductors ($\sigma >> 1$) → One or more electrons/atom are free to move in the material (for metallic conductor).
- Semiconductors → The electrons are not completely free
- Insulators ($\sigma << 1$) → Each electron is tightly attached to the atom and is not free to move in the crystal. Hence no conduction take place.
  Ex. Glass/Rubber

→ In liquid conductors such as salt water it is ions that do not moving through the material.

→ A perfect conductor would be a material containing an unlimited supply of completely free charges.

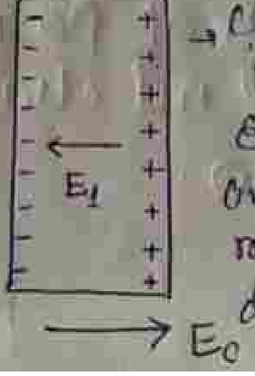
→ In real life there is no perfect conductors.

Properties of Conductors

A. $\vec{E} = 0$ inside the conductor. Why?

Because if there were any field, those free charges would move, and it would not be electrostatic any more.

When a conductor is placed in the electric field $\vec{E}_0$. What happen?
Because of this external field $\vec{E}_0$ the positive →



→ Charge will move in the direction of Electric field (right) and −ive charge move in opposite direction (to the left)

In practice, it's only the negative charges - electrons that do the moving but when they depart right side is left with a net positive charge - (stationary nuclei). When both charges (+ive and -ive) come to the edge of material, these induced charges produce an electric field of own ($\vec{E}_1$) directed from positive charge to negative charge ($\leftarrow$) which is opposite to the external electric field $\vec{E}_0$. The induced electric field $\vec{E}_1$ cancel to the external $\vec{E}_0$ field. Charge will continue until this cancellation is complete and the resultant field inside the conductor is precisely zero.

Note: Outside the conductor, the field is not zero. Here E_0 and E_1 do not cancel.

(B) $\rho = 0$ inside a conductor

We have Gauss' law

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$$

Since $\vec{E} = 0$, Hence $\boxed{\rho = 0}$

There is still charge around, but same number of positive and negative charge. So the net charge density in the interior is zero.

(C) Any net charge resides on the surface.

(D) A conductor is an equipotential.

If a and b are two point within (or at the surface of) a given conductor

$$V(a) - V(b) = -\int_a^b \vec{E} \cdot d\vec{l}$$

Since $\vec{E} = 0$ inside the conductor.

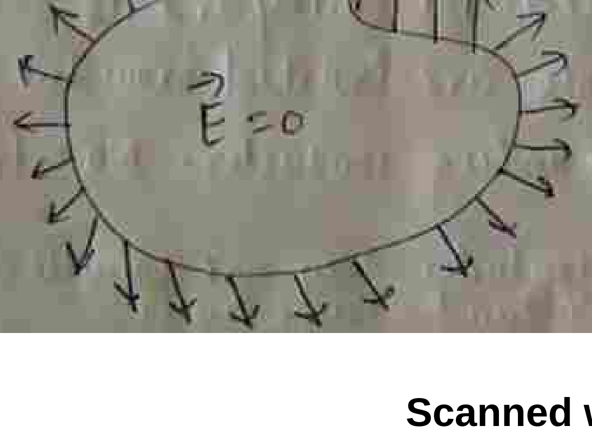
So $V(a) - V(b) = 0$

i.e. $\boxed{V(a) = V(b)}$

(E) $\vec{E}$ is perpendicular to the surface, just outside a conductor.

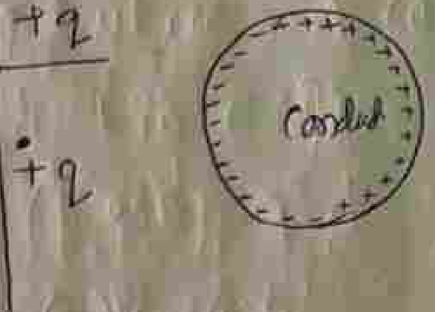
Charge will immidiately flow around the surface until it kills all the tangential component.

