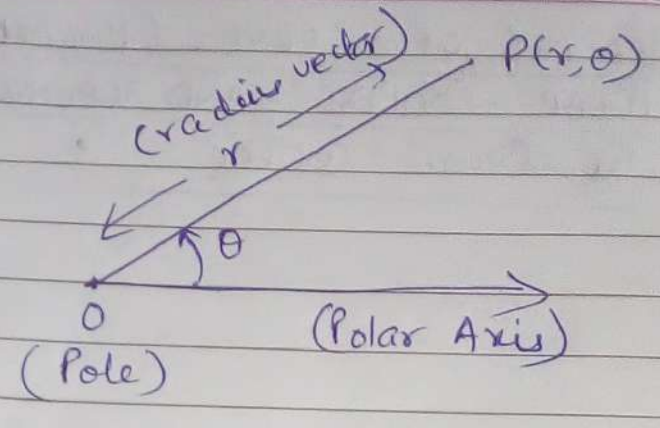
POLAR - COORDINATES

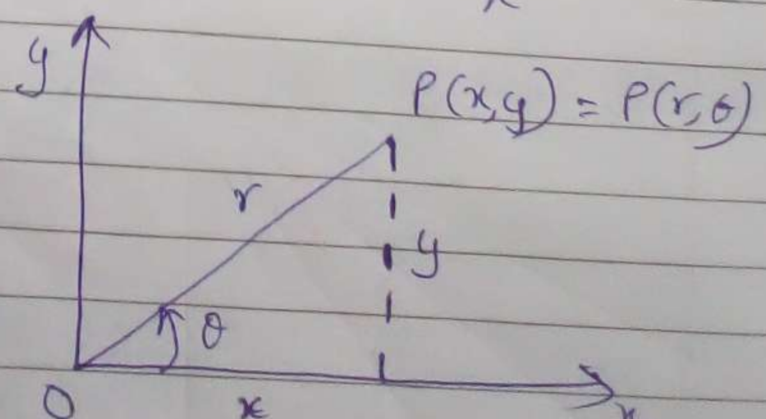


O is the fixed point called Pole & Polar axis is any fixed line through O. Any point P is identified by a set of two coordinates r & θ, r is the length of the radius vector $\vec{OP}$ & θ is the ∠ which OP makes Polar axis, measured in anti-clockwise sense. So, P is identified as P(r,θ). This is called as representing P by polar coordinates r & θ.

Conversion between Polar & Cartesian Coord.s

$x = r\cos\theta$, $y = r\sin\theta$

$r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$.



Questions.

1. Find polar coord.s of $(3,0)$.

Ans Here $x=3$, $y=0$.

$\therefore r=\sqrt{x^2+y^2}=3$

& $\theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{0}{3}\right)=\tan^{-1}(0)$
$=0$.

Hence polar coord.s are $P(r=3, \theta=0)$.

2. Find cartesian coordinates of $(-2, -\pi/3)$

Ans Here $r=-2$, $\theta=-\pi/3$.

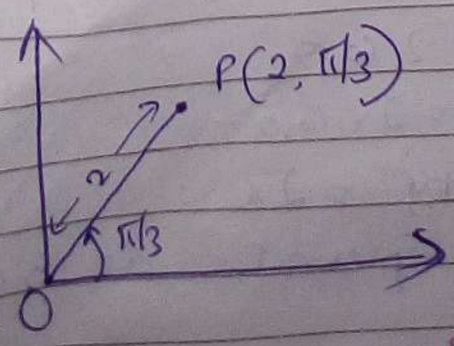
Hence, $x=r\cos\theta = -2\cos(-\pi/3)$
$= -2\cos(\pi/3)$
$= -2\times\frac{1}{2} = -1$

& $y=r\sin\theta = -2\sin(-\pi/3)$
$=+2\sin(\pi/3)$
$=2\times\frac{\sqrt{3}}{2} = \sqrt{3}$

Hence cartesian coords are $(-1, \sqrt{3})$

3. Find all polar coordinates of $(2, \pi/3)$

Ans



All the polar coords are of the form $\left(2, \frac{\pi}{3} + 2n\pi\right)$, $n \in N$
& $\left(-2, \frac{\pi}{3} + (2n+1)\pi\right)$, $n \in N$

4 Convert the foll. polar eqns to Cartesian coords.

(i) $r = \dfrac{4}{2\cos\theta - \sin\theta}$

$\Rightarrow 2r\cos\theta - r\sin\theta = 4.$

$\Rightarrow 2x - y = 4.$

This represents a straight line

(ii) $r = 2\operatorname{cosec}\theta$

$\Rightarrow r\sin\theta = 2.$

$\Rightarrow y = 2 \rightarrow$ This is a straight line.

(iii) $r = \cot\theta \operatorname{cosec}\theta$

$\Rightarrow r = \dfrac{\cos\theta}{\sin\theta} \times \dfrac{1}{\sin\theta}.$

$\Rightarrow r\sin^2\theta = \cos\theta.$

$\Rightarrow r^2\sin^2\theta = r\cos\theta.$

$\Rightarrow y^2 = x.$

This represents a parabola.

