Diffraction

Diffraction: general
Fresnel vs. Fraunhofer diffraction
Several coherent oscillators
Single-slit diffraction

By

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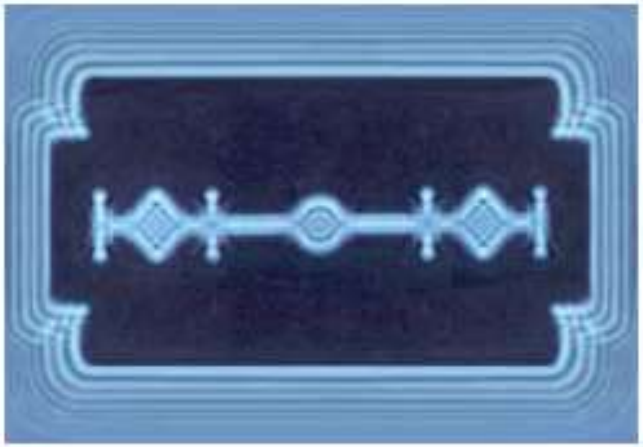
Diffraction

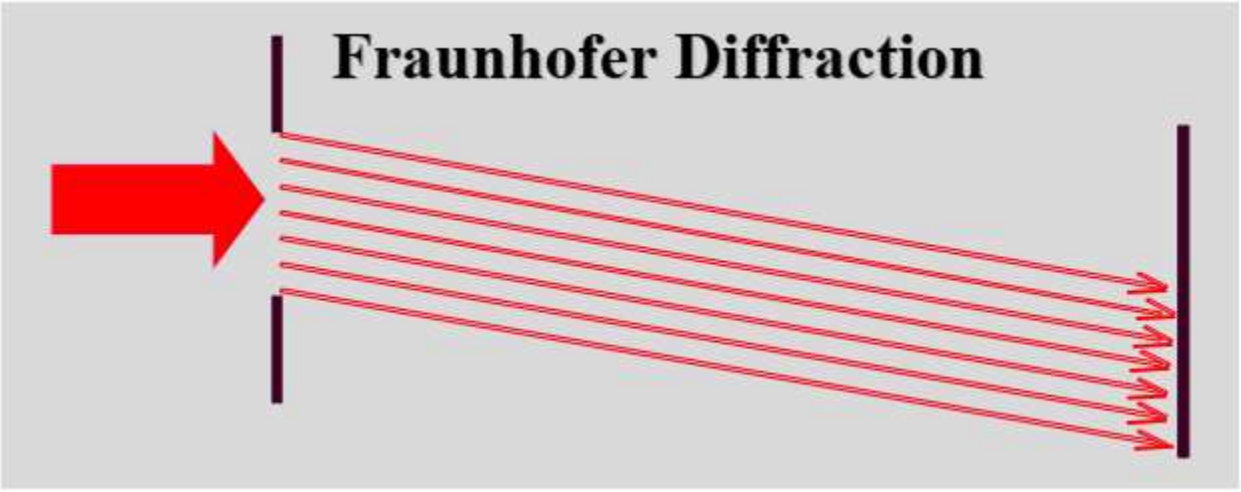
Grimaldi, 1600s: *diffractio*, deviation of light from linear propagation



Diffraction is a consequence of multiple beam interference

Fraunhofer diffraction

- Specific sort of diffraction
  - far-field diffraction
  - plane wavefront
  - Simpler maths



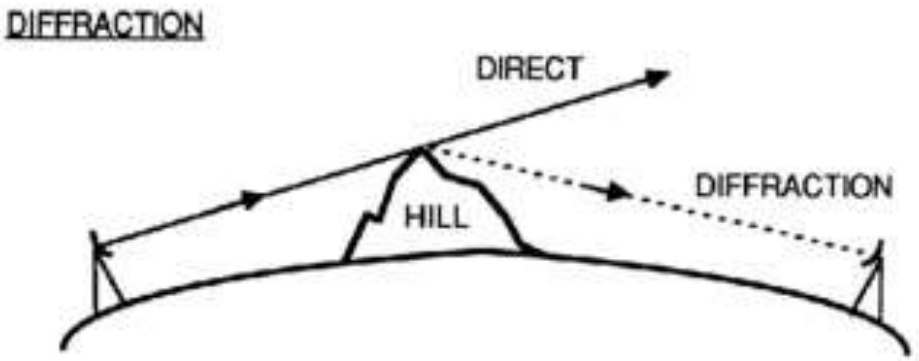
Diffraction of ocean water waves

Ocean waves passing through slits in Tel Aviv, Israel

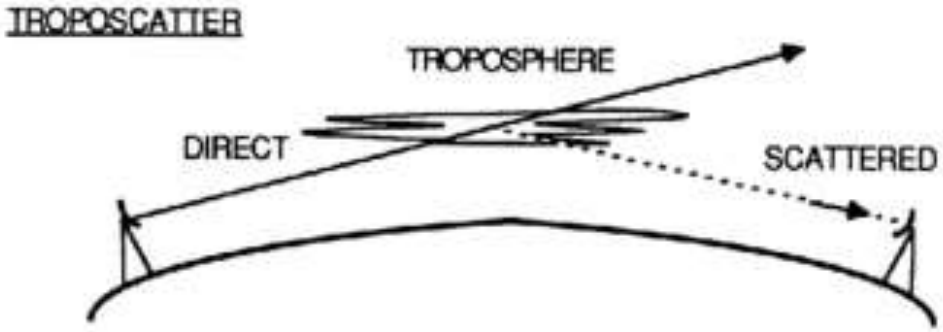
Diffraction occurs for all waves, whatever the phenomenon.