The charges can not flow perpendicular to the surface since it is confined to the conducting object.

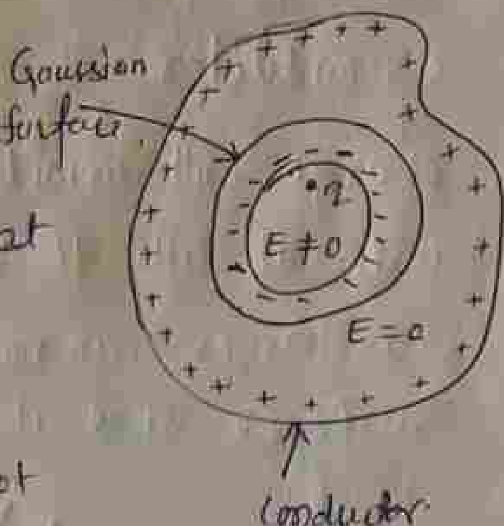


* Induced Charge (A: If cavity carries a charge q)

If you hold a positive charge +q near an uncharged conductor. The two attract one another. This +q charge will attract minus charge to the near side and repel plus charges to the far side. or you can say that it is the charge which moves around the conductor in such a way to cancel off the field of q for points inside the conductor where the total field must be zero. Since the negative charge is closer to q, there is a net force of attraction.



If there is some cavity inside the conductor and within that cavity there is some charge, then the field inside the cavity will not be zero. The cavity and its contents are isolated from the outside word by the surrounding conductor. No external field can penetrate the conductor, they are cancelled at the outer surface by the induced charge there.



Similarly the field due to the charges within the cavity is killed off, for all exterior points, by the induced charge on the inner surface. However the compensating charge left over on the outer surface of the conductor effectively "communicates" the presence of q to the outside world. Incidently, the total charge induced on the cavity wall is equal and opposite to the charge inside.

If we surround the cavity with a Gaussian surface all points of which are in the conductor.

$$\oint \vec{E} \cdot d\vec{s} = 0 = \frac{Q_{enc}}{\epsilon_0}$$

i.e $Q_{enc} = 0$ Net enclosed charge must be zero.

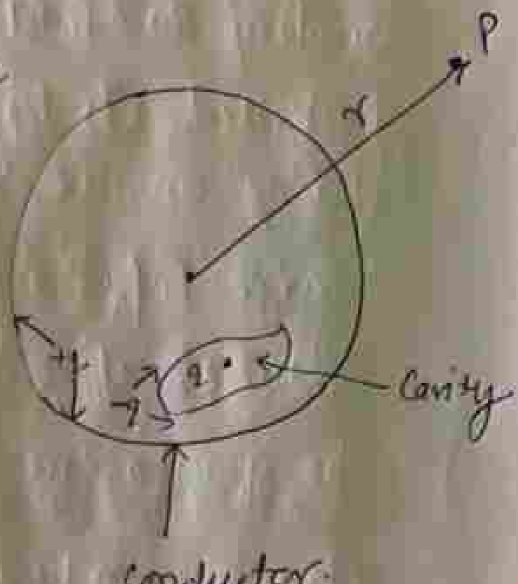
$$Q_{enc} = q + q_{induced} = 0$$

So $\boxed{q_{induced} = -q}$

Example 2.9 An uncharged spherical conductor centered at the origin has a cavity of some weird shape carved out of it. Somewhere within the cavity is a charge q. What is the field outside the sphere.

Solution

The answer depends on the shape of cavity and on the placement of the charge (But that is wrong)



The answer is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

is regardless.