(iv) $r = 2\cos\theta$

$\Rightarrow r^2 = 2r\cos\theta.$

$\Rightarrow x^2 + y^2 = 2x.$

This represents a circle.

5. Represent the cartesian eqns by equivalent polar equation:—

(I) $x^2+y^2=16$.
or, $r^2=16$
or $r=4$.

(II) $x=y$
$\Rightarrow r\cos\theta = r\sin\theta$
$\Rightarrow \tan\theta = 1$
$\Rightarrow \theta = \pi/4$

(III) $x^2+(y-3)^2=9$.
$\Rightarrow x^2+y^2-6y+9=9$.
$\Rightarrow x^2+y^2 = 6y$
$\Rightarrow r^2 = 6r\sin\theta$.
$\Rightarrow r = 6\sin\theta$.

CURVE TRACING OR SKETCHING IN POLAR COORINATES

For this we perform some symmetry tests first:—

(1) Symmetry about x-axis:— If replacing (r,θ) by $(r,-\theta)$ or replacing (r,θ) by $(-r, \pi-\theta)$, gives equi. relation.

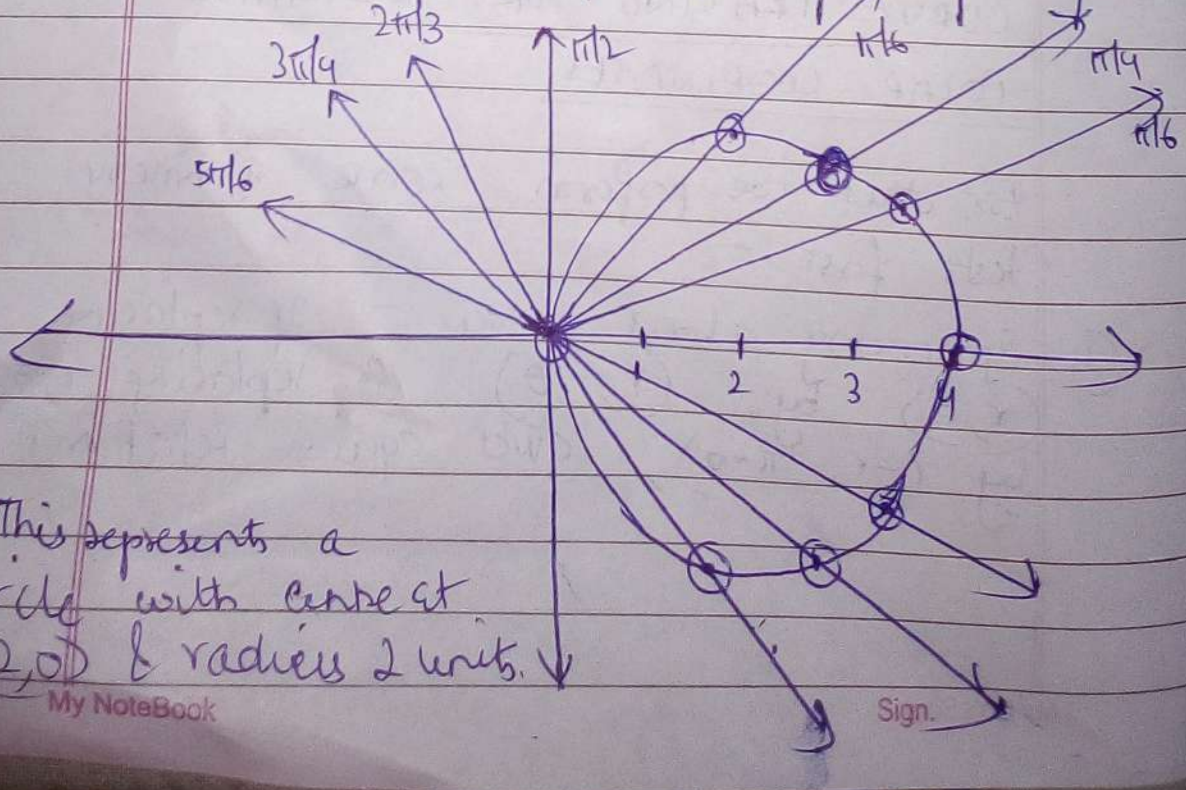
(2) Symmetry about y-axis:-
If replacing (r,θ) by $r, (\pi-\theta)$ or replacing (r,θ) by $(-r,-\theta)$ gives equivalent eqn.

(3) Symmetry about origin
If replacing (r,θ) by $(-r,\theta)$ or replacing (r,θ) by $(r, \pi+\theta)$ gives an equivalent eqn.

Eg! sketch the graph of the eqn $r = 4\cos\theta$ in polar coordinates

Soln: (i) Replacing θ by $-\theta$ does not alter the equation. Hence, it is symmetric about x-axis.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
r	4	3.46	2.82	2	0	-2	-2.82	-3.46	-4



This represents a circle with centre at (2,0) & radius 2 units.