Radio waves diffract around mountains.

When the wavelength is km long, a mountain peak is a very sharp edge!

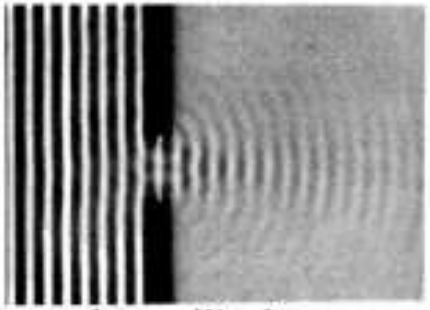


Another effect that occurs is scattering, so diffraction's role is not obvious.

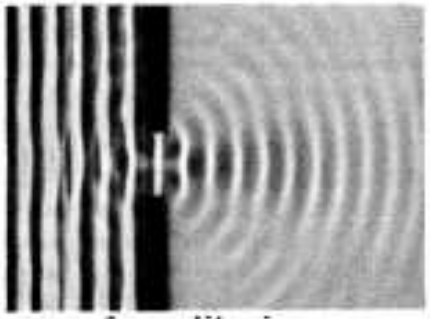


Diffraction of a wave by a slit

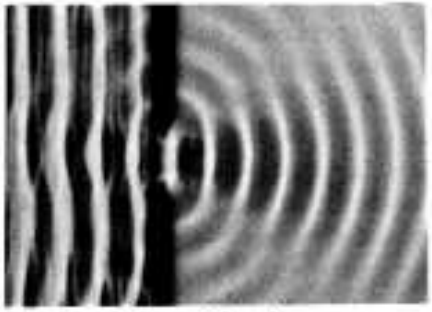
Whether waves in water or electromagnetic radiation in air, passage through a slit yields a diffraction pattern that will appear more dramatic as the size of the slit approaches the wavelength of the wave.