Basically there are three kind of field $\vec{E}_q$, $\vec{E}_{Induced}$ and $\vec{E}_{leftover}$.

and Sum of these inside the conductor must be zero.

i.e. $E_q + E_{induced} + E_{leftover} = 0$

$$\frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\hat{r} + \frac{+q_{ind}}{r^2}\hat{r} + \frac{q_{leftover}}{r^2}\hat{r}\right) = 0$$

$$\frac{1}{4\pi\epsilon_0}\cdot\frac{1}{r^2}\hat{r}\left(\cancel{q} + \cancel{(-q)} + q_{leftover}\right) = 0$$

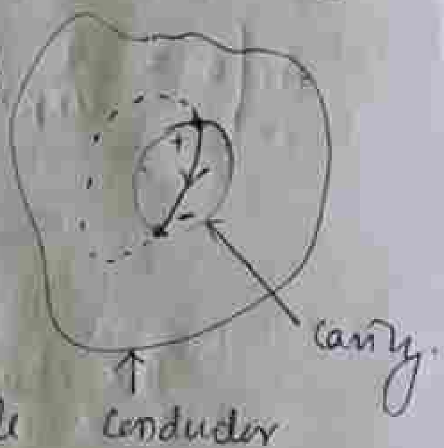
$$\text{i.e. } \frac{1}{4\pi\epsilon_0}\frac{q_{leftover}}{r^2}\hat{r} = 0$$

The charge $+q$ induced an opposite charge $(-q)$ on the wall of cavity, which distribute itself in such a way that its field cancle that of q for all points exterior to the cavity. Since the conductor carrier no net charge, this leaves $+q$ to distribute itself uniformly over the surface of sphere.

Induced charge

B: <u>If cavity carries no charge.</u>

If a cavity surrounded by the conductor is itself empty of charge, then the field within the cavity is zero. For any field line would have to begin and end on the cavity wall going from a plus charge to minus charge. Letting that the field line be part of a closed loop, the rest of which is entirely inside the conductor. Where $\vec{E} = 0$, the integral $\oint \vec{E} \cdot d\vec{l}$ is distinctly positive in violation of equation ($\oint \vec{E} \cdot d\vec{l} = 0$).



It follows that $\vec{E} = 0$ within an empty cavity and there is in fact no charge on the surface of cavity.

Ex. ① This is why you are safe inside a metal car during a thunderstorm.

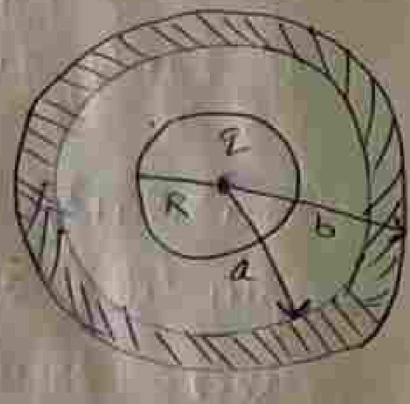
② The same principle applies to the placement of sensitive apparatus inside a grounded Faraday cage to shield out stray electric field.

Sol 2.35

Metal surface of radius R carries a charge q but shell has no net charge.

a)

a) The surface charge density (σ) at R, at a and at b:



$$\sigma = \frac{\text{Charge}}{\text{Surface area}}$$

(i) $\boxed{\sigma = \frac{q}{4\pi R^2}}$ (ii) $\sigma = \frac{q_{induced}}{4\pi a^2}$

$$\boxed{\sigma = \frac{-q}{4\pi a^2}}$$

The charge on outer surface.

$$Q_{net} = 0$$

$$q_{out} + q_{induced} = 0$$

$$q_{out} - q = 0$$

$$q_{out} = +q.$$

Hence $\boxed{\sigma = \frac{+q}{4\pi b^2}}$

b) Potential at the center.

$$V(0) = -\int_{\infty}^{0} \vec{E}\cdot d\vec{l} = -\left[\int_{\infty}^{b} \vec{E}\cdot d\vec{l} + \int_{b}^{a} \vec{E}\cdot d\vec{l} + \int_{a}^{0} \vec{E}\cdot d\vec{l}\right]$$

⑨

$$V(0) = -\int_{\infty}^{b}\left(E/4\pi\epsilon_0\, q/r^2\right)dr - \int_{b}^{a}(0)\,dr - \int_{a}^{R}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)dr - \int_{R}^{0}\vec{E}\cdot d\vec{r}$$

$$V(0) = -\int_{\infty}^{b}\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr - \int_{a}^{R}\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr$$

$$= -\frac{q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)_{\infty}^{b} + \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r}\right)_{a}^{R}$$

$$= +\frac{q}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{\infty}\right) + \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{a}\right)$$

$$V(0) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{b} + \frac{1}{R} - \frac{1}{a}\right)$$

i.e. $\boxed{V(0) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{b} + \frac{q}{R} - \frac{q}{a}\right)}$

c) Now the outer surface is touched to a grounding wire, which lowers its potential to zero. How do your answer to (a) and (b) change.

$$V(0) = -\int_{\infty}^{0}\vec{E}\cdot d\vec{l}$$

$$= \underbrace{-\int_{\infty}^{b}\vec{E}\cdot d\vec{l}}_{0} + \int_{b}^{a}\vec{E}\cdot d\vec{l} - \int_{a}^{R}\vec{E}\cdot d\vec{l} - \int_{R}^{0}\vec{E}\cdot d\vec{l}$$

$$= 0 - \int_{b}^{a}\vec{E}\cdot d\vec{l} - \int_{a}^{R}\vec{E}\cdot d\vec{l} - \int_{R}^{0}\vec{E}\cdot d\vec{l}$$

$$= -\int_{b}^{a}(0)\,dr - \int_{a}^{R}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)dr - \int_{R}^{0}(0)\,dr$$

$$= +\frac{1}{4\pi\epsilon_0}\left[\frac{q}{r}\right]_{a}^{R} = +\frac{1}{4\pi\epsilon_0}\left[\frac{q}{R} - \frac{q}{a}\right]$$

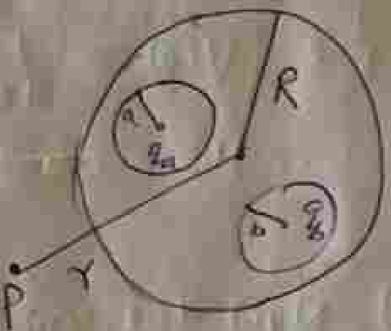
Problem 2.36

Two spherical cavities of radii a and b, are hollowed out from the interior of a (neutral) conducting sphere of radius R. At the center of each cavity a point charge is placed – call these charges q_a and q_b.

a) Find the surface charges σ_a, σ_b and σ_R.

$$\sigma_a = \frac{q_{ind}}{4\pi a^2} = -\frac{q_a}{4\pi a^2}$$

$$\sigma_b = \frac{q_{ind}}{4\pi b^2} = -\frac{q_b}{4\pi b^2}$$



$$\sigma_R = \frac{q_{ind}}{4\pi R^2} = \frac{-(-q_a - q_b)}{4\pi R^2}$$

$$= +\frac{(q_a + q_b)}{4\pi R^2}$$

b) What is the field outside the conductor?

$$\vec{E}_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{(q_a+q_b)}{r^2}\hat{r}$$

c) What is the field within each cavity.

$$\vec{E}_a = \frac{1}{4\pi\epsilon_0}\frac{q_a}{r_a^2}\hat{r}_a$$

$$\vec{E}_b = \frac{1}{4\pi\epsilon_0}\frac{q_b}{r_b^2}\hat{r}_b$$

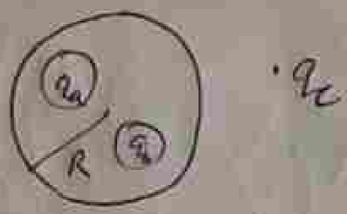
d) What is the force on q_a and q_b.