$\lambda \ll$ slit size



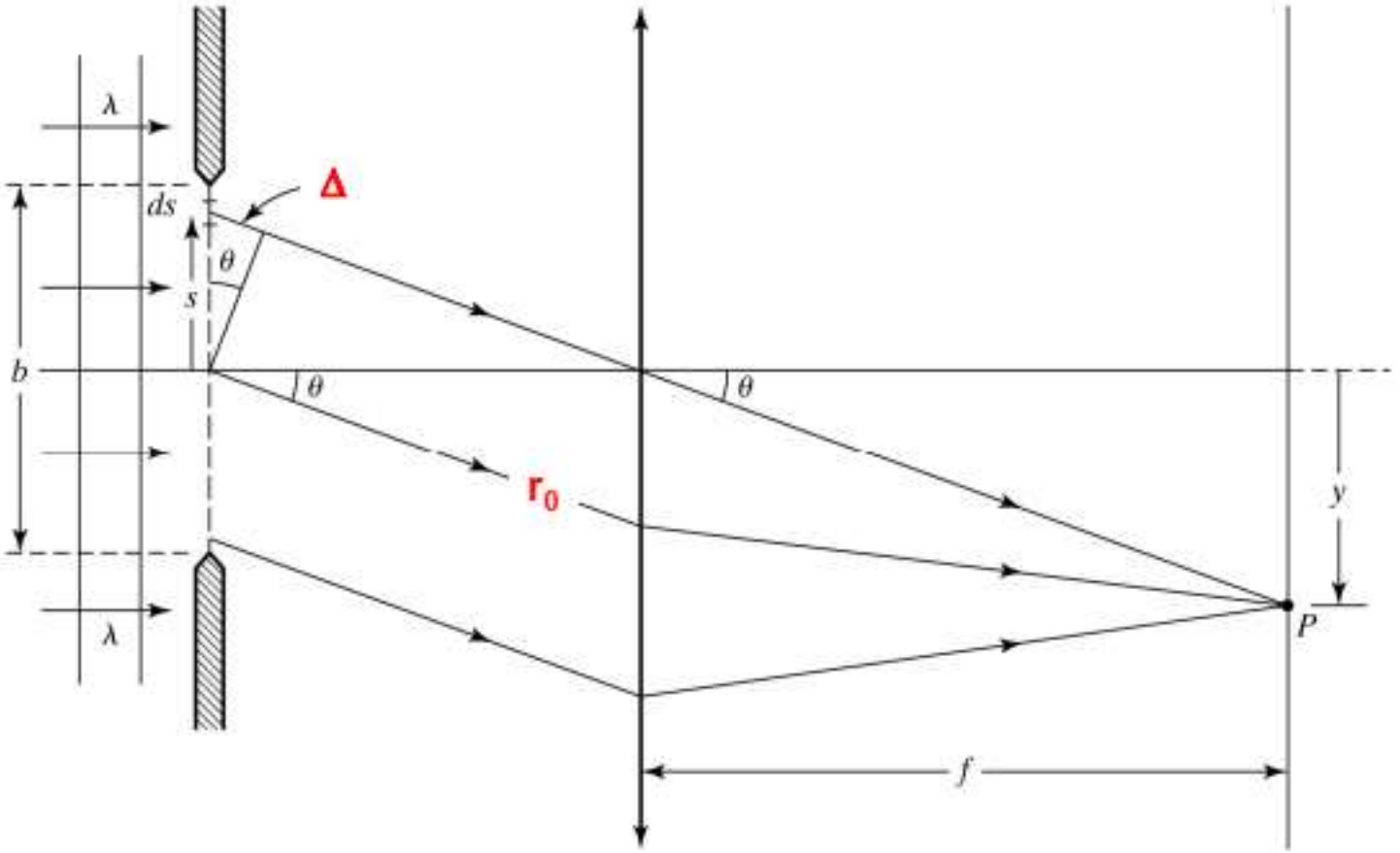
$\lambda <$ slit size



$\lambda \approx$ slit size

11-1. Fraunhofer Diffraction from a Single Slit

- Consider the geometry shown below.
  Assume that the slit is very long in the direction perpendicular to the page so that we can neglect diffraction effects in the perpendicular direction.

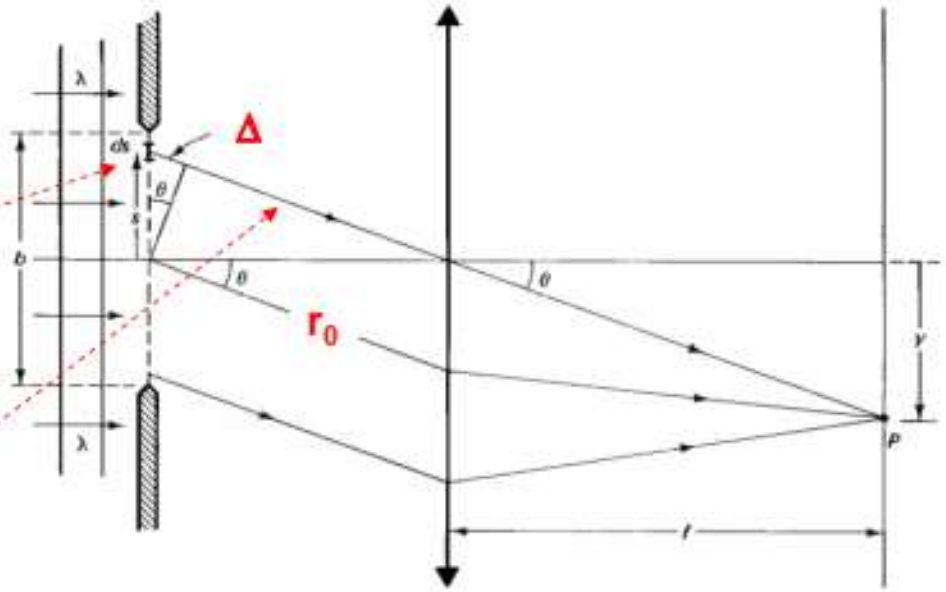


The contribution to the electric field amplitude at point P due to the wavelet emanating from the element ds in the slit is given by

$$dE_P = \left(\frac{dE_0}{r}\right)\exp\left[i(kr-\omega t)\right]$$

Let $r = r_0$ for the source element ds at $s = 0$. Then for any element

$$dE_P = \left(\frac{dE_0}{(r_0+\Delta)}\right)\exp\left\{i\left[k(r_0+\Delta)-\omega t\right]\right\}$$



We can neglect the path difference Δ in the amplitude term, but not in the phase term. why?

We let $dE_0 = E_L\, ds$, where E_L is the electric field amplitude, assumed uniform over the width of the slit.

The path difference $\Delta = s\sin\theta$. Substituting we obtain

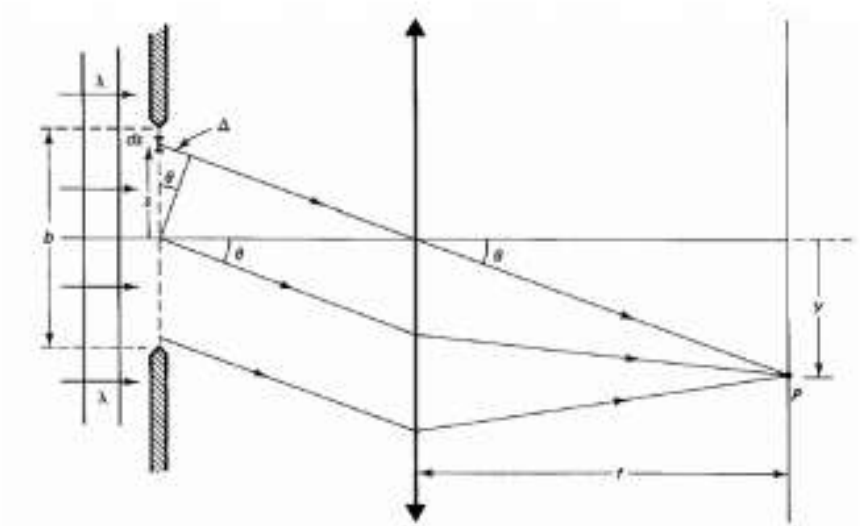
$$dE_P = \left(\frac{E_L\, ds}{r_0}\right)\exp\left\{i\left[k(r_0+s\sin\theta)-\omega t\right]\right\} \qquad E_P = \left(\frac{E_L}{r_0}\right)\exp\left[i(kr_0-\omega t)\right]\int_{-b/2}^{b/2}\exp(ik\,s\sin\theta)\,ds$$

Integrating we obtain

$$E_P = \left(\frac{E_L}{r_0}\right)\exp\left[i(kr_0-\omega t)\right]\left[\frac{\exp(ik\,s\sin\theta)}{ik\sin\theta}\right]_{-b/2}^{b/2}$$

Evaluating with the integral limits we obtain

$$E_P = \left(\frac{E_L}{r_0}\right)\exp\left[i(kr_0 - \omega t)\right]\left[\frac{\exp(i\beta) - \exp(-i\beta)}{ik\sin\theta}\right]$$



where

$$\beta \equiv \frac{1}{2}kb\sin\theta$$

Rearranging we obtain

$$E_P = \left(\frac{E_L}{r_0}\right)\exp\left[i(kr_0 - \omega t)\right]\frac{b}{2i\beta}\left[\exp(i\beta) - \exp(-i\beta)\right]$$

$$= \left(\frac{E_L}{r_0}\right)\exp\left[i(kr_0 - \omega t)\right]\frac{b}{2i\beta}(2i\sin\beta) = \left(\frac{E_L}{r_0}\right)\exp\left[i(kr_0 - \omega t)\right]\frac{b\sin\beta}{\beta}$$

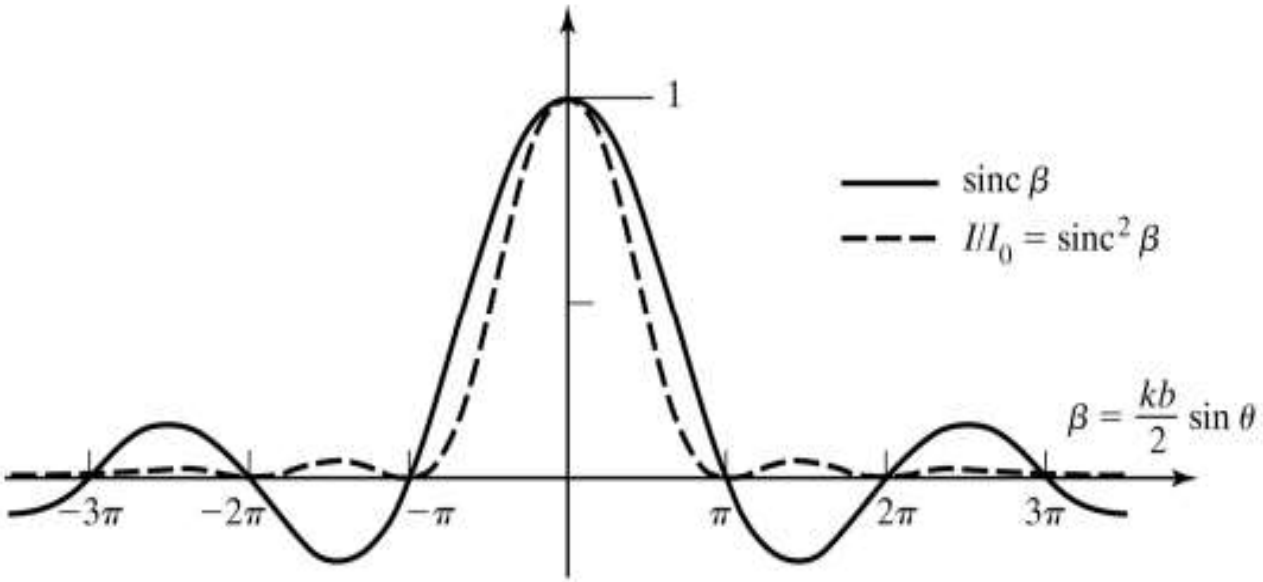
The irradiance at point P is given by

$$I = \frac{1}{2}\varepsilon_0 c E_P E_P^* = \frac{1}{2}\varepsilon_0 c\left(\frac{E_L b}{r_0}\right)^2 \frac{\sin^2\beta}{\beta^2} = I_0\frac{\sin^2\beta}{\beta^2} \quad \Rightarrow \quad I = I_0\,\mathrm{sinc}^2(\beta),\ \beta = \tfrac{1}{2}kb\sin\theta$$

$$I = I_0 \sin c^2(\beta),\ \beta = \tfrac{1}{2} kb \sin\theta$$

The sinc *function is* 1 *for* $\beta = 0$, $\lim_{\beta \to 0} \operatorname{sinc}\beta = \lim_{\beta \to 0} \dfrac{\sin\beta}{\beta} = 1$

The zeroes of irradiance occur when $\sin\beta = 0$, *or when* $\beta = \dfrac{1}{2} k b \sin\theta = m\pi$, $m = \pm 1, \pm 2, \ldots$



The angular width of the central maximum : $\Delta\theta \approx \left(\sin\theta_{m=+1} - \sin\theta_{m=-1}\right) = \dfrac{2\lambda}{b}$

In terms of the length y on the observation screen, $y \cong f \sin\theta$, and in terms of wavelength $\lambda = 2\pi/k$, we can write

$$\beta = \frac{1}{2}\frac{2\pi}{\lambda} b \frac{y}{f} = \frac{\pi b y}{\lambda f}$$