$\vec{F}_a = 0$, $\vec{F}_b = 0$ due to symmetry.

e) Which of these answers would change if a third charge, q_c, were brought near the conductor.

a) → σ_R will change

→ σ_a and σ_b will remains same.



b) $\vec{E}_{out}$ will change but $\vec{E}_a$ and $\vec{E}_b$
c). will be same.

d) Force on q_a and q_b will be still zero.

Convection and Conduction Currents

How the electric field behaves in conductor or dielectric

The current

$$\boxed{I = \frac{dQ}{dt}}$$

electric charge passing through area / time

Current density

$$J = \frac{\Delta I}{\Delta S}$$

$$\boxed{\Delta I = J \Delta S}$$

assuming that current density is perpendicular to the surface. If the current density is not normal to the surface,

$$\Delta I = J \Delta S$$

Thus the total current through a surface S

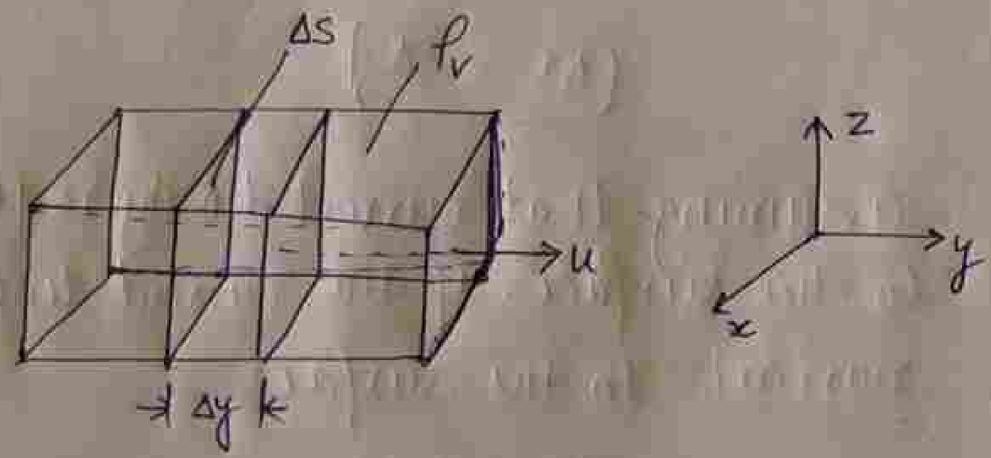
$$\boxed{I = \int_S \vec{J} \cdot d\vec{s}}$$

Depending on how I is produced, there are different kinds of current density

1) Convection current density
2) conduction " "
3) displacement " "

Case - A : Convection Current

Convection current does not involve conductors and consequently does not satisfy the Ohm's law. It occurs when current flow through an insulating medium such as liquid, rarefied gas, or a vacuum. A beam of electrons in a vacuum tube is the example of convection current.



Consider a filament (as shown in fig) If there is a flow of charge, of density ρ_v at velocity $U = u_y \hat{a}_y$ Then the current through the filament

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \, \Delta S \frac{\Delta y}{\Delta t}$$

$$\boxed{\Delta I = \rho_v \, \Delta S (u_y)}$$

$$\rho_v = \frac{\Delta Q}{\text{Volume}} = \frac{\Delta Q}{\Delta S \cdot \Delta y}$$

i.e the current density at a given point is the current through a unit normal area at that point.

The y-directed current density (J_y)

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v v_y$$

Similarly $J_x = \rho_v v_x$, $J_z = \rho_v v_z$

Hence $\boxed{J = \rho_v v}$

The current I is the convection current and J is the convection current density.

B. Conduction current

A conductor is characterized by large no of free electrons that provide conduction current due to an impressed electric field. When an electric field is applied, the force on electron with charge ($-e$) is

$$\vec{F} = -e\vec{E} \quad \text{—— (1)}$$

Since the electron is not in free space, it will not experience an average acceleration under the influence of applied electric field. It suffers constant collisions with the atomic lattice and drifts from one atom to another. If an electron of m is moving with velocity (drift) v in an electric field E then average change in momentum is equal to the applied force.

i.e $$\frac{mu}{\tau} = -eE$$

or $$v = -\frac{eE\tau}{m} \quad \text{—— (2)}$$

Where τ is the average time ~~b/w~~ interval b/w collisions. This indicates that the drift velocity of the $\bar{e}$ is directly proportional to the applied electric field. If there are n electrons per unit volume then

$$\rho_v = -ne \quad \text{——} \ (3)$$

Thus the conduction current density is

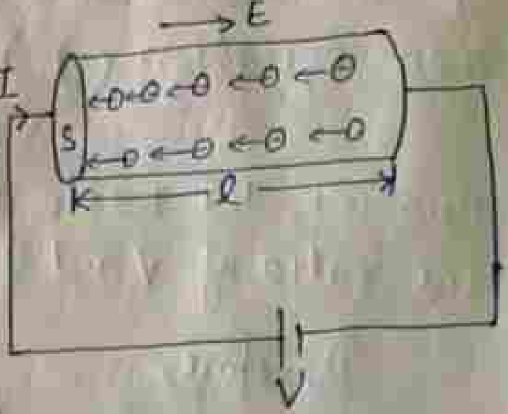
$$\vec{J} = \rho_v U$$
$$= (-ne) U$$
$$= (-ne)\left(-\frac{e\tau}{m}\right)\vec{E}$$
$$\vec{J} = +\left(\frac{ne^2\tau}{m}\right)\vec{E}$$
$$\boxed{\vec{J} = \sigma\vec{E}}$$

Where $\sigma = \dfrac{ne^2\tau}{m}$ is the conductivity of conductor

Properties of conductors

1) A conductor is an equipotential body ie $V_{ab} = 0$ $(\vec{E} = -\nabla V = 0)$
2) $E \to 0$ inside the conductor $(\sigma \to \infty)$ for a finite current density J
3) $\rho_v = 0$

Case: $E \neq 0$ inside the conductor



let us consider a conductor whose ends are maintained at V potential diff.
In this case $\vec{E} \neq 0$ inside the conductor. In this case an electric field (internal) produced inside the conductor to sustain the current. As the electron move, the encounter some damping force called the resistance of conducting material.

The applied electric field

$$\vec{E} = \frac{V}{l} \quad \text{—— (1)}$$

and the current density (for unifor cross section S)

$$J = \frac{I}{S} \quad \text{—— (2)}$$

Now $J = \sigma E$

$$\frac{I}{S} = \sigma\left(\frac{V}{l}\right)$$

$$\frac{V}{I} = \frac{l}{\sigma S}$$

or $$\boxed{R = \frac{V}{I} = \frac{\rho_c l}{S}} \quad \text{—— (3)}$$

Where $\rho_c = \frac{1}{\sigma}$ is the resistivity of conducting material.

If the cross-section of conductor is not uniform then equation (3) is not applicable.

However, the basic definition of resistance R as the ratio of V and I still applies.

Therefore,

$$R = \frac{V}{I} = \frac{\int_l \vec{E} \cdot d\vec{l}}{\int_s \vec{J} \cdot d\vec{s}}$$

$$\boxed{R = \frac{\int_l \vec{E} \cdot d\vec{l}}{\int_s (\sigma \vec{E}) \cdot d\vec{s}}} \quad \text{— (4)}$$

For non-uniform cross section.

Note that (-ive) sign before $V = -\int_l \vec{E} \cdot d\vec{l}$ because $\int \vec{E} \cdot d\vec{l} < 0$ if $I > 0$ equation (4) will not be utilized.

→ Power (in watts) is defined as the rate of change of energy (W) in joules (force × velocity):

Hence,

$$P = \text{Force} \times \text{velocity}$$

$$= Q\vec{E} \cdot \vec{u}$$

$$= \int \rho_v \, dv \, \vec{E} \cdot \vec{U}$$

$$= \int \vec{E} \cdot (\rho_v \vec{U}) \, dv \qquad J = \rho_v U$$

$$\boxed{P = \int \vec{E} \cdot \vec{J} \, dv} \quad \text{— (5)}$$

Equation 5 is known as Joule's law.
The power density ω_p (W/m³) is given by the integrand in equation 5.