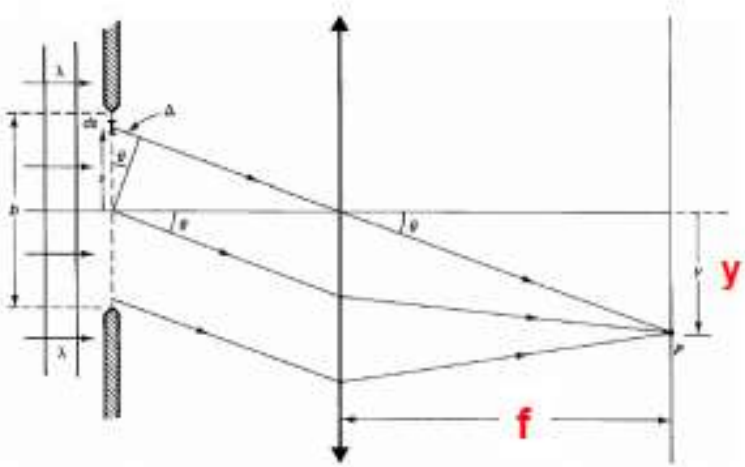
Zeroes in the irradiance pattern will occur when

$$\frac{\pi b y}{\lambda f} = m\pi \quad \Rightarrow \quad y = \frac{m\lambda f}{b}$$

The maximum in the irradiance pattern is at $\beta = 0$. Secondary maxima are found from

$$\frac{d}{d\beta}\left(\frac{\sin\beta}{\beta}\right) = \frac{\cos\beta}{\beta} - \frac{\sin\beta}{\beta^2} = \frac{\beta\cos\beta - \sin\beta}{\beta^2} = 0$$

$$\Rightarrow \quad \beta = \frac{\sin\beta}{\cos\beta} = \tan\beta$$



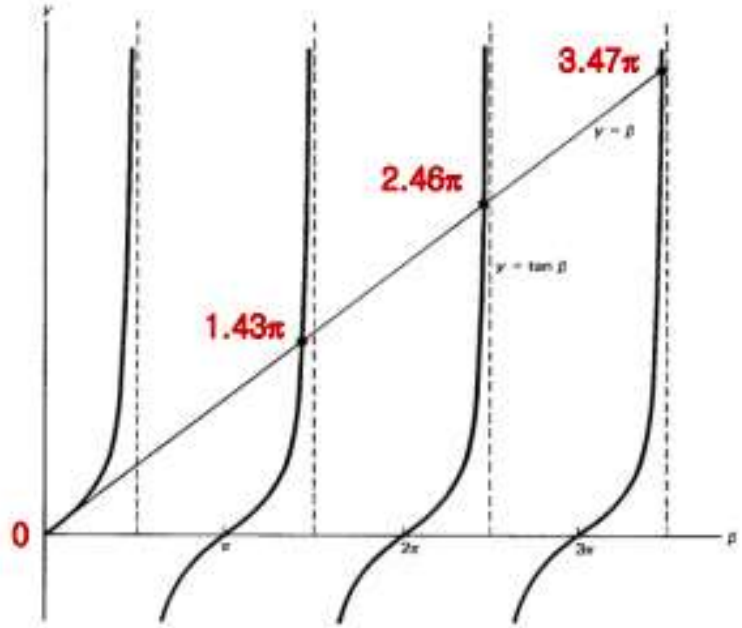
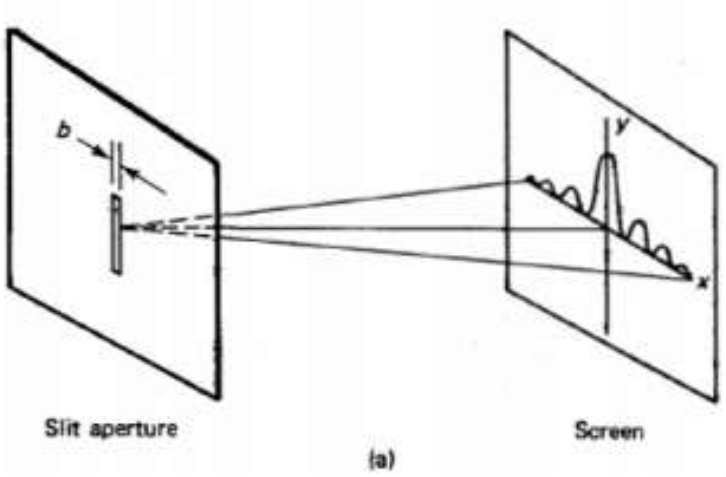
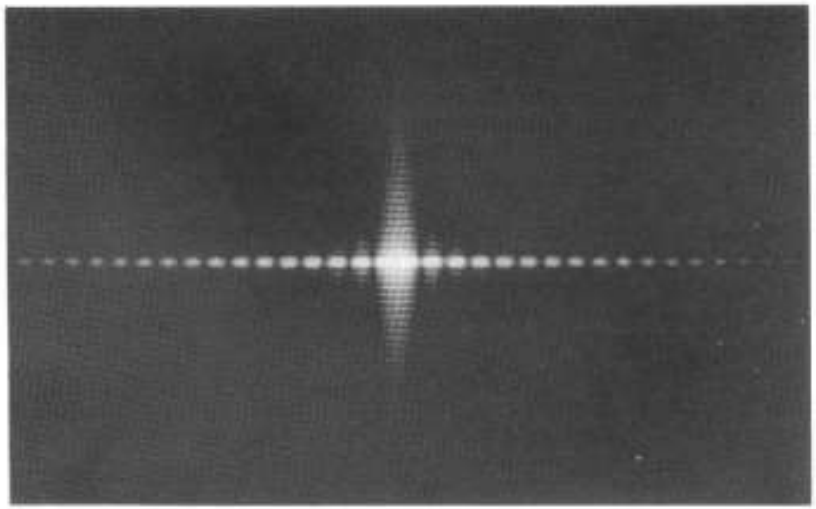


Figure 16-3 Intersections of the curves $y = \beta$ and $y = \tan\beta$ determine the angles β at which the sinc function is a maximum.

Fraunhofer Diffraction pattern from a Single Slit



$$I = I_0 \sin c^2(\tfrac{1}{2} kb \sin\theta)$$



Single slit diffraction?

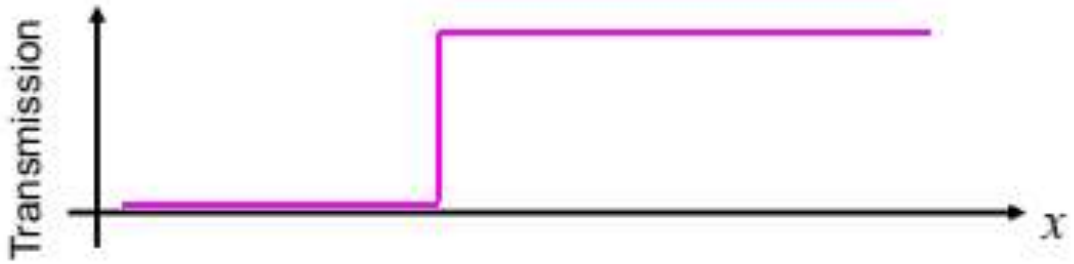
Expectation:

No diffraction

Reality:

With diffraction

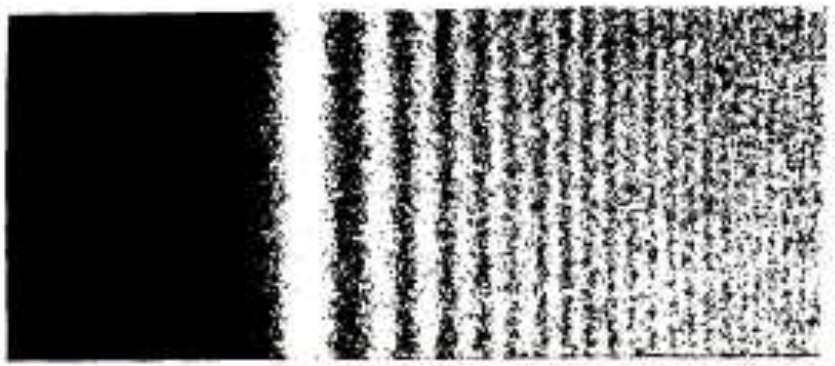
Diffraction by an Edge



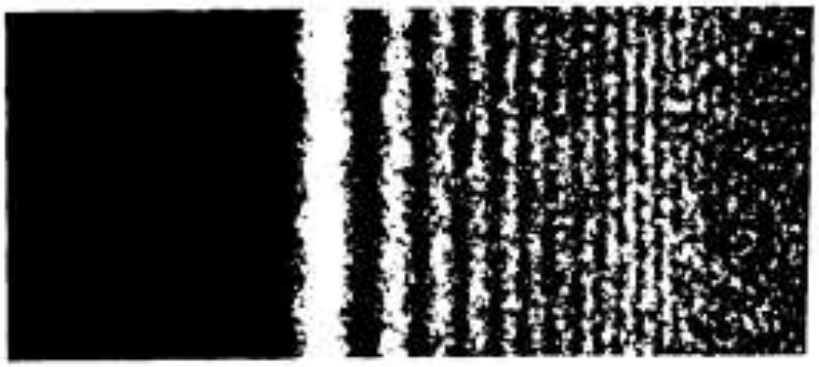
Even without a small slit, diffraction can be strong.

Simple propagation past an edge yields an unintuitive irradiance pattern.