Hence $\omega_p = \frac{dP}{dV} = \vec{E} \cdot \vec{J} = \vec{E} \cdot (\sigma \vec{E})$

$$\boxed{\omega_p = \sigma |E|^2} \quad \text{--- (6)}$$

For a conductor with uniform cross-section
$$dv = s\,dl$$

Then power $P = \int_l E \cdot J \; dV$

$$= \int_l \vec{E} \cdot d\vec{l} \int \vec{J} \cdot d\vec{s}$$

$$P = V \; I$$

$$\boxed{P = VI = I^2R}$$

Q. If $\vec{J} = \frac{1}{r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)$ A/m^2. Calculate the current passing through

a) A hemispherical shell of radius 20 cm, $0<\theta<\pi/2$, $0<\phi<2\pi$

b) A spherical shell of radius 10 cm.

Solution $I = \int J \cdot ds$ and $ds = r^2\sin\theta\, d\theta\, d\phi\, \hat{a}_r$

a) $$I = \int \frac{1}{r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)\cdot r^2\sin\theta\, d\theta\, d\phi\, \hat{a}_r \Big|_{r=0.2}$$

$$= \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \frac{1}{r^3}(2\cos\theta \times r^2\sin\theta)\, d\theta\, d\phi\, (\hat{a}_r\cdot\hat{a}_r) + 0\Big|_{r=0.2}$$

$$= \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \frac{2\cos\theta\sin\theta}{r}\, d\theta\, d\phi \Big|_{r=0.2}$$

$$= \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \frac{2\cos\theta\sin\theta}{0.2}\, d\theta\, d\phi$$

$$= \frac{4\pi}{0.2}\int_{\theta=0}^{\pi/2} \cos\sin\theta\, d\theta = \frac{4\pi}{0.2}\left[\int_0^1 y\, dy\right]$$

$$\cancel{= \frac{4\pi}{0.2}\int_0^{\pi/2} \sin\theta\, d(\sin\theta)}$$ let $\sin\theta = y$

$\cos\theta\, d\theta = dy$

$$= \frac{4\pi}{0.2}\left[\frac{y^2}{2}\right]_0^1$$

$$= \frac{4\pi}{0.2}\left[\frac{\sin^2\theta}{2}\right]_0^{\pi/2} = 10\pi = 31.4 \text{ Amp.}$$

b) The only difference here is that we have $0 \le \theta \le \pi$ instead of $0 < \theta < \pi/2$ and $r = 0.1\,m$.

Hence $$I = \frac{4\pi}{0.1}\left|\frac{\sin^2\theta}{2}\right|_0^{\pi} = 0$$

$$I = \oint J \cdot ds = \int (\nabla \cdot J) \cdot dv = 0$$

Since $$\nabla \cdot \vec{J} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{2}{r}\cos\theta\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{1}{r^3}\sin^2\theta\right]$$

$$= \frac{1}{r^2} \times -\frac{2}{r^2}\cos\theta + \frac{1}{r\sin\theta} \times \frac{1}{r^3} \times 2\sin\theta\cos\theta$$

$$= -\frac{2}{r^4}\cos\theta + \frac{2}{r^4}\frac{\cancel{\sin\theta}\cos\theta}{\cancel{\sin\theta}}$$

$$= -\cancel{\frac{2}{r^4}\cos\theta} + \cancel{\frac{2}{r^4}\cos\theta}$$

$$\boxed{\vec{\nabla} \cdot \vec{J} = 0}$$

Q. For the current density
$$J = 10z\sin^2\phi\,\hat{a}_\rho \text{ A/m}^2.$$
Find the current through cylindrical surface. $\rho = 2$, $1 \le z \le 5$ m.