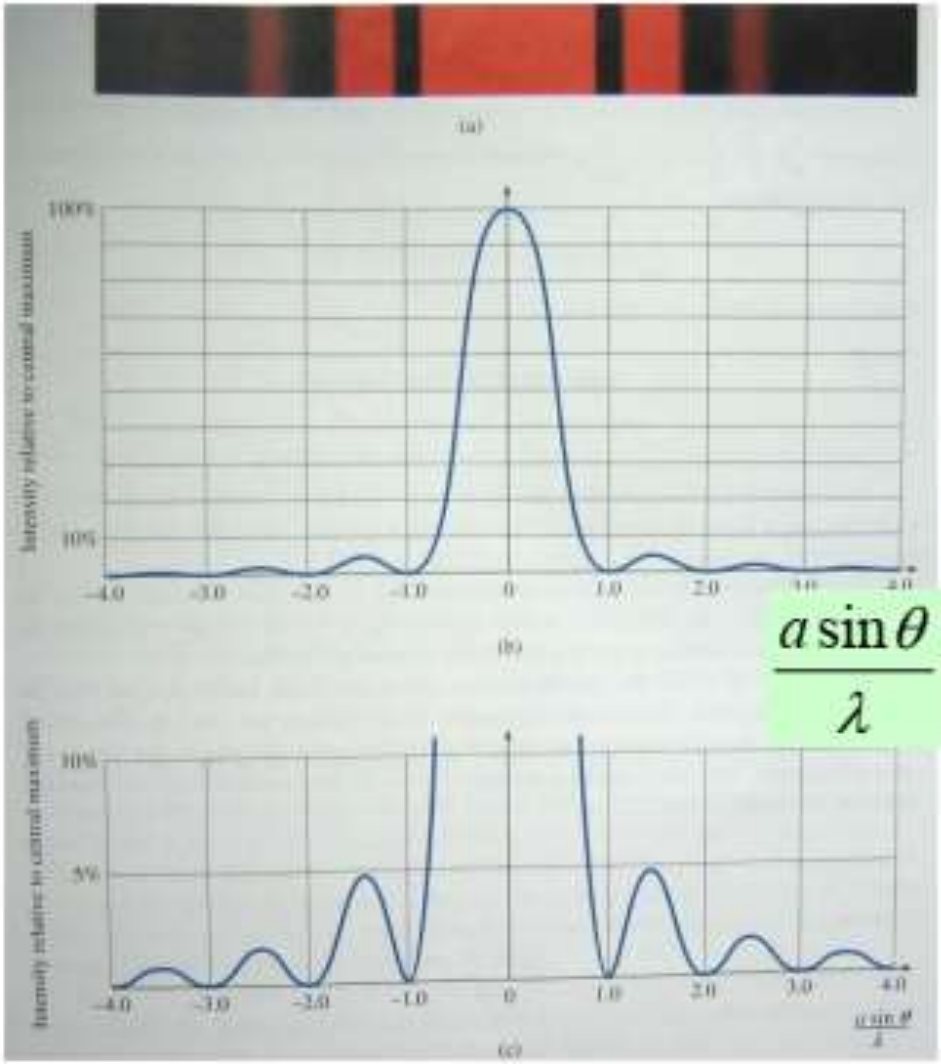
Light passing by edge



Electrons passing by an edge (MgO crystal)



Single slit: diffraction minimum



$$\frac{a \sin\theta}{\lambda}$$

Single slit diffraction minima:

$a \sin\theta = m\lambda \quad (m = \pm 1, \pm 2, \ldots)$

or

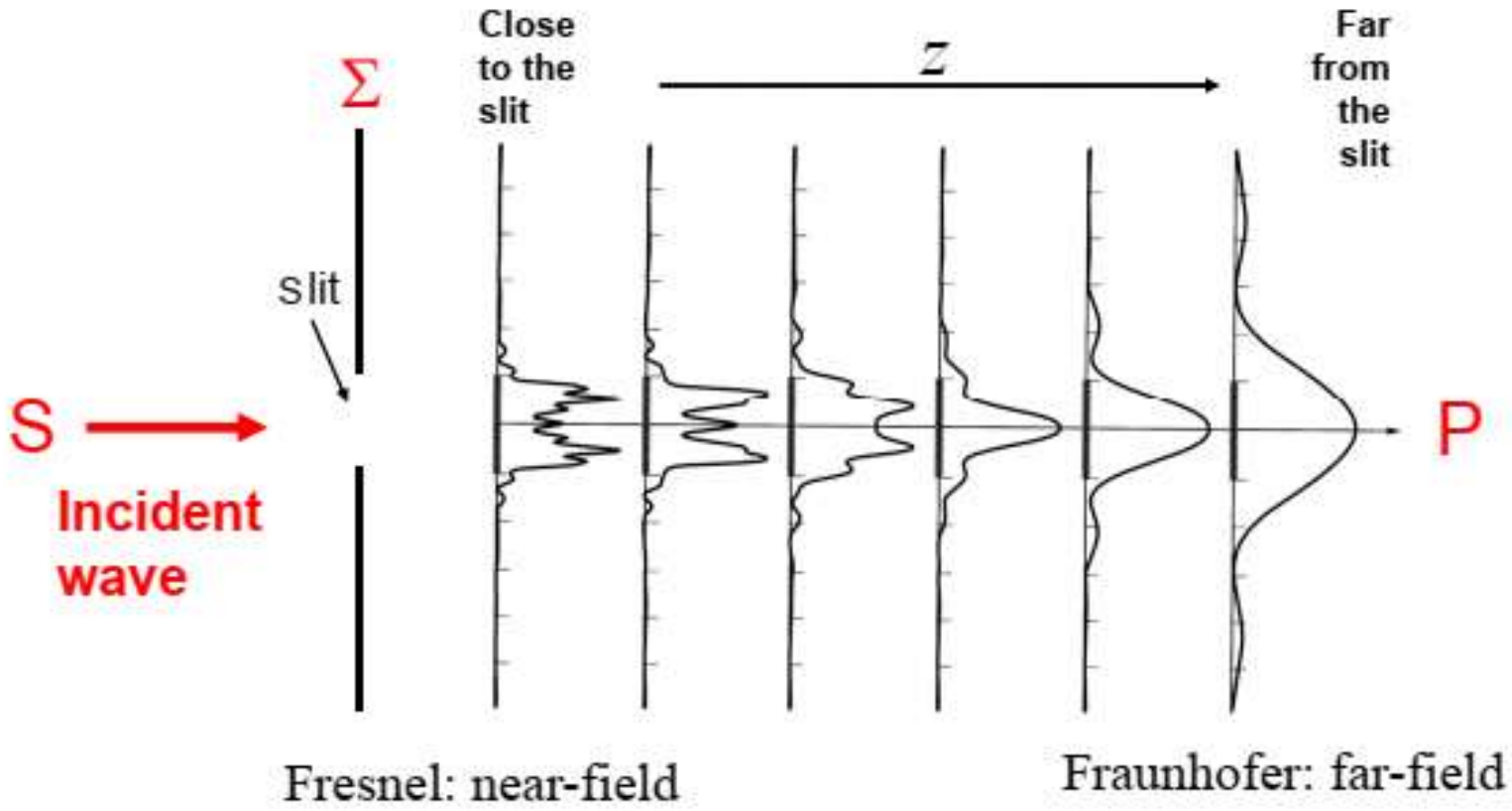
$\sin\theta = m\lambda / a$

Narrower slits - broader maxima

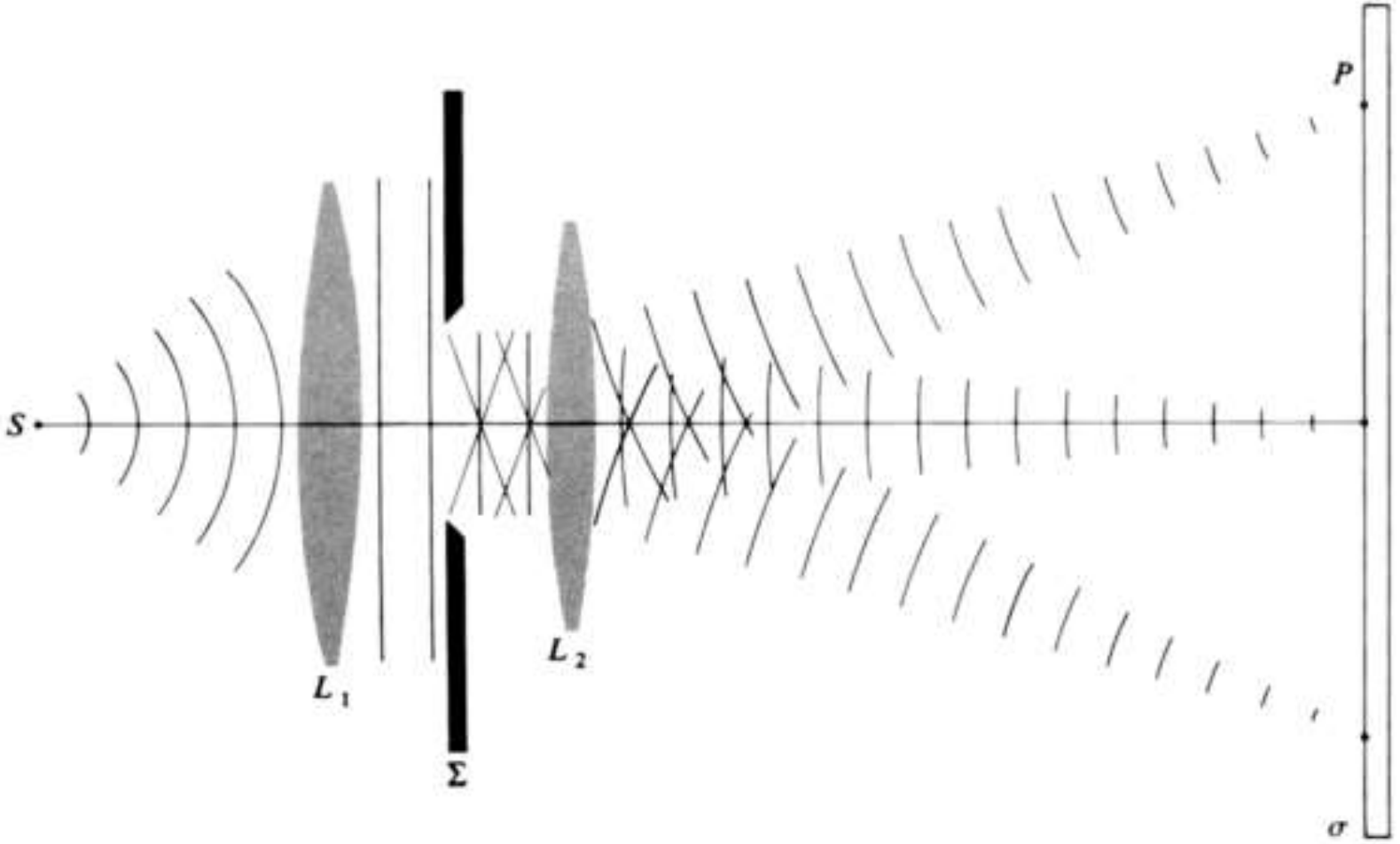
Note: minima only occur when

$a > \lambda$

Fraunhofer vs. Fresnel Diffraction



Fraunhofer diffraction: single slit



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